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UNIVERSITY

*Shaping Lives...
Empowering Communities...*

System Modeling & Control

Presented by

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Bode Plots

A **Bode plot** is a (semilog) plot of the transfer function magnitude and phase angle as a function of frequency

The gain magnitude is many times expressed in terms of decibels (dB)

$$dB = 20 \log_{10} A$$

where A is the amplitude or gain

a *decade* is defined as any 10-to-1 frequency range

an *octave* is any 2-to-1 frequency range

$$20 \text{ dB/decade} = 6 \text{ dB/octave}$$

Bode Plot Overview

Technique for estimating a complicated transfer function (several poles and zeros) quickly

$$H(\omega) = G_0(j\omega)^K \frac{(1 + j\omega\tau_{z1})(1 + j\omega\tau_{z2}) \cdots (1 + j\omega\tau_{zn})}{(1 + j\omega\tau_{p1})(1 + j\omega\tau_{p2}) \cdots (1 + j\omega\tau_{pm})}$$

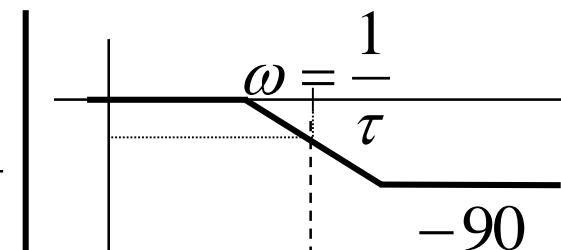
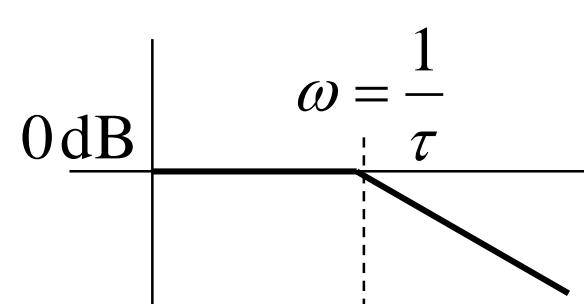
Break frequencies :

$$\omega_i = \frac{1}{\tau_i}$$

Summary of Individual Factors

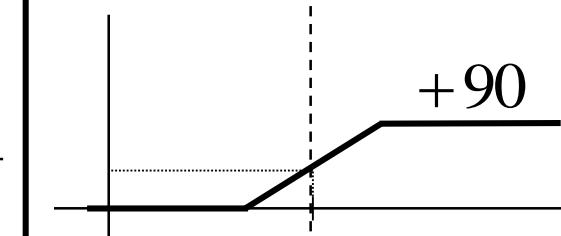
Simple Pole:

$$\frac{1}{1 + j\omega\tau}$$



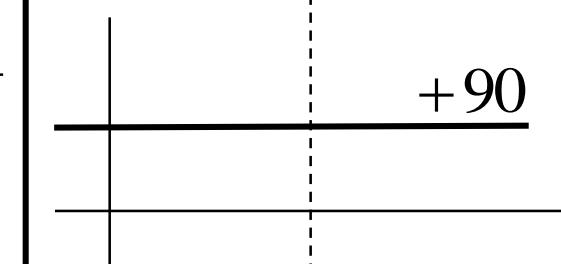
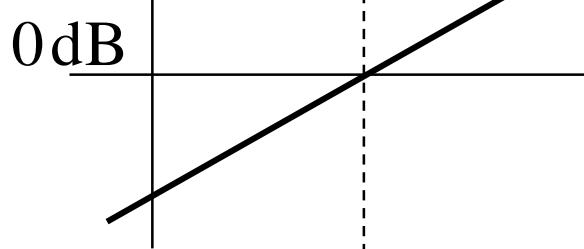
Simple Zero:

$$1 + j\omega\tau$$



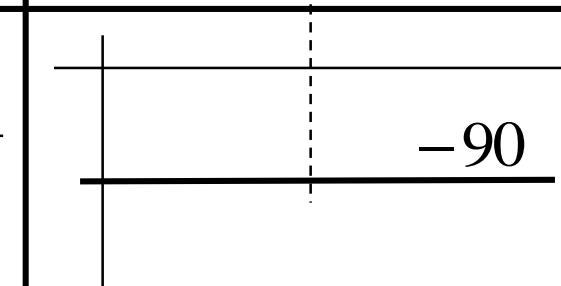
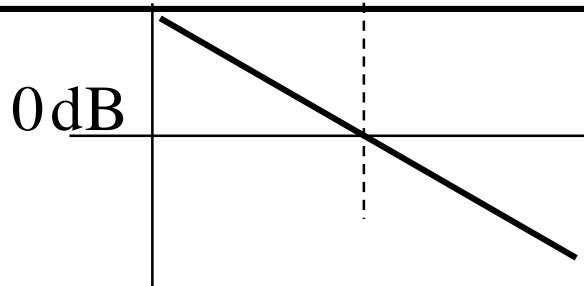
DC Zero:

$$j\omega\tau$$



DC Pole:

$$\frac{1}{j\omega\tau}$$



Example

Consider the following transfer function

$$H(j\omega) = \frac{10^{-5} j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$
$$\begin{aligned}\tau_1 &= 100 \text{ ns} \\ \tau_2 &= 10 \text{ ns}\end{aligned}$$

Break frequencies: invert time constants $\tau_3 = 100 \text{ ps}$

$$\omega_1 = 10 \text{ Mrad/s} \quad \omega_2 = 100 \text{ Mrad/s} \quad \omega_3 = 10 \text{ Grad/s}$$

$$H(j\omega) = \frac{\frac{j\omega}{10^5} \left(1 + j\frac{\omega}{\omega_2}\right)}{\left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + j\frac{\omega}{\omega_3}\right)}$$

Breaking Down the Magnitude

Recall log of products is sum of logs

$$\begin{aligned}|H(j\omega)|_{\text{dB}} &= 20 \log \left| \frac{\frac{j\omega}{10^5} (1 + j \frac{\omega}{\omega_2})}{(1 + j \frac{\omega}{\omega_1})(1 + j \frac{\omega}{\omega_3})} \right| \\&= 20 \log \left| \frac{j\omega}{10^5} \right| + 20 \log \left| 1 + j \frac{\omega}{\omega_2} \right| \\&\quad - 20 \log \left| 1 + j \frac{\omega}{\omega_1} \right| - 20 \log \left| 1 + j \frac{\omega}{\omega_3} \right|\end{aligned}$$

Let's plot each factor separately and add them graphically

Breaking Down the Phase

Since

$$\angle a \cdot b = \angle a + \angle b$$

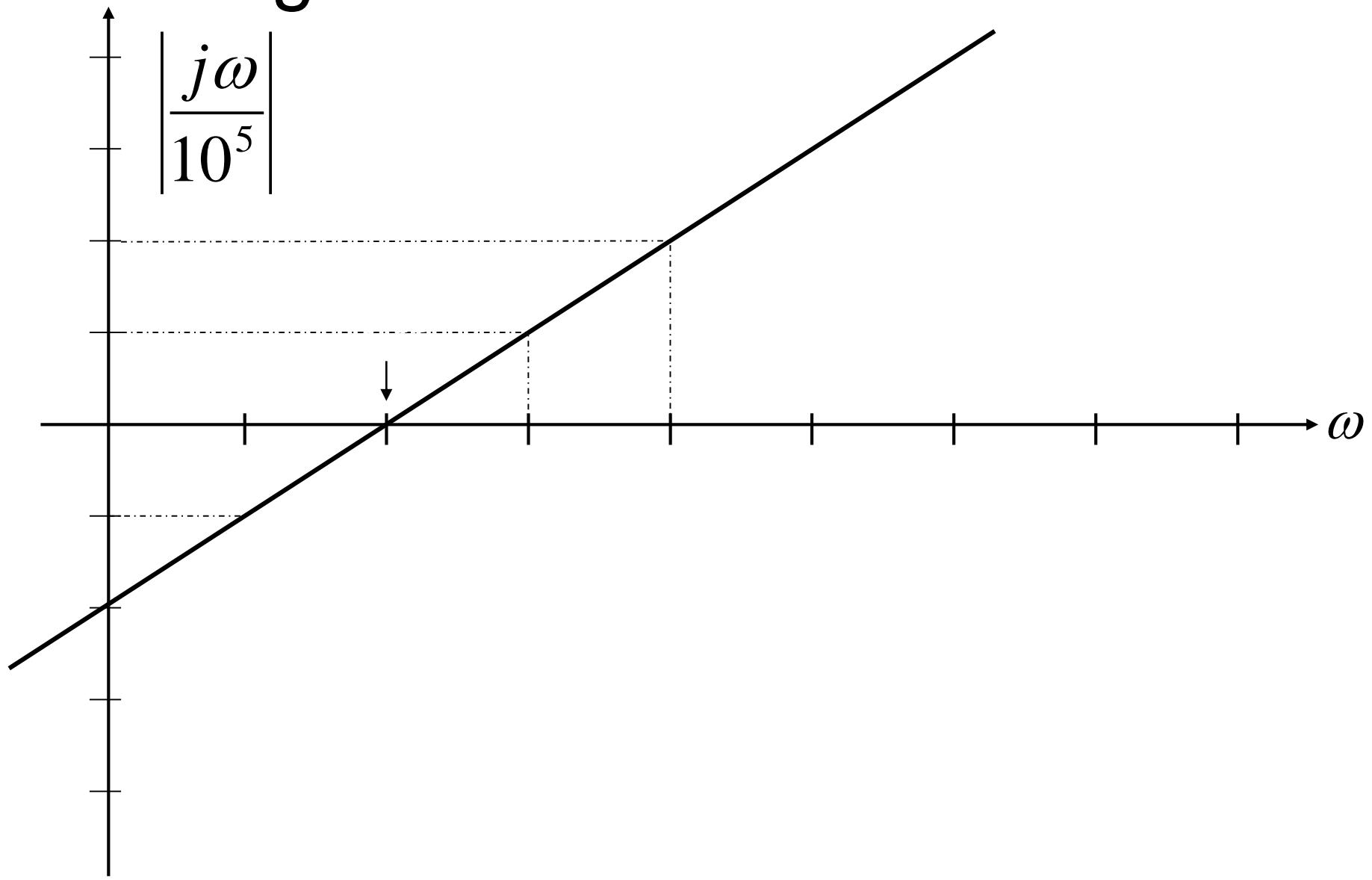
$$\angle H(j\omega) = \angle \frac{10^{-5} j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

$$\angle H(j\omega) = \angle \frac{j\omega}{10^5} + \angle 1 + j\frac{\omega}{\omega_2}$$

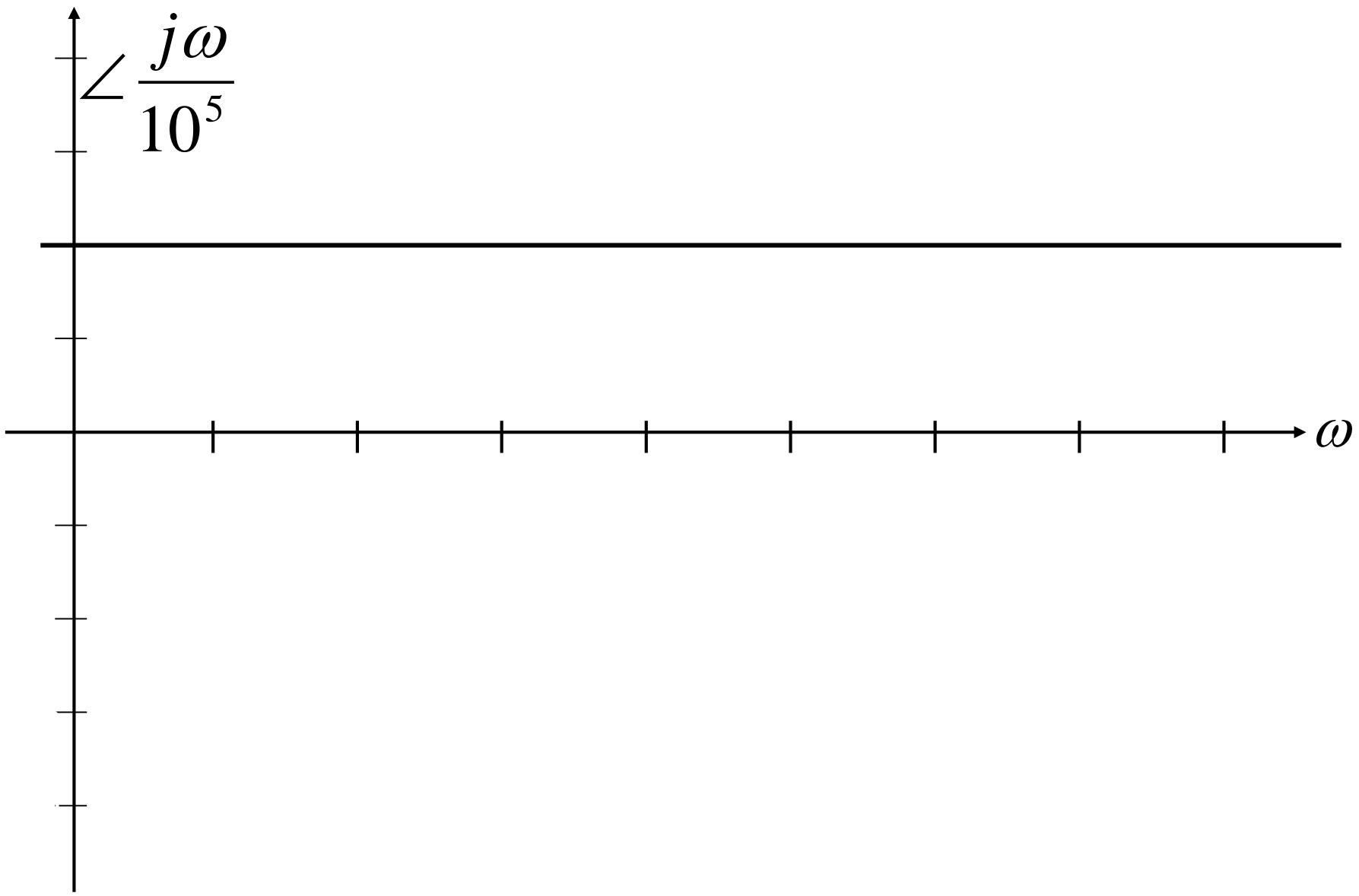
$$- \angle 1 + j\frac{\omega}{\omega_1} - \angle 1 + j\frac{\omega}{\omega_3}$$

Let's plot each factor separately and add them graphically

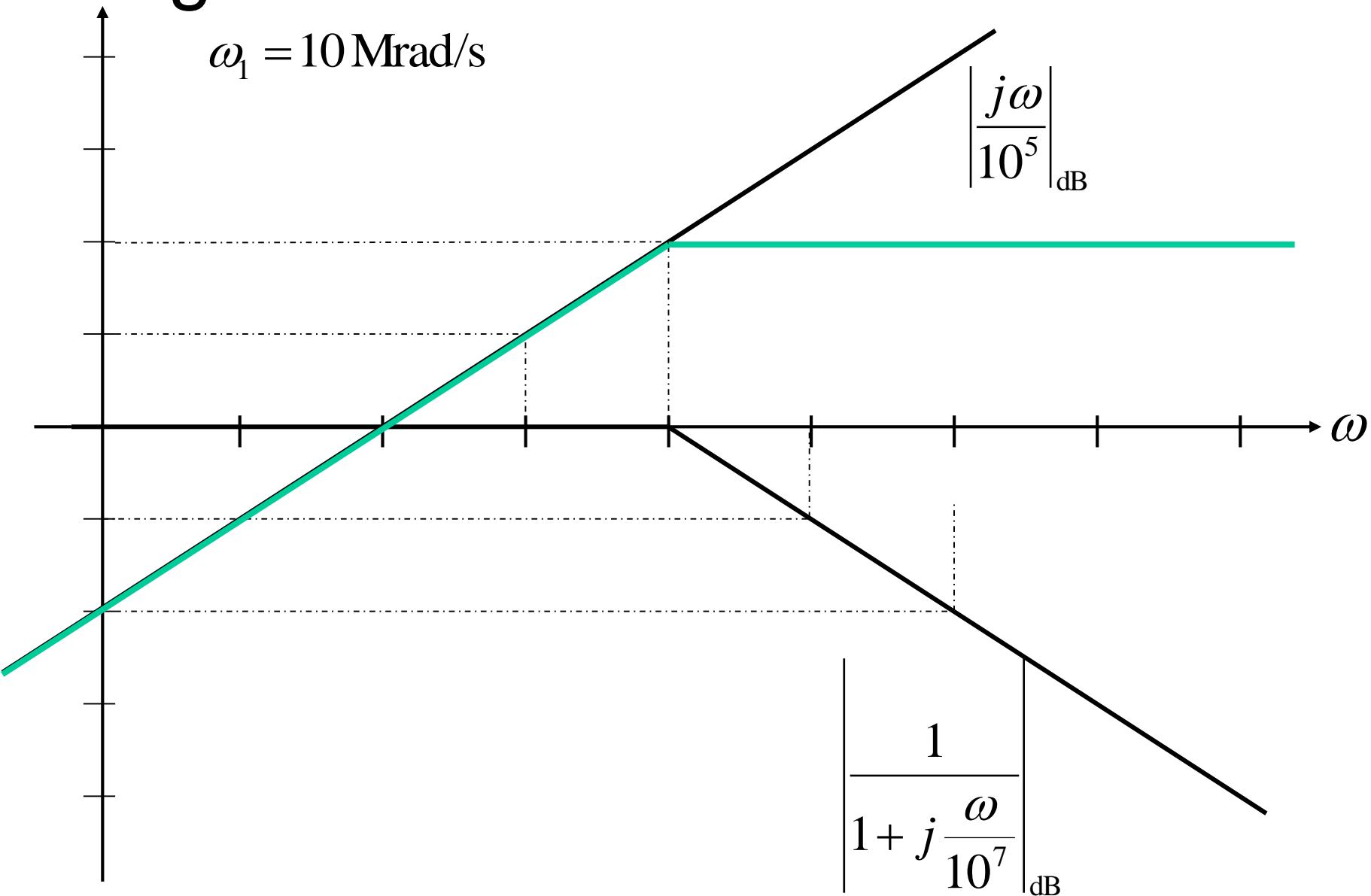
Magnitude Bode Plot: DC Zero



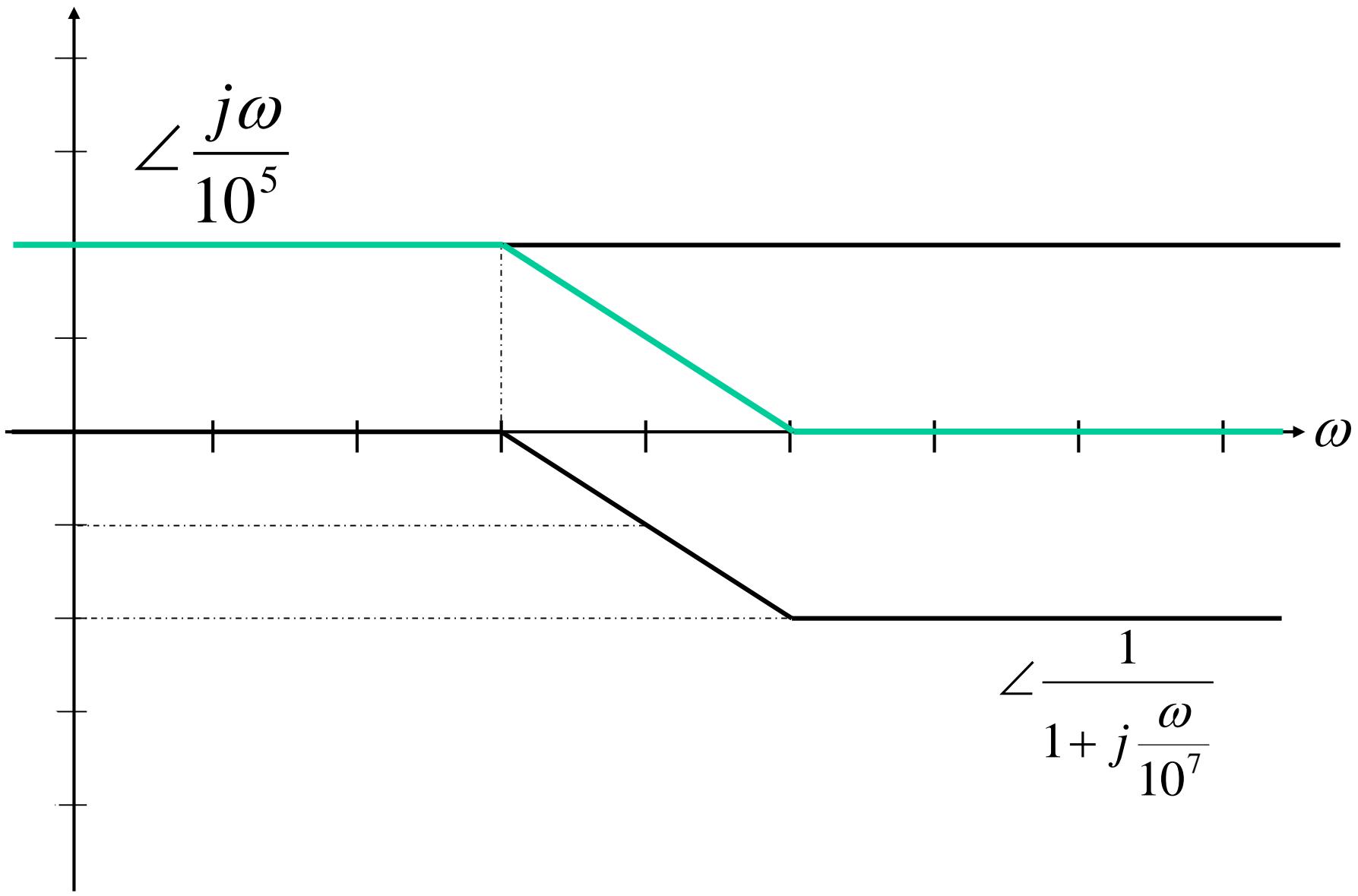
Phase Bode Plot: DC Zero



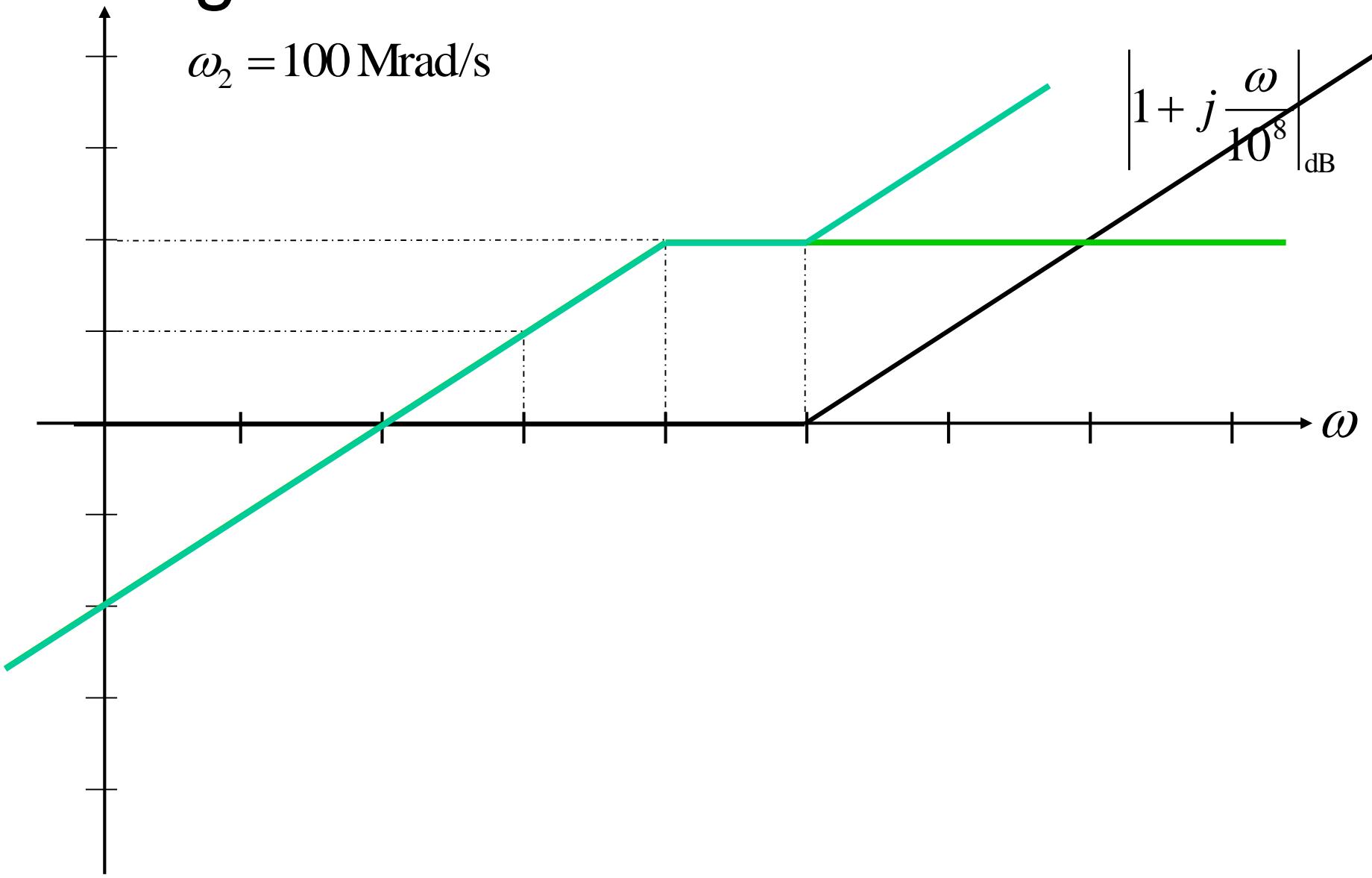
Magnitude Bode Plot: Add First Pole



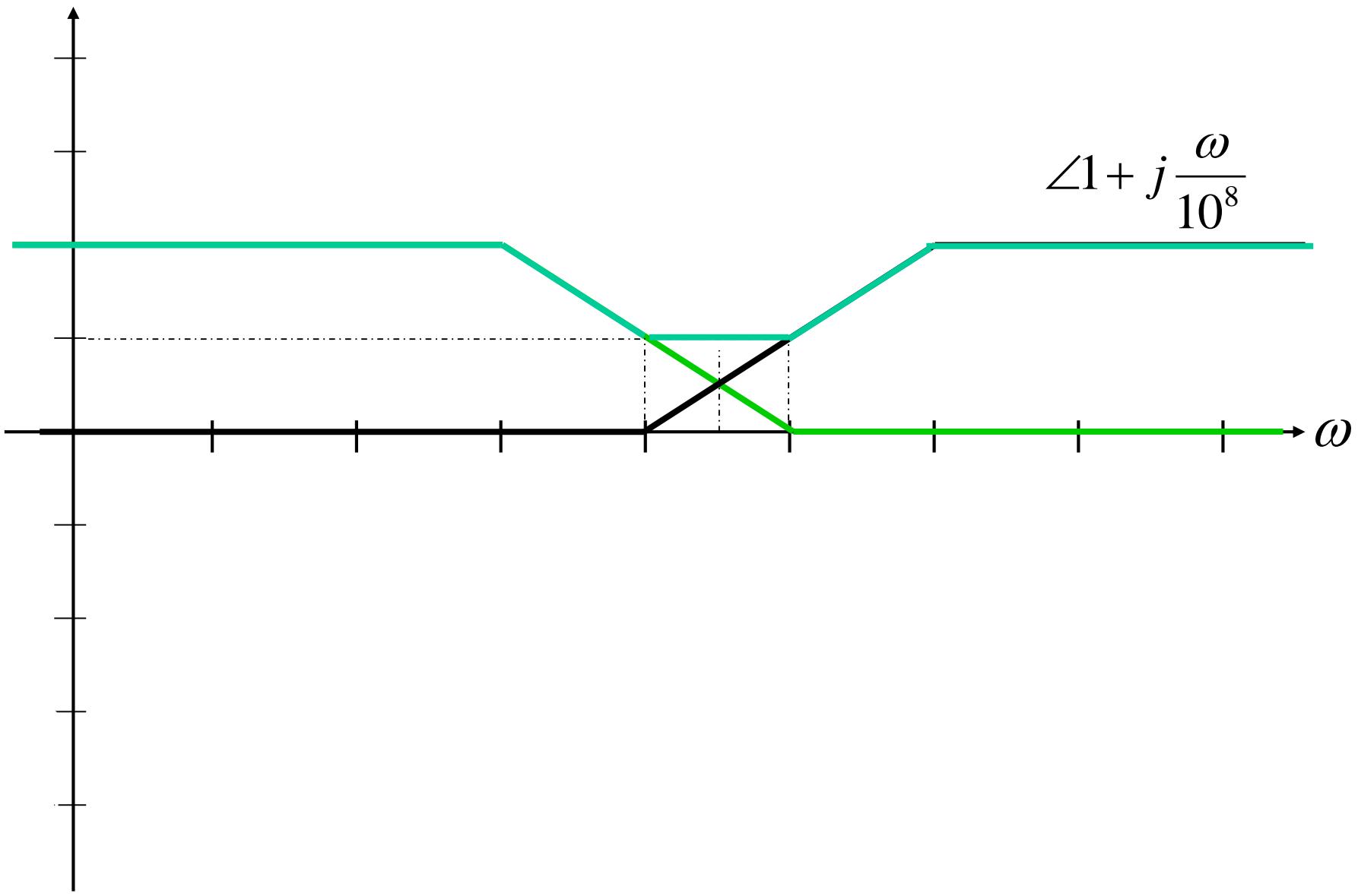
Phase Bode Plot: Add First Pole



Magnitude Bode Plot: Add 2nd Zero

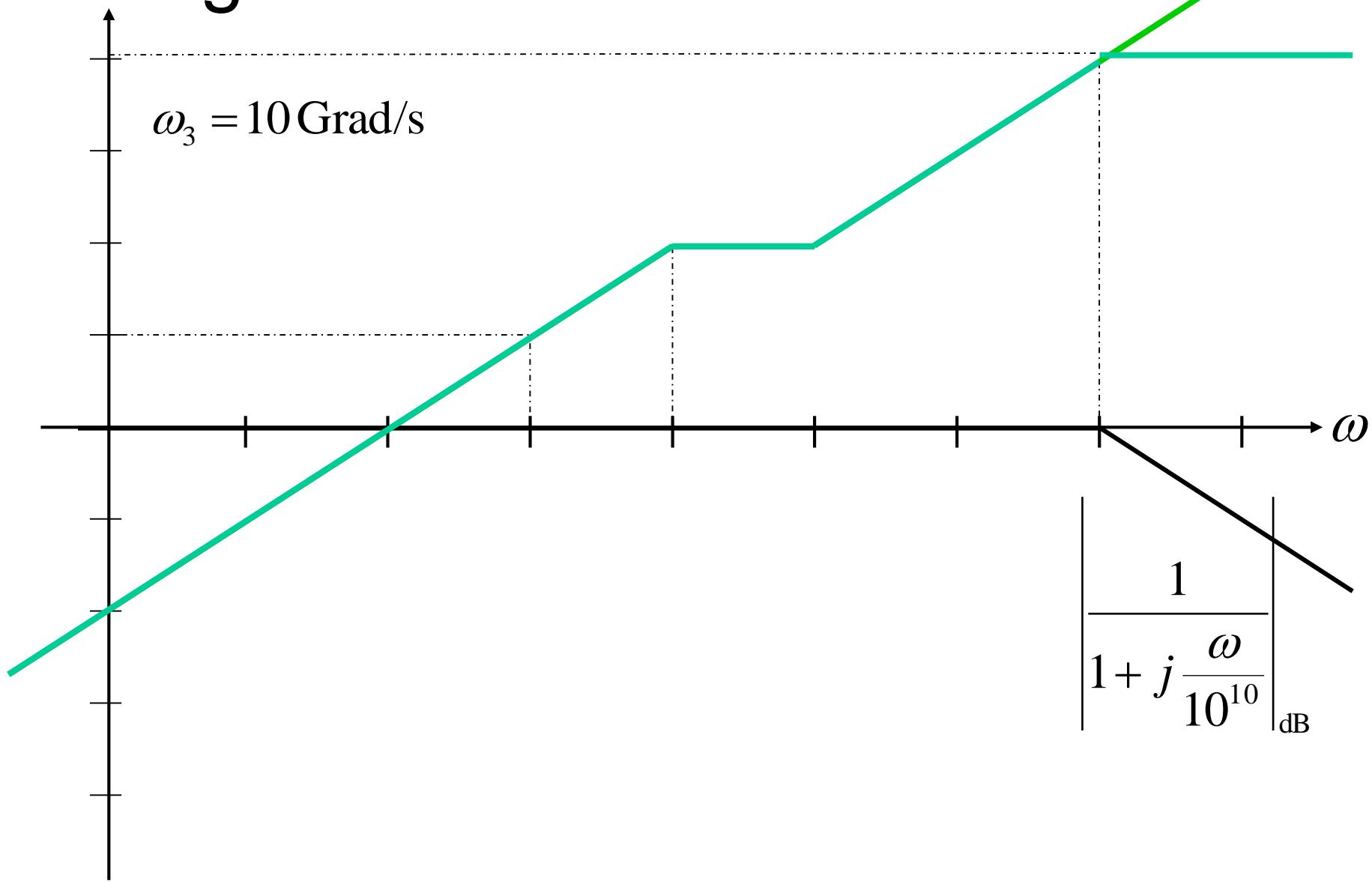


Phase Bode Plot: Add 2nd Zero

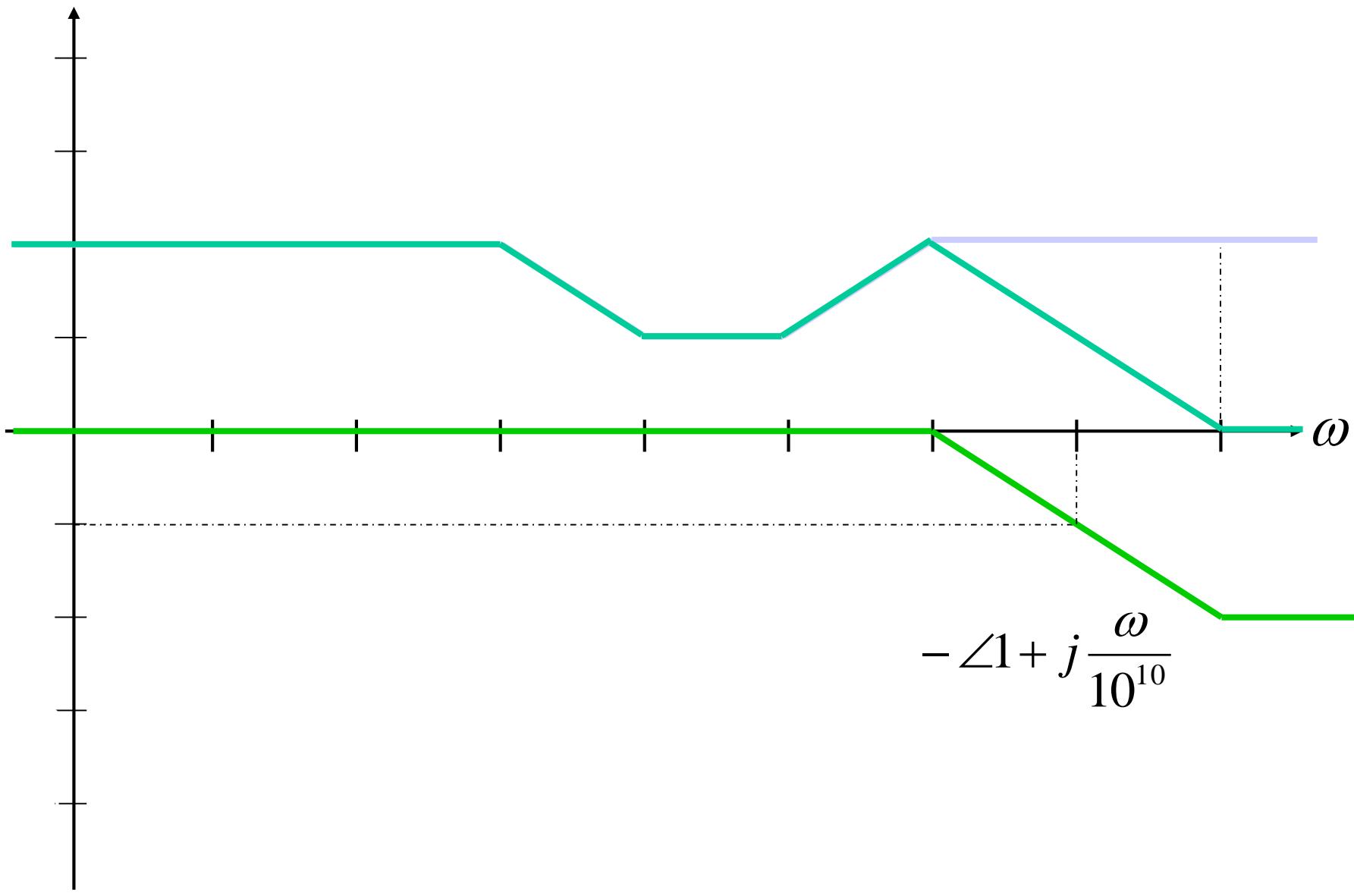


Magnitude Bode Plot: Add 2nd Pole

$$\omega_3 = 10 \text{ Grad/s}$$

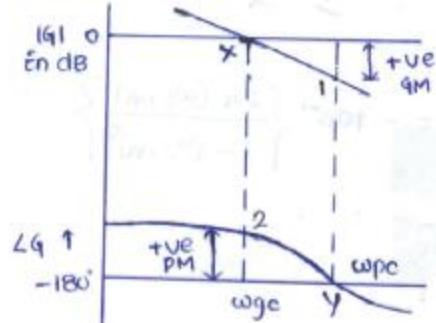


Phase Bode Plot: Add 2nd Pole

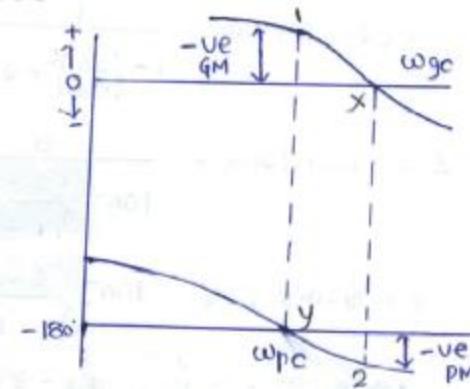




* Calculation of GM and PM from Bode plot :-



(Stable System)



(Unstable System)

Draw a line downwards from gain cross over point 'x' till it intersects phase angle plot. The point of intersection is replaced by '2' the distance b/w point 2 and -180° line is the phase margin. If point 2 on phase angle plot is above -180° line, phase margin is +ve and if point 2 is below -180° line the phase margin is -ve.

From phase cross over point 'y' draw a line upwards till it intersects the magnitude plot at point 1. The difference b/w magnitude corresponding to point 1 and 0 dB is gain margin. If point 1 is below 0 dB GM is +ve and if point 1 is above 0 dB line GM is -ve.



* Calculation of GM & PM mathematically :-

e.g. → The transfer function is given by $G(s)H(s) = \frac{2}{s(1+0.5s)(1+0.05s)}$

As per the defn the phase cross over frequency is the frequency at which phase angle of $G(j\omega)H(j\omega)$ is -180° .

For the given function $\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1} \frac{0.5\omega}{1} - \tan^{-1} \frac{0.05\omega}{1}$
According to definition

$$-180^\circ = -90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.05\omega$$

$$\Rightarrow \tan^{-1} 0.5\omega + \tan^{-1} 0.05\omega = -90^\circ \Rightarrow$$

Taking tangent on both the sides

$$\tan \{\tan^{-1} 0.5\omega + \tan^{-1} 0.05\omega\} = \tan 90$$

$$\Rightarrow \frac{0.5\omega + 0.05\omega}{1 - 0.05\omega \times 0.05\omega} = \infty \Rightarrow 1 - 0.25\omega^2 = 0 \Rightarrow \omega = 6.32 \text{ rad/s} \\ = \omega_{c2}$$

This is the phase cross over frequency.

$$G(j\omega)H(j\omega) = \frac{2}{\sqrt{6.32(1+0.5 \times 6.32)}(1+0.05 \times 6.32)}$$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{c2}} = 0.0918$$

$$GM = -20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{c2}} = 20 \log 0.0918 = 20.74 \text{ dB}$$

$$|G(j\omega)H(j\omega)| = \frac{2}{\omega \sqrt{1+0.25\omega^2} \sqrt{1+0.0025\omega^2}}$$

$$\Rightarrow 1 = \frac{2}{\omega \sqrt{1+0.25\omega^2} \sqrt{1+0.0025\omega^2}} \Rightarrow \omega = 2 \text{ rad/s} = \omega_{gc}$$

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 0.707$$

$$\angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = -140.71^\circ$$

$$\text{Now } PM = 180 + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 180 + (-140.71) \\ = 39.28^\circ$$