Stability and Control
Stick Free Characteristics

Hinge Moments

Each control surface on an aircraft has a hinge of some sort. By deflecting the control, there is an aerodynamic moment about that hinge. The pilot (or some power augmentation system) must provide the moment to counter that hinge moment if s/he is to be able to deflect the surface. So the study of hinge moments is important to be able to predict the forces (moments) required by the human pilot or the hydraulic or electric actuator system. Further, the feedback of this required force or moment to the pilot is an additional cue to help her/him to fly the aircraft.

Here we will be primarily interested in the elevator hinge moment, but the procedure is the same for the aileron or the rudder (or most other flapped control). If the hinge moment is designated by H_e, then the ***hinge moment coefficient*** is defined as:

$$C_{H_e} = \frac{H_e}{1/2\, \rho\, V^2 S_e \bar{c}_e} \tag{1}$$

where S_e = the elevator (control surface) area behind the hinge line
$\bar{c}_e$ = the mean aerodynamic chord of the elevator (control surface) of the same area

Generally we assume that the hinge moment depends linearly on the tail angle-of-attack, the elevator deflection, and the deflection of an additional surface at the end of the elevator called the tab. Thus the hinge moment can be represented as:

$$\boxed{\begin{aligned} C_{H_e} &= C_{H_{e_0}} + \frac{\partial C_{H_e}}{\partial \alpha_{ht}} \alpha_{ht} + \frac{\partial C_{H_e}}{\partial \delta_e} \delta_e + \frac{\partial C_{H_e}}{\partial \delta_t} \delta_t \\ &= b_0 + b_1 \alpha_{ht} + b_2 \delta_e + b_3 \delta_t \end{aligned}} \tag{2}$$

where $b_0,\ b_1,\ b_2,$ and b_3 are defined accordingly.

Reversible and Irreversible Controls

At this stage we must introduce the concept of reversible and irreversible control systems. In a ***reversible system***, the pilot controls are directly connected (using pulleys, cables, and push-rods) to the control surface. Therefore if the pilot controls are deflected, the corresponding control surface is deflected. Also, if the control surface is deflected, then the pilot controls will be deflected (one of the standard pre-flight checks).

On the other hand, in an ***irreversible system***, the controls may be directly connected, but there is an additional boost system the supplies additional forces to the controls. Therefore when the pilot moves the cockpit controls, the control surface moves. However, if an attempt to move the control surface is made, it will not move, or if it does move, the pilots controls will not move. Generally, the boost system will hold the control surface in a fixed position once it is set at that position.

For a reversible system, if the pilot lets go of the stick, then the force or moment applied to the control surface disappears (neglecting friction) and the control surface will "float" to the position where there is no hinge moment. The condition where the hinge moment is equal to zero is called the "***stick free***" condition. Under these conditions, the aerodynamic characteristics including the neutral point change. Previous calculations assumed a constant elevator deflection and hence are called "***stick fixed***" characteristics. Even for irreversible control systems it is important to calculate the stick free characteristics.

It turns out that it is relatively easy to calculate the stick free characteristics from the stick fixed ones developed previously. The secret is to note that the stick free condition only affects the contribution from the tail surface. All stick free conditions are obtained by setting the hinge moment equal to zero. For simplicity we will assume $b_0 = 0$ and that the tab deflection $\delta_t = 0$. Then, if we set the hinge moment coefficient equal to zero, we can solve for the elevator float angle. From Eq (2) we obtain:

$$\delta_{e_{float}} = -\frac{b_1}{b_2}\alpha_{ht} \tag{3}$$

If we examine the tail lift coefficient we can observe that:

$$\begin{aligned} C_{L_{ht}} &= a_{ht}\alpha_{ht} + a_e\delta_e \\ &= a_{ht}\alpha_{ht} + a_e\left(-\frac{b_1}{b_2}\right)\alpha_{ht} \\ &= a_{ht}\left(1 - \frac{a_e}{a_{ht}}\frac{b_1}{b_2}\right)\alpha_{ht} \\ &= F\,a_{ht}\alpha_{ht} \end{aligned} \tag{4}$$

where

$$F = \left(1 - \frac{a_e}{a_{ht}}\frac{b_1}{b_2}\right) \tag{5}$$

is called the ***free elevator factor***.

We can now observe that the effective lift-curve slope of the horizontal tail is modified by freeing the stick. Therefore, to convert the stick-fixed results to stick free results, all we have to do is to replace the stick fixed horizontal tail lift curve slope with the horizontal tail stick free lift curve slope, that is replace a_{ht} with $\hat{a}_{ht} = F a_{ht}$, where the $\hat{a}_{ht}$ indicates stick free conditions. As a result, we have:

$$\hat{a} = a_{wb}\left[1 + \frac{F a_{ht}}{a_{wb}} \eta_{ht} \forall_{ht}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right] \tag{6}$$

$$\hat{C}_{m_\alpha} = a_{wb}\left[(h - h_{n_{wb}}) - \frac{F a_{ht}}{a_{wb}} \eta_{ht} \forall_{ht}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right] \tag{7}$$

and

$$\hat{C}_{m_\alpha} = \hat{a}(h - \hat{h}_n) \tag{8}$$

where $\hat{h}_n$ is the ***stick free neutral point***.

Stick Free Neutral Point

Just as we did for the stick-fixed case (elevator position fixed) we can calculate the stick free neutral point directly. We use the same equations as for the stick-fixed case but replace the tail lift curve slope with the stick-free tail lift curve slope:

$$a_{ht} \quad \Rightarrow \quad F a_{ht}$$

The two expressions for the stick free neutral point become:

$$\begin{aligned} \hat{h}_n &= h_{n_{wb}} + F \frac{a_{ht}}{\hat{a}} \eta_{ht} \frac{S_{ht}}{S} (h_{ht} - h_{n_{wb}})\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \\ &= \frac{h_{n_{wb}} + F \dfrac{a_{ht}}{a_{wb}} \eta_{ht} \dfrac{S_{ht}}{S} h_{ht}\left(1 - \dfrac{\partial \epsilon}{\partial \alpha}\right)}{1 + F \dfrac{a_{ht}}{a_{wb}} \eta_{ht} \dfrac{s_{ht}}{S}\left(\dfrac{\partial \epsilon}{\partial \alpha}\right)} \end{aligned} \tag{9}$$

Stick Forces

The stick forces are directly related to the hinge moments. In reversible controls, the hinge moment is conveyed to the pilot through the push rods, cables and pulleys. In an irreversible control system, these forces are transmitted to the hydraulic boost system. In either case, we need to know what they are so that we know a pilot can control the aircraft or that the boost system has enough power. Also in irreversible systems, we like to supply the pilot with an artificial feel so that s/he can use these cues to help control the plane.

The stick force is given by:

$$F_s = G\, C_{H_e}\, \eta_{ht}\, \frac{1}{2}\, \rho\, V^2\, S_e\, \bar{c}_e \tag{10}$$

where G is a gearing factor (with units) to convert moments to force and includes the geometry of the control mechanisms, pulleys, push-rods and cables. It can also include a contribution from the boost system. In any case the important thing to note is that the stick force is related directly to the hinge moment. If the hinge moment is zero, so is the stick force. If there is no stick force, then (for a reversible system) there will be no applied moment to counter the hinge moment so the elevator will float as discussed earlier.

We can define balanced, trimmed flight as equilibrium flight with zero stick force. Ato see how this is accomplished, we can look at the basic governing equations;

$$\begin{aligned} \frac{W}{\bar{q}\,S} &= C_L = C_{L_\alpha}\,\bar{\alpha} + C_{L_{\delta_e}}\,\delta_e \\ 0 &= C_m = C_{m_{0L}} + C_{m_\alpha}\,\bar{\alpha} + C_{m_{\delta_e}}\,\delta_e \\ 0 &= C_{H_e} = b_1\,\alpha_{ht} + b_2\,\delta_e + b_3\,\delta_t \end{aligned} \tag{11}$$

We can note that $\alpha_{ht} = \alpha + i_{ht} - \epsilon$. Then Eq. (11) represents three equations in the three unknowns, $\alpha,\ \delta_e,\ \text{and}\ \delta_t$. Consequently, we can see that it is possible to balance the aircraft with the stick force equal to zero. It generally is not required to solve for the trim tab angle. It is sufficient to know that it exists. The pilot will adjust the trim tab using a trim button until s/he feels no stick force.

It is convenient to think of this problem in a slightly different way. Although the stated result is approximate it would allow us to solve for the trim tab angle using a simpler set of equations. We can sove for the required elevator angle for balance in the usual way to get:

$$\delta_{e_{balance}} = -\frac{C_{m_{0L}}\,C_{L_\alpha} + C_{m_\alpha}\,C_L}{C_{m_{\delta_e}}\,C_{L_\alpha} - C_{m_\alpha}\,C_{L_{\delta_e}}} \tag{12}$$

We can solve the hinge moment equation for the float angle in terms of the trim tab deflection:

$$\delta_{e_{float}} = -\frac{b_1 \alpha_{ht} + b_3 \delta_t}{b_2} \tag{13}$$

We now require that the trim tab, δ_t to be selected so that $\delta_{e_{balance}} = \delta_{e_{float}}$.

If we pick a flight speed, in principle the above equations can be solved to determine a trim tab setting that will give zero stick force. If we fly at a different speed, the elevator deflection will have to be changed (for balance) and the stick force will no longer be zero. We can determine the stick force required to fly at a different airspeed from that at which we trimmed the vehicle. We can do this by looking at the change in hinge moment. If we hold the trim tab fixed, we can subtract the original hinge moment ($C_{H_e} = 0$) from the new hinge moment and note that the angle-of-attack and elevator angle must change. We obtain:

$$\Delta C_{H_e} = b_1 \Delta \alpha_{ht} + b_2 \Delta \delta_e \tag{14}$$

where $\Delta \alpha_{ht} = \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \Delta \alpha$. In addition, from the equations for balanced flight we can difference the balanced and trim angle-of-attack and elevator angle from the new flight condition and arrive at expressions for $\Delta \alpha$ and $\Delta\delta_e$. These are given by:

$$\Delta\alpha = \frac{C_{m_{\delta_e}}}{\Delta} \Delta C_L \qquad \text{and} \qquad \Delta \delta_e = -\frac{C_{m_\alpha}}{\Delta} \Delta C_L$$

In addition

$$\Delta C_L = \frac{W}{1/2 \rho S}\left[\frac{1}{V^2} - \frac{1}{V_{trim}^2}\right]$$

If we put all this together and do a little algebra, we can arrive at the stick force for off-trim conditions:

$$\boxed{F_s = G S_e \bar{c}_e \eta_{ht} \frac{W}{S} \frac{b_2}{\Delta} \hat{a} (h - \hat{h}_n) \left(\frac{V^2}{V_{trim}^2} - 1\right)} \tag{15}$$

Stick Force Gradient

The stick force gradient describes how the stick force changes as we move away from the trimmed flight speed. It is determined by taking the derivative of Eq. (15) with respect to the velocity and evaluating it at the trim speed. The only term that contains the velocity is one in the parenthesis. Taking the derivative gives $2\ V/V_{trim}^2 \,|_{V\ =\ V_{trim}}\ = \dfrac{2}{V_{trim}}$. Consequently the stick force gradient is given by:

$$\boxed{\left.\frac{dF_s}{dV}\right|_{V_{trim}} = 2\, G\, S_e\, \bar{c}_e\, \eta_{ht} \frac{W}{S} \frac{b_2}{\Delta} \hat{a}\,(h - \hat{h}_n)\, \frac{1}{V_{trim}} \qquad (16)}$$

The stick force gradient varies as

1) The size of the aircraft ($S_e\, \bar{c}_e$)
2) The inverse of the trim speed
3) directly with wing loading (W/S)
4) directly with the stick free static margin