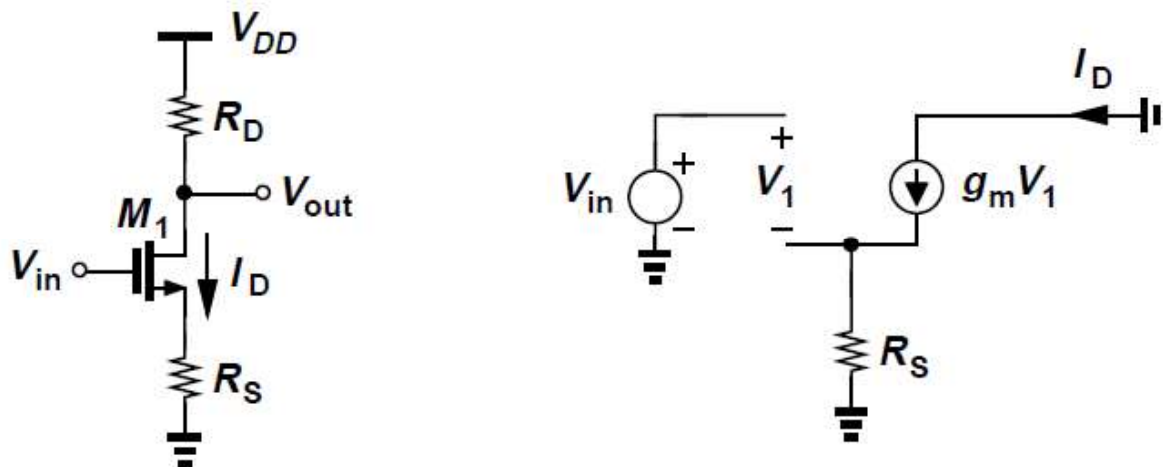
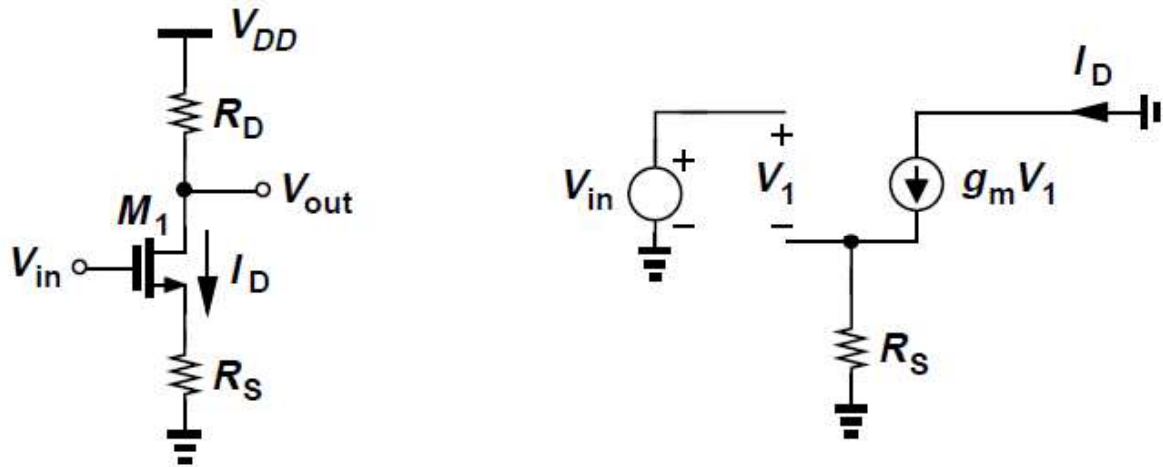


CS Stage with Source Degeneration



- Degeneration resistor R_S in series with source terminal makes input device more linear
 - As V_{in} increases, so do I_D and the voltage drop across R_S
 - Part of the change in V_{in} appears across R_S rather than gate-source overdrive, making variation in I_D smoother
- Gain is now a weaker function of g_m

CS Stage with Source Degeneration



- Nonlinearity of circuit is due to nonlinear dependence of I_D upon V_{in}
- Equivalent transconductance G_m of the circuit can be defined as $G_m = \partial I_D / \partial V_{in}$

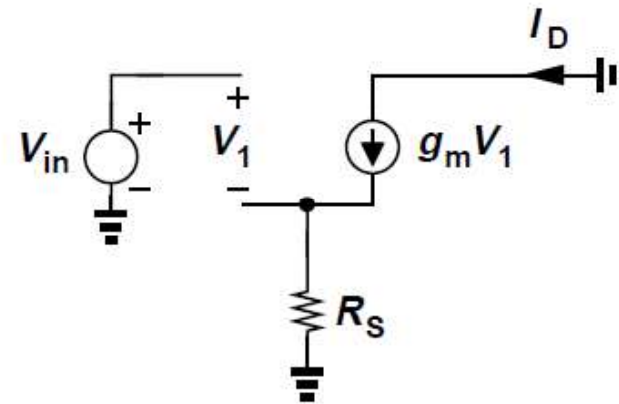
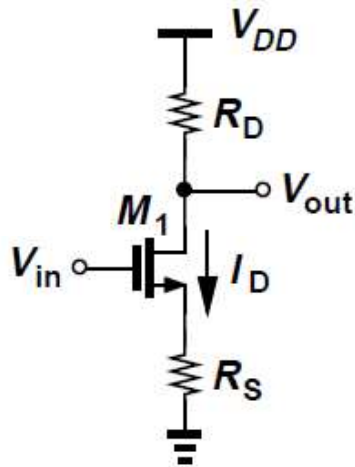
$$V_{out} = V_{DD} - I_D R_D$$

$$\partial V_{out} / \partial V_{in} = -(\partial I_D / \partial V_{in}) R_D$$

$$I_D = f(V_{GS})$$

$$G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{\partial f}{\partial V_{GS}} \frac{\partial V_{GS}}{\partial V_{in}}$$

CS Stage with Source Degeneration



$$V_{GS} = V_{in} - I_D R_S$$

$$\frac{\partial V_{GS}}{\partial V_{in}} = 1 - R_S \frac{\partial I_D}{\partial V_{in}}$$

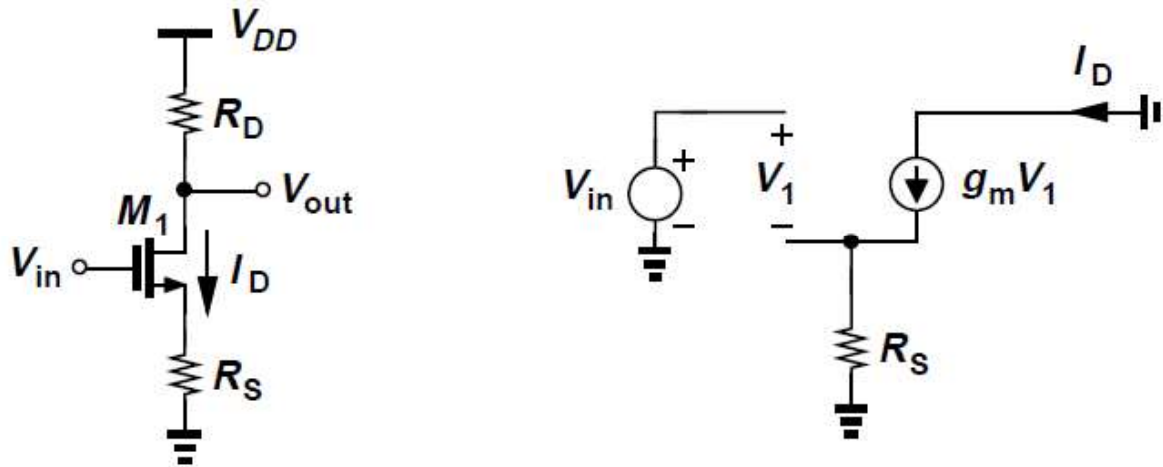
$$G_m = \left(1 - R_S \frac{\partial I_D}{\partial V_{in}} \right) \frac{\partial f}{\partial V_{GS}}$$

$$G_m = \frac{g_m}{1 + g_m R_S}$$

- g_m is the transconductance of M_1
- Small-signal voltage gain A_v is then given by

$$\begin{aligned} A_v &= -G_m R_D \\ &= \frac{-g_m R_D}{1 + g_m R_S} \end{aligned}$$

CS Stage with Source Degeneration



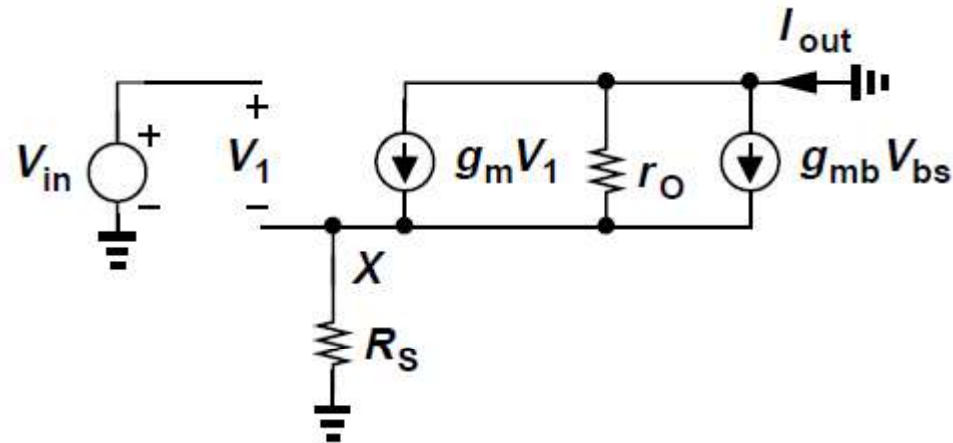
- Same result for G_m is obtained from small-signal equivalent circuit, by noting that

$$V_{in} = V_1 + I_D R_S$$

$$I_D = g_m V_1$$

- As R_S increases, G_m becomes a weaker function of g_m and hence I_D
- For $R_S \gg 1/g_m$, $G_m \approx 1/R_S$, i.e., $\Delta I_D \approx \Delta V_{in}/R_S$
- Most of the change in V_{in} across R_S and drain current becomes a “linearized” function of input voltage

CS Stage with Source Degeneration

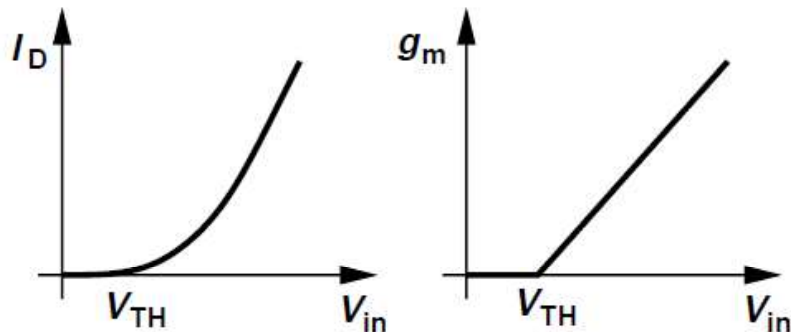


- Including body-effect and channel-length modulation, G_m is found from modified small-signal equivalent circuit

$$\begin{aligned}
 V_{in} &= V_1 + I_{out} R_S \\
 I_{out} &= g_m V_1 - g_{mb} V_X - \frac{I_{out} R_S}{r_O} \\
 &= g_m (V_{in} - I_{out} R_S) + g_{mb} (-I_{out} R_S) - \frac{I_{out} R_S}{r_O} \\
 G_m &= \frac{I_{out}}{V_{in}} \\
 &= \frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O}
 \end{aligned}$$

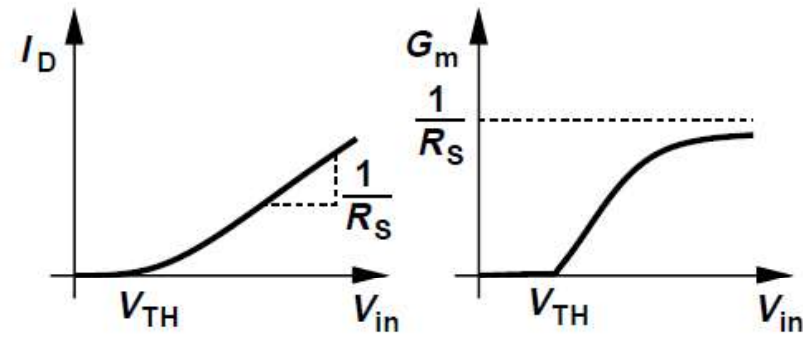
CS Stage with Source Degeneration

Large-signal behavior



$R_S = 0$

- I_D and g_m vary with V_{in} as derived in calculations in Chapter 2



$R_S \neq 0$

- At low current levels, turn-on behavior is similar to when $R_S = 0$ since

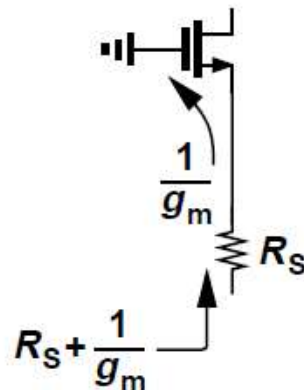
$$1/g_m \gg R_S$$
- As overall $G_m \approx g_m$ and g_m increase, effect of R_S becomes more significant

CS Stage with Source Degeneration

- Small-signal derived previously can be written as

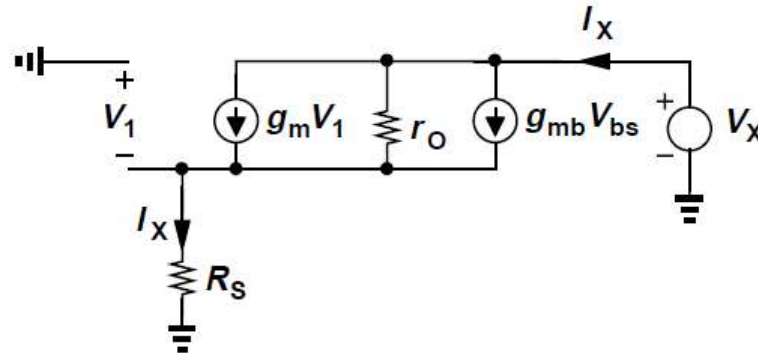
$$A_v = -\frac{R_D}{\frac{1}{g_m} + R_S}$$

- Denominator = Series combination of inverse transconductance + explicit resistance seen from source to ground
- Called “resistance seen in the source path”
- Magnitude of gain = Resistance seen at the drain/ Total resistance seen in the source path



CS Stage with Source Degeneration

- Degeneration causes increase in output resistance



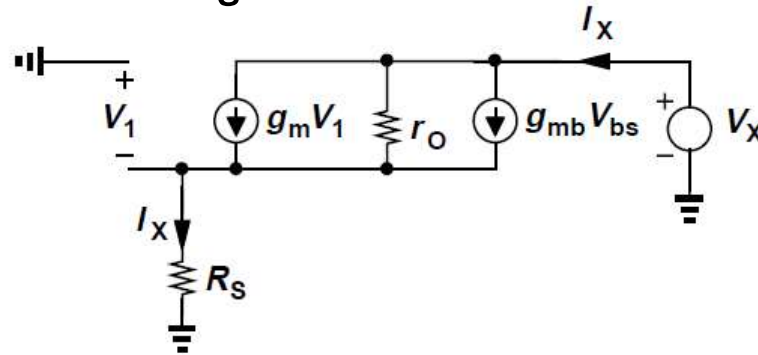
- Ignoring R_D and including body effect in small-signal equivalent model,

$$\begin{aligned}
 V_1 &= -I_X R_S \\
 I_X - (g_m + g_{mb})V_1 &= I_X + (g_m + g_{mb})R_S I_X \\
 R_{out} &= [1 + (g_m + g_{mb})R_S]r_O + R_S \\
 &= [1 + (g_m + g_{mb})r_O]R_S + r_O.
 \end{aligned}$$

- r_o is boosted by a factor of $\{1 + (g_m + g_{mb})R_S\}$ and then added to R_S
- Alternatively, R_S is boosted by a factor of $\{1 + (g_m + g_{mb})r_o\}$ and then added to r_o

CS Stage with Source Degeneration

- Compare $R_S = 0$ with $R_S > 0$

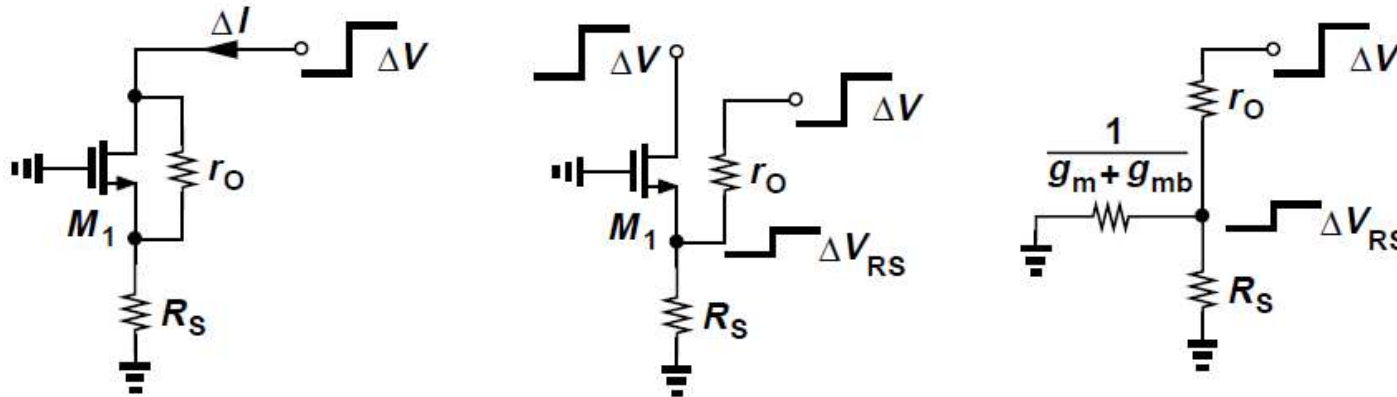


- If $R_S = 0$, $g_m V_1 = g_{mb} V_{bs} = 0$ and $I_X = V_X / r_o$
- If $R_S > 0$, $I_X R_S > 0$ and $V_1 < 0$, obtaining negative $g_m V_1$ and $g_{mb} V_{bs}$
- Thus, current supplied by V_X is less than V_X / r_o and hence output impedance is greater than r_o

CS Stage with Source Degeneration

Intuitive understanding of increased output impedance

- Apply voltage change ΔV at output and measure resulting change ΔI in output current, which is also the change in current through R_S

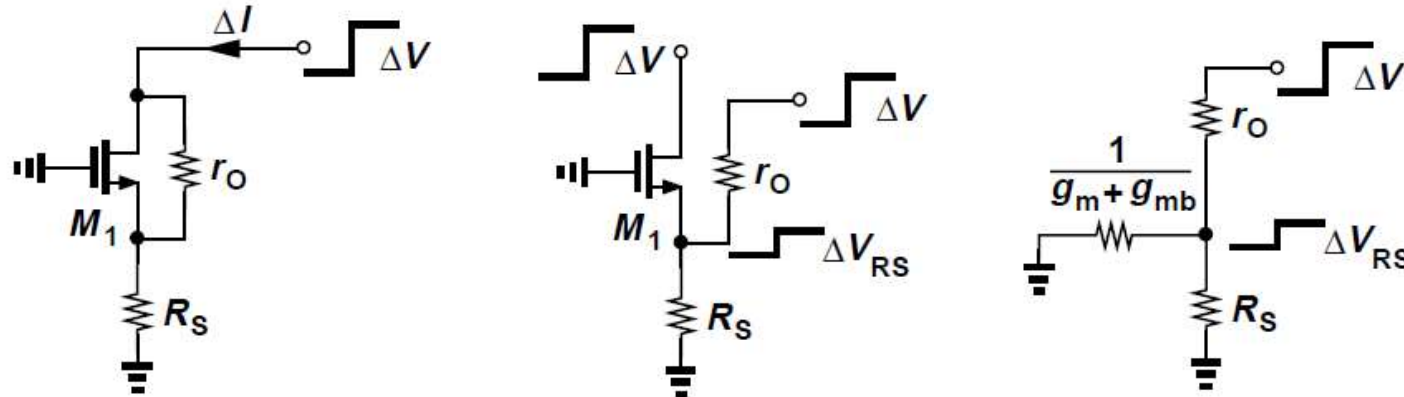


- Resistance seen looking into the source of M_1 is $1/(g_m + g_{mb})$
- Voltage change across R_S is

$$\Delta V_{RS} = \Delta V \frac{\frac{1}{g_m + g_{mb}} \parallel R_S}{\frac{1}{g_m + g_{mb}} \parallel R_S + r_O}$$

CS Stage with Source Degeneration

Intuitive understanding of increased output impedance

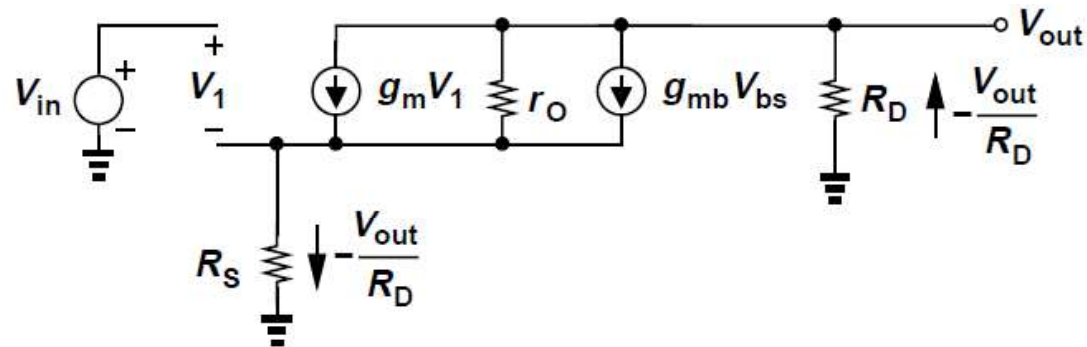


- Change in current across R_S is

$$\begin{aligned}\Delta I &= \frac{\Delta V_{RS}}{R_S} \\ &= \Delta V \frac{1}{[1 + (g_m + g_{mb})R_S]r_O + R_S}\end{aligned}$$

- Output resistance $\frac{\Delta V}{\Delta I} = [1 + (g_m + g_{mb})R_S]r_O + R_S$

CS Stage with Source Degeneration



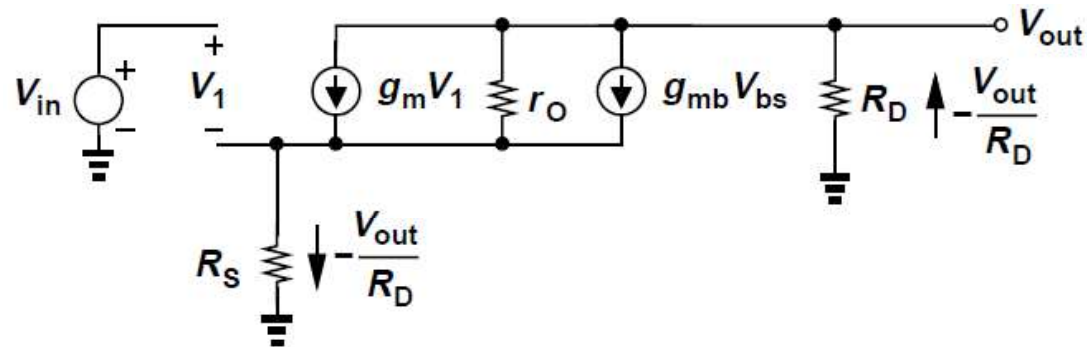
- To compute gain in the general case including body effect and channel-length modulation, consider above small-signal model
- From KVL at input,

$$V_1 = V_{in} + V_{out} R_S / R_D$$

- KCL at output gives

$$\begin{aligned} I_{rO} &= -\frac{V_{out}}{R_D} - (g_m V_1 + g_{mb} V_{bs}) \\ &= -\frac{V_{out}}{R_D} - \left[g_m \left(V_{in} + V_{out} \frac{R_S}{R_D} \right) + g_{mb} V_{out} \frac{R_S}{R_D} \right] \end{aligned}$$

CS Stage with Source Degeneration



- Since voltage drops across r_o and R_S must add up to V_{out} ,

$$\begin{aligned} V_{out} &= I_{r_o} r_o - \frac{V_{out}}{R_D} R_S \\ &= -\frac{V_{out}}{R_D} r_o - \left[g_m \left(V_{in} + V_{out} \frac{R_S}{R_D} \right) + g_{mb} V_{out} \frac{R_S}{R_D} \right] r_o - V_{out} \frac{R_S}{R_D} \end{aligned}$$

- Voltage gain is therefore

$$\frac{V_{out}}{V_{in}} = \frac{-g_m r_o R_D}{R_D + R_S + r_o + (g_m + g_{mb}) R_S r_o}$$

Lemma

- In a linear circuit, the voltage gain is equal to $-G_m R_{out}$
 - G_m denotes the transconductance of the circuit when output is shorted to ground
 - R_{out} represents the output resistance of the circuit when the input voltage is set to zero
- Norton equivalent of a linear circuit

