

Session-10:

Some Properties of Eigen Values (Continued):

4. If λ is an eigenvalue of a matrix A , then λ^m be an eigenvalue of a matrix $A^m, m > 0$.

Proof: If X be the eigenvector corresponding to eigenvalue λ then
(Premultiplying A)
 $AX = \lambda X \Rightarrow A(AX) = A(\lambda X)$
 $\Rightarrow A^2 X = \lambda(A X) \Rightarrow A^2 X = \lambda(\lambda X)$
 $\Rightarrow A^2 X = \lambda^2 X$

Again premultiplying by A , we obtain
 $A(A^2 X) = A(\lambda^2 X) \Rightarrow (AA^2)X = \lambda^2(AX)$
 $\Rightarrow A^3 X = \lambda^2(\lambda X) \Rightarrow A^3 X = \lambda^3 X$

Similarly

$$A^m X = \lambda^m X$$

Thus λ^m is the eigenvalue of the matrix $A^m (m > 0)$.

In general, if $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ be the eigenvalues of A , then $\lambda_1^m, \lambda_2^m, \lambda_3^m, \dots, \lambda_n^m$ be the eigenvalues of $A^m (m > 0)$.

5. If λ be an eigenvalue of a non-singular matrix A , then $\frac{|A|}{\lambda}$ is an eigenvalue of the matrix $\text{Adj } A$.

Proof: If X be the eigenvector corresponding to eigenvalue λ , then
(Premultiplying $\text{Adj } A$)
 $AX = \lambda X \Rightarrow \text{Adj } A(AX) = \text{Adj } A(\lambda X)$
 $\Rightarrow ((\text{Adj } A)A)X = \lambda((\text{Adj } A)X) \Rightarrow |A|IX = \lambda((\text{Adj } A)X)$
 $\Rightarrow \frac{|A|}{\lambda} X = (\text{Adj } A)X$

Thus $\frac{|A|}{\lambda}$ is an eigenvalue of the matrix $\text{Adj } A$.

6. If λ is an eigenvalue of a matrix A , then $\frac{1}{\lambda}$ is the eigenvalue of A^{-1} .

Proof: If X be the eigenvector corresponding to eigenvalue λ , then
(Premultiplying A^{-1})
 $AX = \lambda X \Rightarrow A^{-1}(AX) = A^{-1}(\lambda X)$
 $\Rightarrow (A^{-1}A)X = \lambda(A^{-1}X) \Rightarrow IX = \lambda(A^{-1}X)$
 $\Rightarrow \frac{1}{\lambda}X = A^{-1}X$

Thus $\frac{1}{\lambda}$ is the eigenvalue of A^{-1} .

In general, if $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ be the eigenvalues of A , then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$ be the eigenvalues of A^{-1} .