

Real Matrices: Symmetric, Skew-Symmetric & Orthogonal Matrices:

Symmetric Matrix: A real square matrix $A = [a_{ij}]$ is said to be symmetric if its transposition leaves unchanged.

That is,

$$A^T = A$$

Here

$$a_{ij} = a_{ji}$$

Skew-Symmetric Matrix: A real square matrix $A = [a_{ij}]$ is said to be skew-symmetric if its transposition gives the negative of A .

That is,

Here
$$A^T = -A$$

$$a_{ij} = -a_{ji}$$

Orthogonal Matrix: A real square matrix $A = [a_{ij}]$ is said to be Orthogonal if its transposition gives the inverse of A.

That is,

$$A^T = A^{-1}$$

Example1:

Let

$$A = \begin{bmatrix} -5 & 2 & 3 \\ 2 & 0 & -4 \\ 3 & -4 & 8 \end{bmatrix}$$

Then

$$A^T = \begin{bmatrix} -5 & 2 & 3 \\ 2 & 0 & -4 \\ 3 & -4 & 8 \end{bmatrix}$$

Thus

$$A^T = A$$

Hence A is said to be symmetric.

Example2:

Let

$$A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix}$$

Then
$$A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$$

Thus
$$A^T = -A$$

Hence A is said to be skew-symmetric.

Solved Example3:

Let
$$A = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

Then
$$A^T = A^{-1}$$

Hence A is said to be Orthogonal.