

## Session -1

### VECTOR CALCULUS

#### Introduction:

In our day to day life, we come across many queries such as – What is your height? What is your weight? We observe that a possible answer to the first query may be 1.6 meters and second query may be 45 kg. That means a quantity that involves only one value (magnitude) which is a real number. Such quantity are called *scalars*.

How should a football player hit the ball to give a pass to another player of his team an answer to this third query is a quantity (called force) which involves muscular strength (magnitude) and direction (in which another player is positioned). Such quantities are called *vectors*.

In mathematics, physics and engineering, we frequently come across with both types of quantities, namely, *scalar quantities* such as length, mass, time, distance, speed, area, volume, temperature, work, money, voltage, density, resistance etc. and *vector quantities* like displacement, velocity, acceleration, force, weight, momentum, electric field intensity etc.

In this chapter, we will study some of the basic concepts about vectors, various operations on vectors, algebra of vectors in 3-space, application of vectors and Vectors differential calculus. We finally discuss three physically and geometrically important concepts related to scalar and vector fields like gradient, divergent and curl of a vector field.

#### Some Important Concept of Vectors:

- A quantity that has magnitude as well as direction is called a vector.
- The distance between initial and terminal points of a vector is called the *magnitude* (or length) of the vector, denoted as  $|\overrightarrow{AB}|$  or  $|\vec{a}|$  or  $a$ ,  $|\vec{a}| > 0$ .

#### Types of Vectors

**Zero Vector:** A vector whose initial and terminal points coincide, is called a zero vector (or null vector), and denoted as  $\vec{0}$ . Zero vector cannot be assigned a definite direction as it has zero magnitude. Or, alternatively otherwise, it may be regarded as having any direction. The vectors  $\overrightarrow{AA}$ ,  $\overrightarrow{BB}$  represent the zero vector,

**Unit Vector:** A vector whose magnitude is unity (i.e., 1 unit) is called a unit vector. The unit vector in the direction of a given vector  $\vec{a}$  is denoted by  $\hat{a}$ .

**Coinitial Vectors:** Two or more vectors having the same initial point are called coinital vectors.

**Collinear Vectors** Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

**Equal Vectors:** Two vectors  $\vec{a}$  and  $\vec{b}$  are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points, and written as

$$\vec{a} = \vec{b}.$$

**Negative of a Vector:** A vector whose magnitude is the same as that of a given vector (say  $\overrightarrow{AB}$ ), but direction is opposite to that of it, is called *negative* of the given vector.

For example, vector  $\overrightarrow{BA}$  is negative of the vector  $\overrightarrow{AB}$  and written as  $\overrightarrow{BA} = -\overrightarrow{AB}$ ,

#### Addition of Vectors

A vector  $\overrightarrow{AB}$  simply means the displacement from a point A to the point B. Now consider a situation that a girl moves from A to B and then from B to C (Fig 170). The net displacement made by the girl from point A to the point C, is given by the vector  $\overrightarrow{AC}$  and expressed as

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

This is known as the *triangle law of vector addition*

### Properties of vector addition

**Property 1** For any two vectors  $a$  and  $b$

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \text{ (Commutative property).}$$

**Property 2** For any three vectors  $a, b$  and  $c$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \text{ (Associative property)}$$

### Addition of Vectors

The **sum**  $\mathbf{a} + \mathbf{b}$  of two vectors  $\mathbf{a} = [a_1, a_2, a_3]$  and  $\mathbf{b} = [b_1, b_2, b_3]$  is obtained by adding the corresponding components,

$$(3) \quad \mathbf{a} + \mathbf{b} = [a_1 + b_1, a_2 + b_2, a_3 + b_3].$$

Geometrically, place the vectors as in Fig. 170 (the initial point of  $\mathbf{b}$  at the terminal point of  $\mathbf{a}$ ); then  $\mathbf{a} + \mathbf{b}$  is the vector drawn from the initial point of  $\mathbf{a}$  to the terminal point of  $\mathbf{b}$ .

**Basic Properties of Vector Addition.** Familiar laws for real numbers give immediately

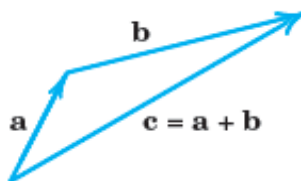
- |     |   |                        |
|-----|---|------------------------|
| (a) | $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$                               | <i>(Commutativity)</i> |
| (b) | $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ | <i>(Associativity)</i> |
| (c) | $\mathbf{a} + \mathbf{0} = \mathbf{0} + \mathbf{a} = \mathbf{a}$                  |                        |
| (d) | $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}.$  |                        |

### Scalar Multiplication (Multiplication by a Number)

The product  $c\mathbf{a}$  of any vector  $\mathbf{a} = [a_1, a_2, a_3]$  and any scalar  $c$  (real number  $c$ ) is the vector obtained by multiplying each component of  $\mathbf{a}$  by  $c$ ,

$$(5) \quad c\mathbf{a} = [ca_1, ca_2, ca_3].$$

Geometrically, if  $\mathbf{a} \neq \mathbf{0}$ , then  $c\mathbf{a}$  with  $c > 0$  has the direction of  $\mathbf{a}$  and with  $c < 0$  the direction opposite to  $\mathbf{a}$ . In any case, the length of  $c\mathbf{a}$  is  $|c\mathbf{a}| = |c||\mathbf{a}|$ , and  $c\mathbf{a} = \mathbf{0}$  if  $\mathbf{a} = \mathbf{0}$  or  $c = 0$  (or both). (See Fig. 175.)



**Fig. 170.** Vector addition



**Fig. 175.** Scalar multiplication [multiplication of vectors by scalars (numbers)]