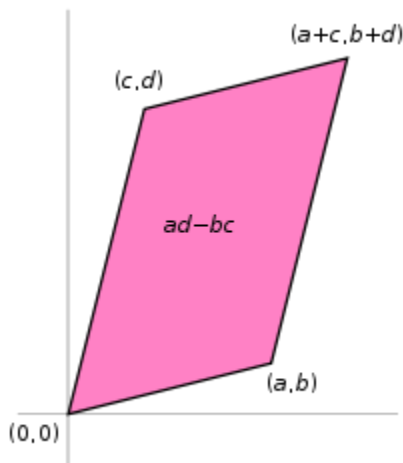


## Session-5:

### Determinants

The determinant of a  $2 \times 2$  matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is defined by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$



The area of the parallelogram is the absolute value of the determinant of the matrix formed by the vectors representing the parallelogram's sides.

The determinant of a  $3 \times 3$  matrix  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$  is defined by

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ = aei + bfg + cdh - ceg - bdi - afh.$$

### Determinant of a $3 \times 3$ Matrix by Diagonals Method:

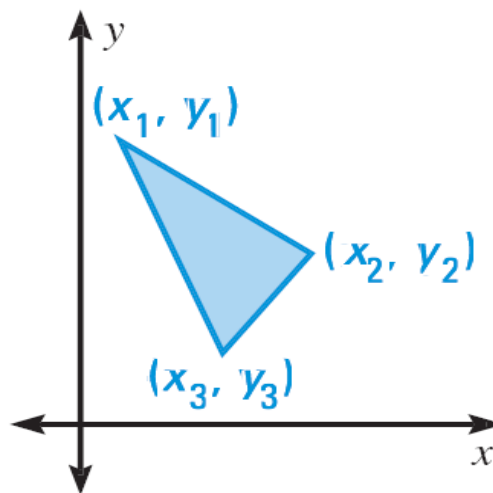
This one is a little crazier. First you have to duplicate the first two columns and then write them after the last column.

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

## Area of a Triangle:

The area of a triangle with vertices at  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is given by:

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



**(Figure for calculation of the Area of a Triangle)**

## Properties of the Determinant:

The determinant has many properties. Some basic properties of determinants are:

- $\det(I_n)=1$ , where  $I_n$  is an  $n \times n$  unity matrix.
- $\det(A^T)=\det(A)$ .
- $\det(A^{-1})=\frac{1}{\det(A)}$ .
- $\det(AB)=\det(A)\det(B)$ .
- $\det(cA)=c^n\det(A)$  for an  $n \times n$  matrix  $A$  and  $c$  is a constant.