

Session-9:

Some Properties of Eigen Values:

1. The product of the eigenvalues of a matrix $A_{n \times n}$ is equal to its determinant.

Proof: If $\lambda_1, \lambda_2, \dots, \lambda_n$ are n eigenvalues of A

$$\text{So, } |A - \lambda I| = (-1)^n (\lambda - \lambda_1)(\lambda - \lambda_2) \dots (\lambda - \lambda_n)$$

$$\text{Put } \lambda = 0 \text{ so } |A| = (-1)^n (-\lambda_1)(-\lambda_2) \dots (-\lambda_n)$$

$$= (-1)^{2n} \lambda_1 \dots \lambda_n = \lambda_1 \dots \lambda_n$$

From this we can say that a matrix is singular if it has at least one zero eigenvalue and non-singular if all its eigenvalues are non-zero.

2. The sum of eigenvalues of a matrix is the trace of the matrix.

Proof: Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{So } |A - \lambda I| = \begin{vmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ a_{21} & a_{22} - \lambda & a_{23} \\ a_{31} & a_{32} & a_{33} - \lambda \end{vmatrix}$$

$$\Rightarrow -\lambda^3 + \lambda^2(a_{11} + a_{22} + a_{33}) + \dots = 0 \dots \dots \dots (1)$$

If $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of A

$$\text{Then } |A - \lambda I| = (-1)^3 (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = 0$$

$$\Rightarrow -\lambda^3 + \lambda^2(\lambda_1 + \lambda_2 + \lambda_3) = 0 \dots \dots \dots (2)$$

From (1) and (2) we get

$$\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{22} + a_{33}$$

Thus the sum of eigenvalues of a matrix A is the trace of the matrix.

3. If λ is an eigenvalue of a matrix A , then $\frac{1}{\lambda}$ is the eigenvalue of A^{-1} .

Proof: If X be the eigenvector corresponding to eigenvalue λ , then
(Premultiplying A^{-1})

$$\Rightarrow (A^{-1}A)X = \lambda(A^{-1}X) \Rightarrow IX = \lambda(A^{-1}X)$$

$$\Rightarrow \frac{1}{\lambda}X = A^{-1}X$$

Thus $\frac{1}{\lambda}$ is the eigenvalue of A^{-1} .

In general, if $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ be the eigenvalues of A , then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$ be the eigenvalues of A^{-1} .