

STATICS

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Center of Gravity and Centroid



Chapter Objectives

- Concept of the center of gravity, center of mass, and the centroid
 - Determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape
 - Theorems of Pappus and Guldinus
 - Method for finding the resultant of a general distributed loading
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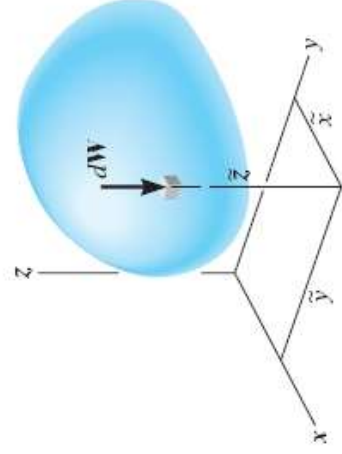
Chapter Outline

1. Center of Gravity and Center of Mass for a System of Particles
 2. Composite Bodies
 3. Theorems of Pappus and Guldinus
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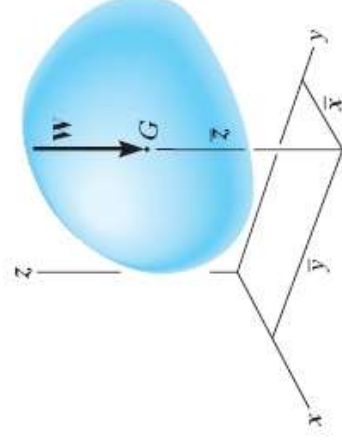
Center of Gravity and Center of Mass for a System of Particles

Center of Gravity

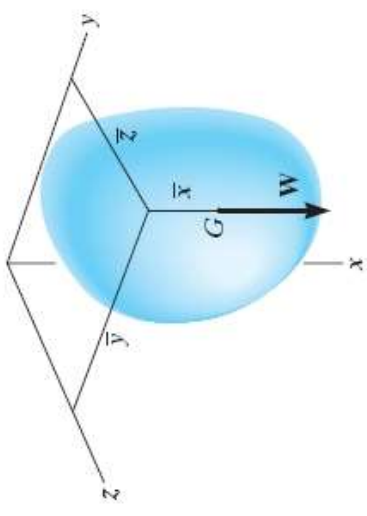
- Locates the resultant weight of a system of particles
- Consider system of n particles fixed within a region of space
- The weights of the particles can be replaced by a single (equivalent) resultant weight having defined point G of application



(a)



(b)



(c)

Center of Gravity and Center of Mass for a System of Particles

Center of Gravity

- Resultant weight = total weight of n particles

$$W_R = \sum W$$

- Sum of moments of weights of all the particles about x , y , z axes = moment of resultant weight about these axes
- Summing moments about the x axis,

$$\bar{x}W_R = \tilde{x}_1W_1 + \tilde{x}_2W_2 + \dots + \tilde{x}_nW_n$$

- Summing moments about y axis,

$$\bar{y}W_R = \tilde{y}_1W_1 + \tilde{y}_2W_2 + \dots + \tilde{y}_nW_n$$

Center of Gravity and Center of Mass for a System of Particles

Center of Gravity

- Although the weights do not produce a moment about z axis, by rotating the coordinate system 90° about x or y axis with the particles fixed in it and summing moments about the x axis,

$$\bar{z}W_R = \tilde{z}_1W_1 + \tilde{z}_2W_2 + \dots + \tilde{z}_nW_n$$

- Generally,

$$\bar{x} = \frac{\sum \tilde{x}m}{\sum m}; \bar{y} = \frac{\sum \tilde{y}m}{\sum m}, \bar{z} = \frac{\sum \tilde{z}m}{\sum m}$$

Center of Gravity and Center of Mass for a System of Particles

Center Mass

- Provided acceleration due to gravity g for every particle is constant, then $W = mg$

$$\bar{x} = \frac{\sum \tilde{x}m}{\sum m}; \bar{y} = \frac{\sum \tilde{y}m}{\sum m}, \bar{z} = \frac{\sum \tilde{z}m}{\sum m}$$

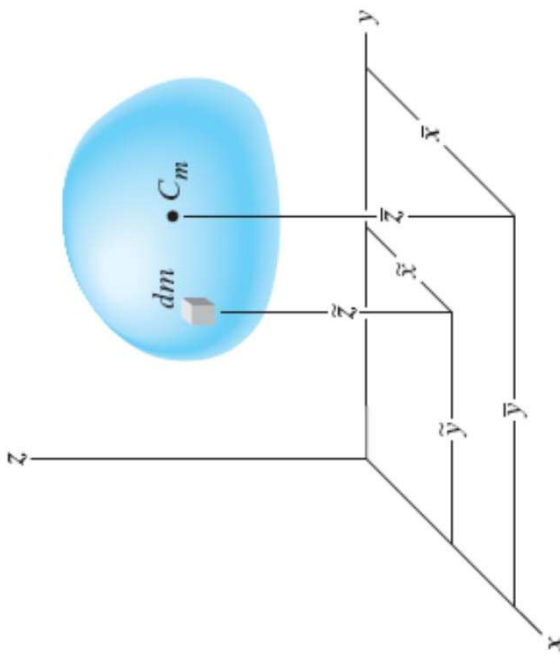
- By comparison, the location of the center of gravity coincides with that of center of mass
 - Particles have weight only when under the influence of gravitational attraction, whereas center of mass is independent of gravity
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Center of Gravity and Center of Mass for a System of Particles

Center Mass

- A rigid body is composed of an infinite number of particles
- Consider arbitrary particle having a weight of dW

$$\bar{x} = \frac{\int \tilde{x}dW}{\int dW}; \bar{y} = \frac{\int \tilde{y}dW}{\int dW}; \bar{z} = \frac{\int \tilde{z}dW}{\int dW}$$

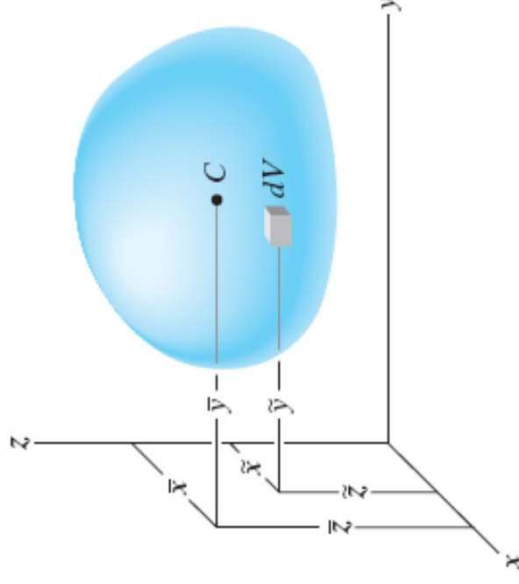


Center of Gravity and Center of Mass for a System of Particles

Centroid of a Volume

- Consider an object subdivided into volume elements dV , for location of the centroid,

$$\bar{x} = \frac{\int \tilde{x}dV}{\int dV}; \bar{y} = \frac{\int \tilde{y}dV}{\int dV}; \bar{z} = \frac{\int \tilde{z}dV}{\int dV}$$

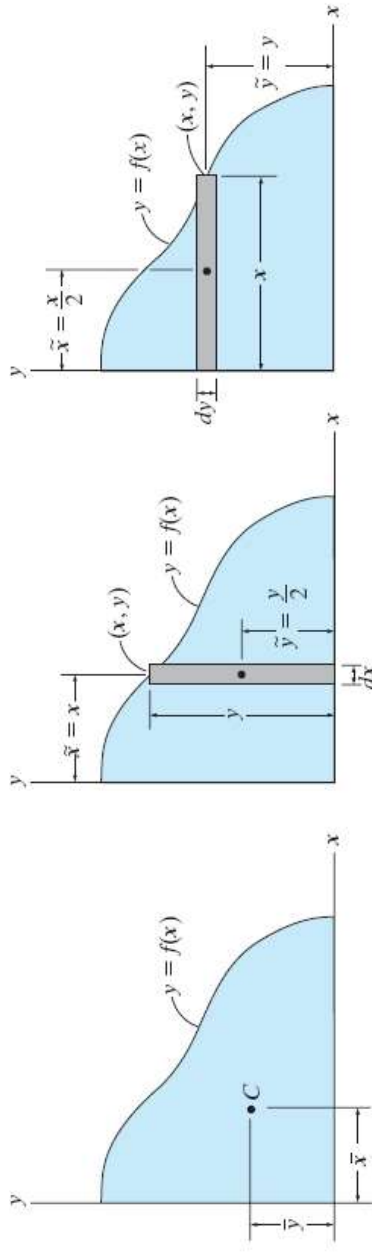


Center of Gravity and Center of Mass for a System of Particles

Centroid of an Area

- For centroid for surface area of an object, such as plate and shell, subdivide the area into differential elements dA

$$\bar{x} = \frac{\int \tilde{x} dA}{A}; \bar{y} = \frac{\int \tilde{y} dA}{A}; \bar{z} = \frac{\int \tilde{z} dA}{A}$$



(a)

(b)

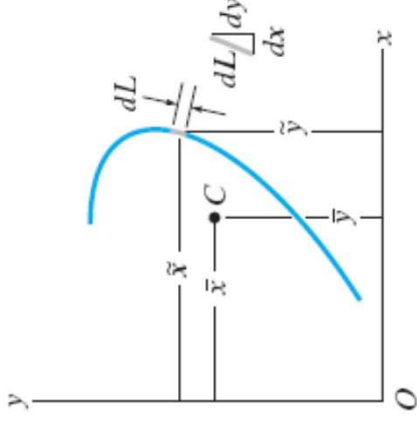
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Center of Gravity and Center of Mass for a System of Particles

Centroid of a Line

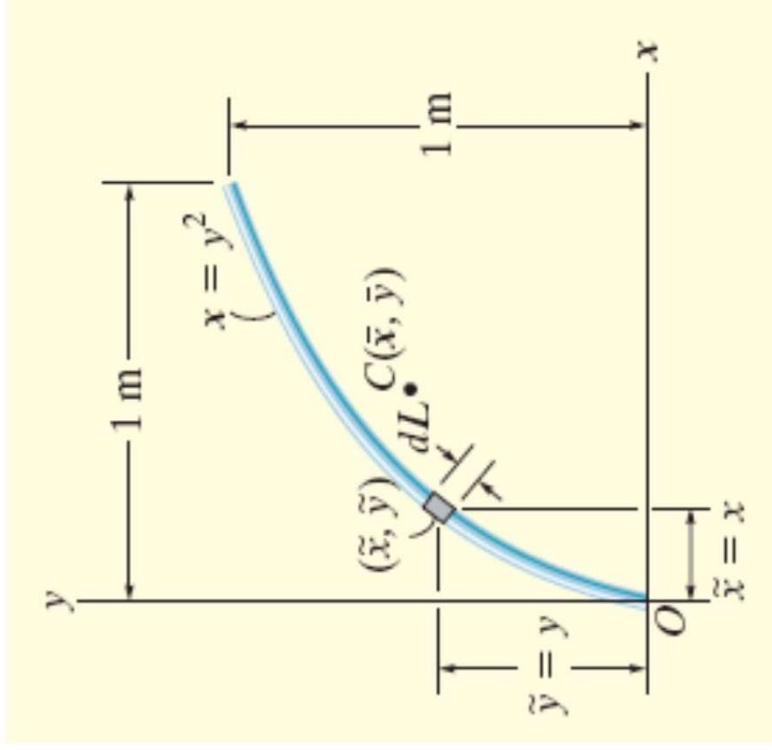
- If the geometry of the object takes the form of a line, the balance of moments of differential elements dL about each of the coordinate system yields

$$\bar{x} = \frac{\int \tilde{x} dL}{\int dL}; \bar{y} = \frac{\int \tilde{y} dL}{\int dL}; \bar{z} = \frac{\int \tilde{z} dL}{\int dL}$$



Example

Locate the centroid of the rod bent into the shape of a parabolic arc.



Example

Differential element

Located on the curve at the arbitrary point (x, y)

Area and Moment Arms

For differential length of the element dL

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} dy$$

Since $x = y^2$ and then $dx/dy = 2y$

$$dL = \sqrt{(2y)^2 + 1} dy$$

The centroid is located at

$$\tilde{x} = x, \tilde{y} = y$$

Example

Integrations

$$\begin{aligned}\bar{x} &= \frac{\int_L \tilde{x} dL}{\int_L dL} = \frac{\int_0^1 x \sqrt{4y^2 + 1} dy}{\int_0^1 \sqrt{4y^2 + 1} dy} = \frac{\int_0^1 y^2 \sqrt{4y^2 + 1} dy}{\int_0^1 \sqrt{4y^2 + 1} dy} \\ &= \frac{0.6063}{1.479} = 0.410m\end{aligned}$$

$$\begin{aligned}\bar{y} &= \frac{\int_L \tilde{y} dL}{\int_L dL} = \frac{\int_0^1 y \sqrt{4y^2 + 1} dy}{\int_0^1 \sqrt{4y^2 + 1} dy} \\ &= \frac{0.8484}{1.479} = 0.574m\end{aligned}$$

Composite Bodies

- Consists of a series of connected “simpler” shaped bodies, which may be rectangular, triangular or semicircular
- A body can be sectioned or divided into its composite parts
- Accounting for finite number of weights

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \quad \bar{y} = \frac{\sum \tilde{y}W}{\sum W} \quad \bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$



Composite Bodies

Procedure for Analysis

Composite Parts

- Divide the body or object into a finite number of composite parts that have simpler shapes
- Treat the hole in composite as an additional composite part having negative weight or size

Moment Arms

- Establish the coordinate axes and determine the coordinates of the center of gravity or centroid of each part

Composite Bodies

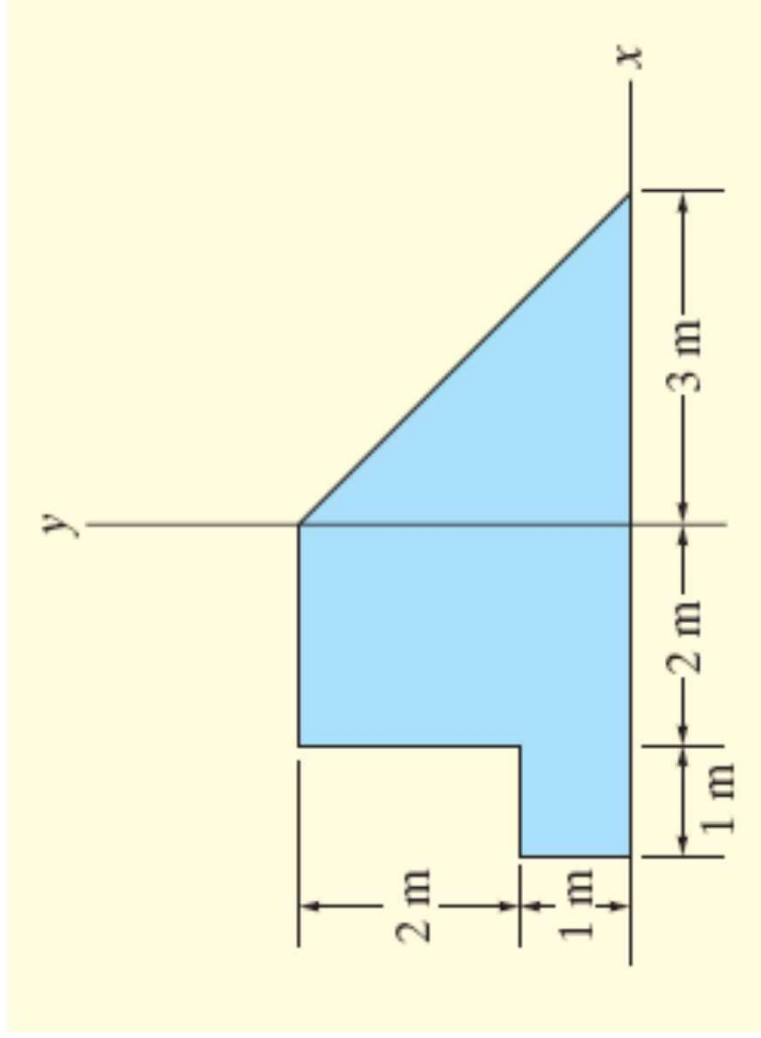
Procedure for Analysis

Summations

- Determine the coordinates of the center of gravity by applying the center of gravity equations
 - If an object is symmetrical about an axis, the centroid of the objects lies on the axis
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Example

Locate the centroid of the plate area.

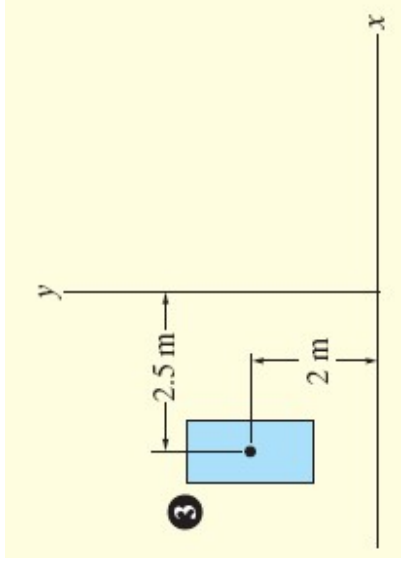
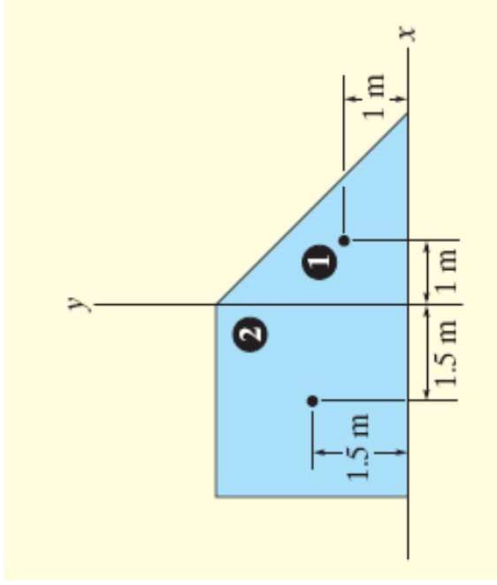


Solution

Composite Parts

Plate divided into 3 segments.

Area of small rectangle considered "negative".



Solution

Moment Arm

Location of the centroid for each piece is determined and indicated in the diagram.

Segment	A (m ²)	\tilde{x} (m)	\tilde{y} (m)	$\tilde{x}A$ (m ³)	$\tilde{y}A$ (m ³)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	-1.5	1.5	-13.5	13.5
3	$-(2)(1) = -2$	-2.5	2	5	-4
	$\Sigma A = 11.5$			$\Sigma \tilde{x}A = -4$	$\Sigma \tilde{y}A = 14$

Summations

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348mm$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22mm$$