

Session 21

Moment of Inertia

- It is the double moment of area or mass about an axis so that one is called area moment of inertia and the other is called mass moment of inertia.
- Area moment of inertia is essentially the capacity of a cross-section to resist bending and is applied to bending problems to calculate deflection or deformation of members.
- Mass moment of inertia is the resistance of the object to changes in its rotation and is applied to rotation of rigid bodies.
- Area moment of inertia = Area x (distance)² = $A.x^2$
- Mass moment of inertia = Mass x (distance)² = $m.x^2$
- It must be specified with respect to a chosen axis of rotation.
- It is usually quantified in m⁴ or kgm².

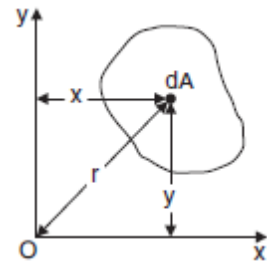
Rectangular and Polar Moment of Inertia

Moment of inertia about an axis lying in the plane of an area is known as *rectangular moment of inertia*. It may be denoted as I_{xx} or I_{yy} . Moment of inertia about an axis perpendicular to the plane of an area is known as *polar moment of inertia*. It may be denoted as J or I_{zz} . Thus different type of moment of inertia of a plane area is given by;

$$I_{xx} = \sum dA.y^2$$

$$I_{yy} = \sum dA.x^2$$

$$I_{zz} = \sum dA.r^2$$



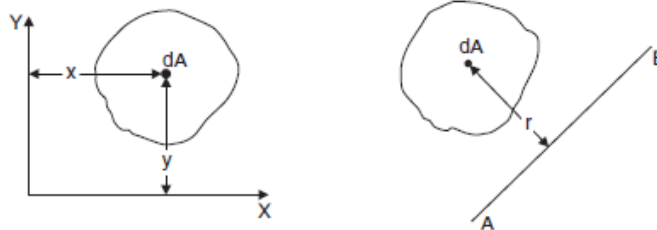
Moment of Inertia by Integration

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA = Area of the strip

x = Distance of the centre of gravity of the strip on X-X axis

y = Distance of the centre of gravity of the strip on Y-Y axis.



We know that the moment of inertia of the strip about Y-Y axis

$$= dA \cdot x^2$$

The moment of inertia of the whole area is found out by integrating above equation. *i.e.*,

$$I_{YY} = \sum dA \cdot x^2$$

Similarly $I_{XX} = \sum dA \cdot y^2$

In general, if r is the distance of elemental area dA from the axis AB , the sum of the terms $\sum dA \cdot r^2$ is the moment of inertia of the entire area about the axis AB . Thus

$$I_{AB} = \sum r_i^2 dA_i = \int r^2 dA$$