Session 22

Perpendicular Axis Theorem

The moment of inertia (MI) of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis. Mathematically, $I_{zz} = I_{xx} + I_{yy}$



The above theorem can be easily proved. Let us consider an elemental area dA at a distance r from O. Let the coordinates of dA be x and y. Then from definition:

$$I_{zz} = \sum r^2 dA$$

= $\sum (x^2 + y^2) dA$
= $\sum x^2 dA + \sum y^2 dA$
 $I_{zz} = I_{xx} + I_{yy}$

Parallel Axis Theorem

The moment of area of an object about any axis parallel to the centroidal axis is the sum of MI about its centroidal axis and the product of area with the square of distance from the reference axis. Mathematically, $I_{AB} = I_G + Ah^2$

where I_{AB} = moment of inertia about axis AB

 I_{GG} = moment of inertia about centroidal axis GG parallel to AB.

A = the area of the plane figure given and

h = the distance between the axis AB and the parallel centroidal axis GG.

Note: The above equation cannot be applied to any two parallel axes. One of the axes (GG) must be centroidal axis only.

