

---

# Chapter 3

## Equilibrium of concurrent forces

# Objective

---

- ◆ To know how to apply the principle of force equilibrium in static analysis
- ◆ To practice how to construct free-body diagrams

# Some assumptions (just for this chapter)

---

- ◆ The loaded object can be reduced into a dimensionless particle
- ◆ The shape of the object does not affect the response
- ◆ The mass of the object is concentrated to this particle
- ◆ The forces are concurrent
- ◆ The lines of action pass the particle point

# Force equilibrium (mechanical eql.)

---

- ◆ (Mechanical) equilibrium requires that the concurrent forces that act on the body satisfy

$$\mathbf{R} = \sum \mathbf{F} = 0$$

- ◆ The particle in a equilibrium system must satisfy

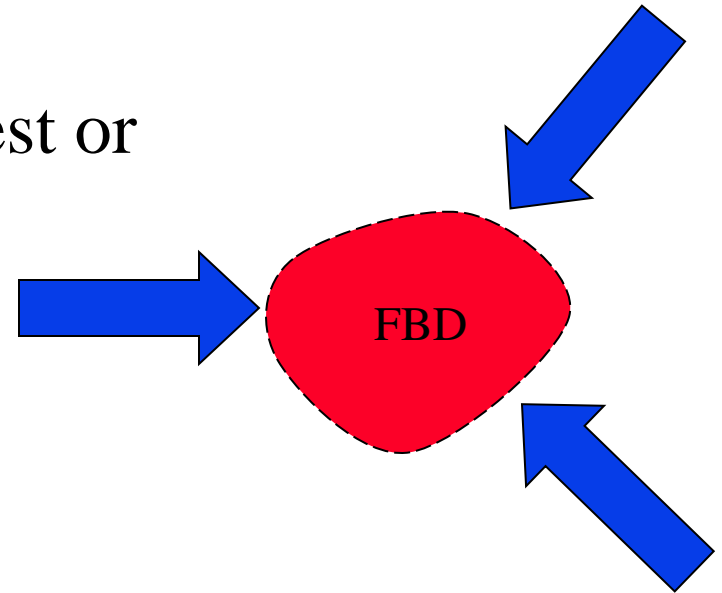
$$\mathbf{R} = \sum \mathbf{F} = m \cdot \mathbf{a}$$

- ◆ Since both must be satisfied, the material point then must have zero acceleration,  $\mathbf{a} = 0$

# Free body diagrams

---

- ◆ Inside the FBD
  - A portion of the body of interest or
  - A full body of interest or
  - A group of bodies of interest
  - Body forces
- ◆ On the boundary of the FBD
  - “Replacement” forces/distributed forces



# How to make the FBD

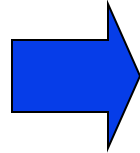
---

- ◆ Select and draw bodies or parties of interest to be shown in the FBD
- ◆ Identify and draw the body and surface forces applied on the bodies
- ◆ Discard contacting bodies that are not wanted and replace them with the forces they exert to the body of interest
- ◆ Cut the body parts that are not wanted and replace them with internal forces they exert to the body of interest
- ◆ Decide and locate a coordinate system

# Discarded bodies and the replacement forces

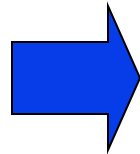
---

◆ Contacting bodies



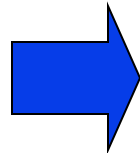
◆ Forces normal and/or tangential to the FBD surfaces

◆ String/cable/axially loaded members



◆ Forces with lines of actions that coincide with the member axis

◆ Body parts



◆ A distributed force called stress

# Example: basic problem

---

Block  $A$  of Fig. P3-3 rests on a smooth (frictionless) surface. If the block weighs 25 lb, determine the force exerted on the block by the surface and the force  $P$  parallel to the surface that is required to prevent motion of the block.

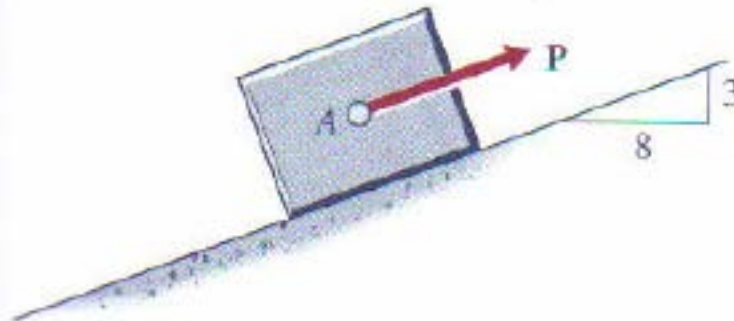
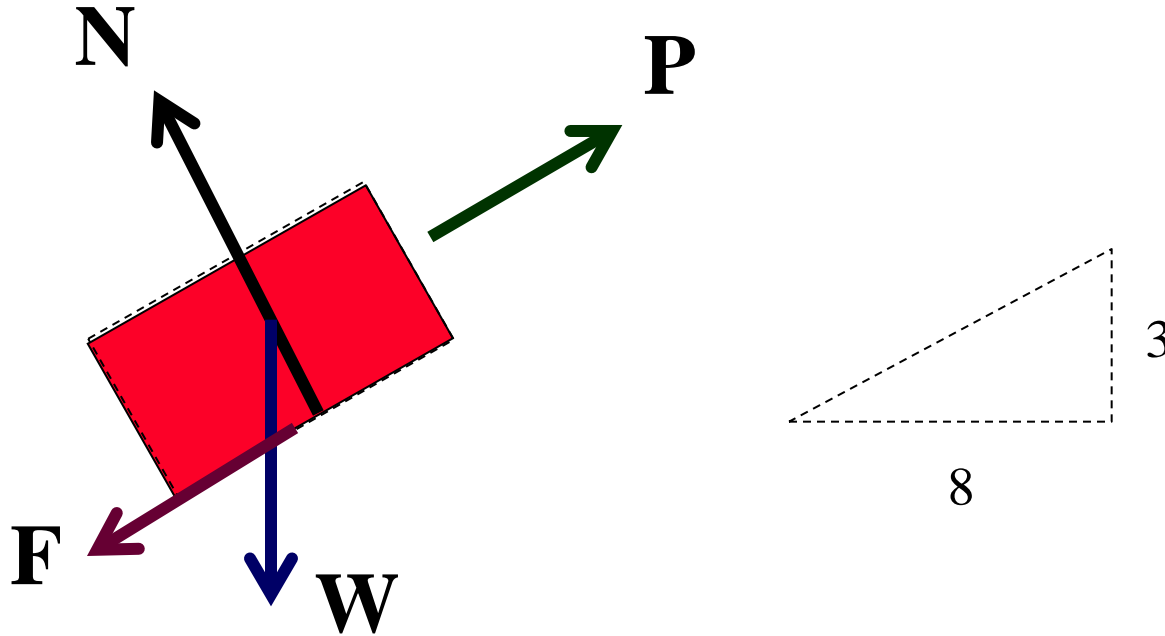


Figure P3-3



# The free body diagram

---



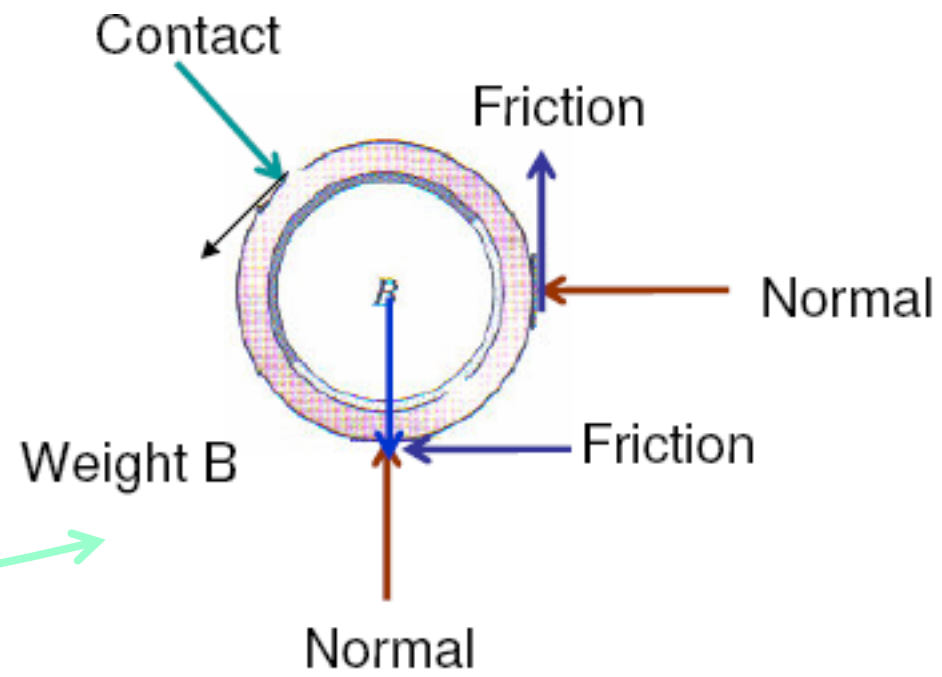
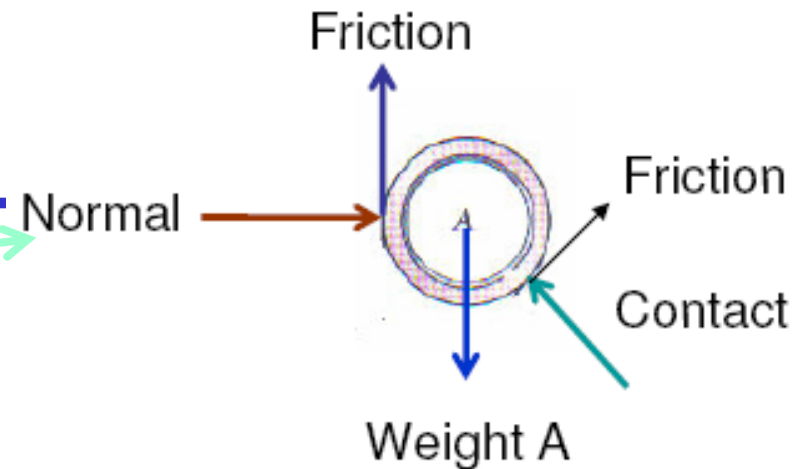
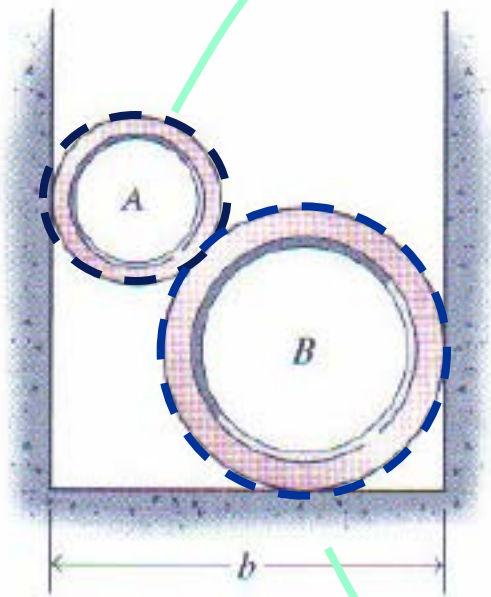
$N$  = normal force  $\rightarrow$  perpendicular to the removed surface

$W$  = weight of the box  $\rightarrow$  downward

$P$  = the applied force

$F$  = friction force  $\rightarrow$  opposite to the direction of the motion/applied forces

# Example: contact problem



Frictions on the same contact point but different FBDs are in the opposite direction.

# Example: cable tension

A 750-kg body is supported by the flexible cable system shown in Fig. P3-4. Determine the tensions in cables AC, BC, and CD.

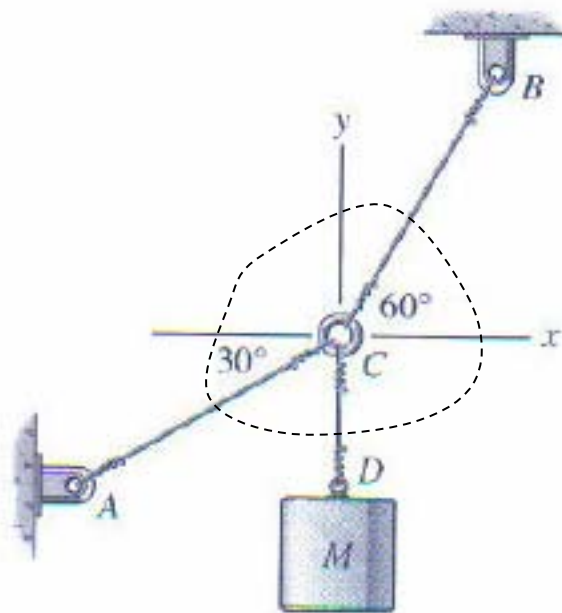
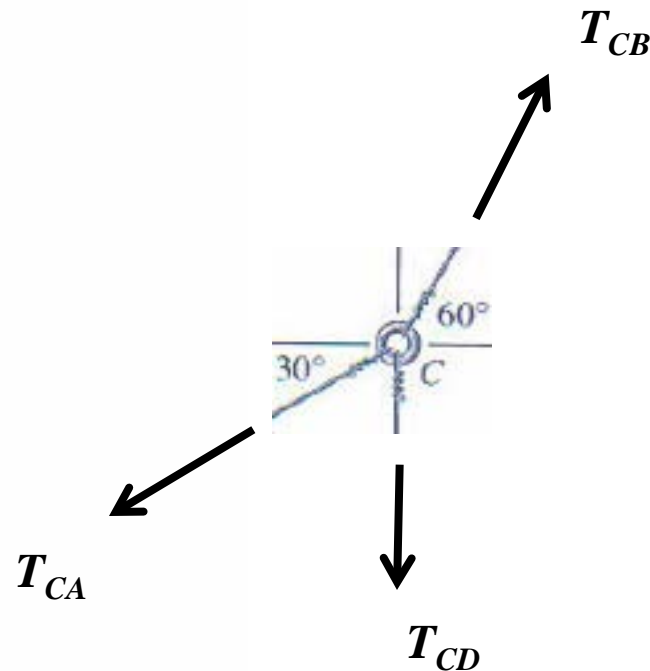


Figure P3-4

Cable can only sustain tension



$T = \text{Cable tension} \rightarrow$  outward from the FBD

# Example: Pulley problem

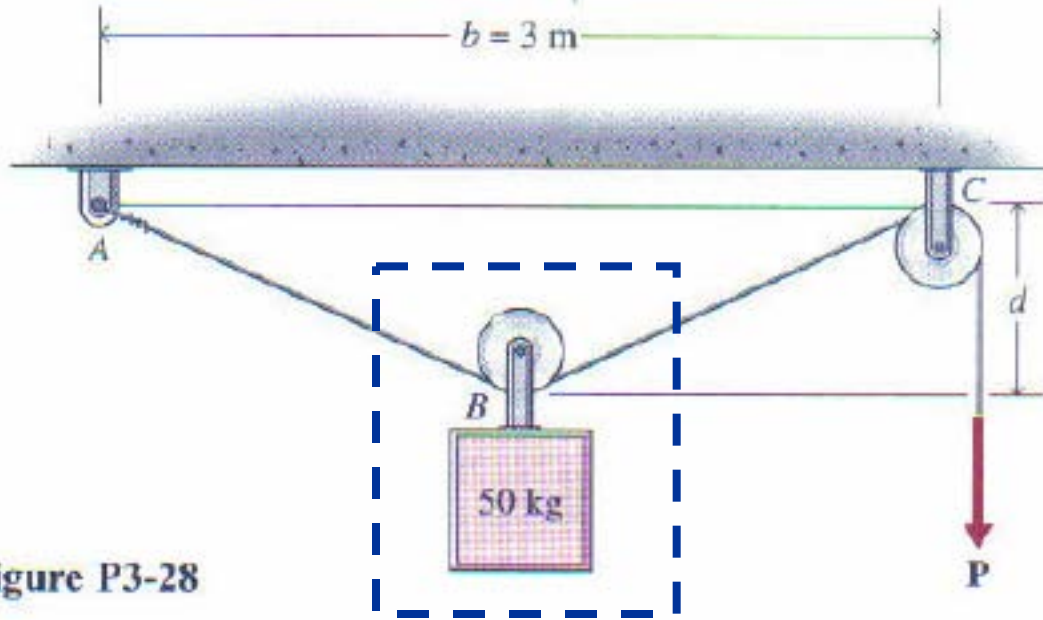
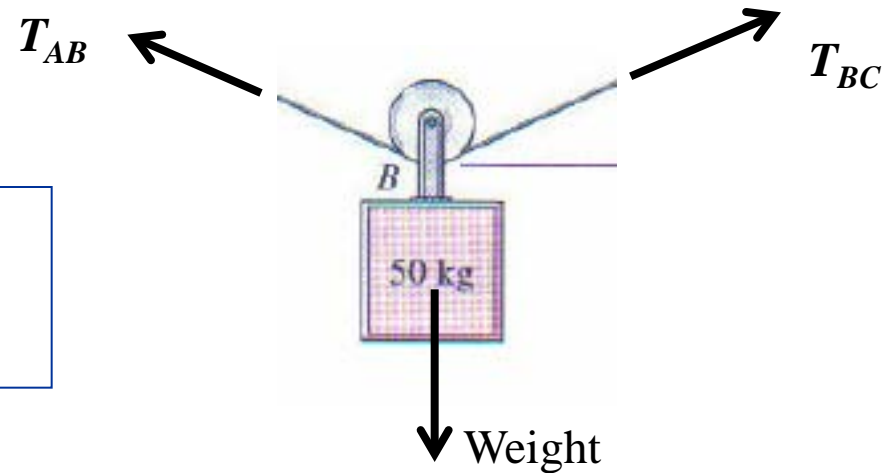


Figure P3-28



For pulley:

$$T_{AB} = T_{BC} = T$$

# Equilibrium of a Particle

---

$$\bar{R} = \sum \bar{F} = 0$$



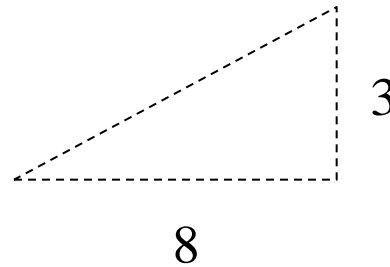
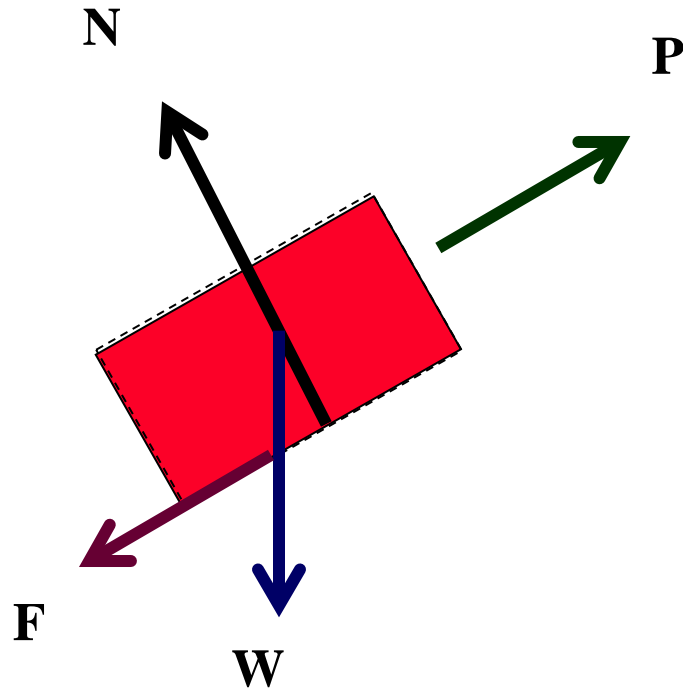
$$\begin{aligned} R &= R_x \bar{i} + R_y \bar{j} + R_z \bar{k} \\ &= \sum F_x \bar{i} + \sum F_y \bar{j} + \sum F_z \bar{k} \\ &= 0 \end{aligned}$$



$$R_x = \sum F_x = 0 \quad R_y = \sum F_y = 0 \quad R_z = \sum F_z = 0$$

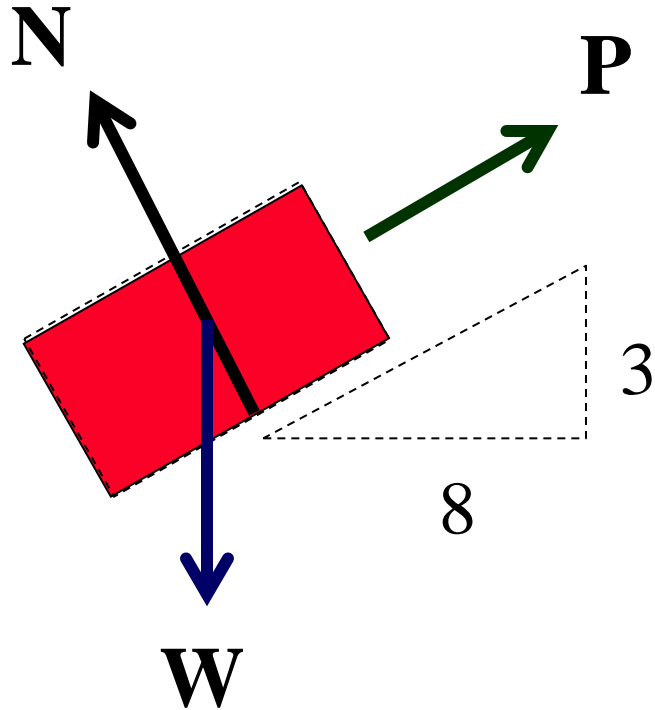
# Example 1: Static analysis

---



# Static analysis

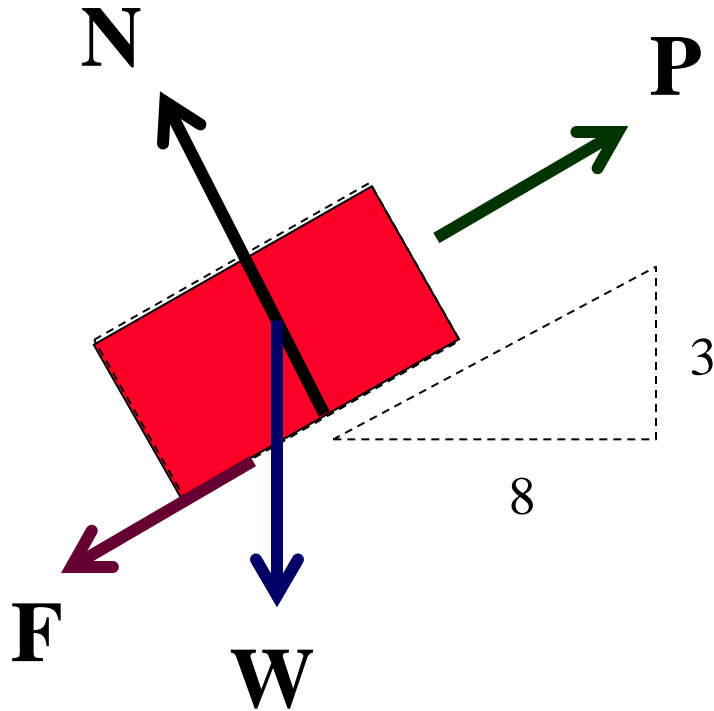
---



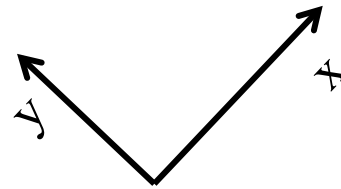
- ◆ Some simplification:
  - Note that in this case,  $F = 0$  (frictionless/smooth surface)
  - The problem is 2D

# Static analysis

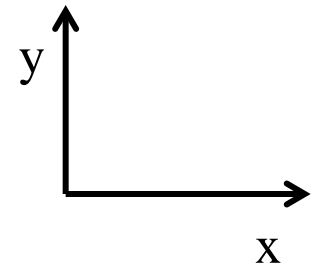
---



- ◆ Select and position a coordinate system

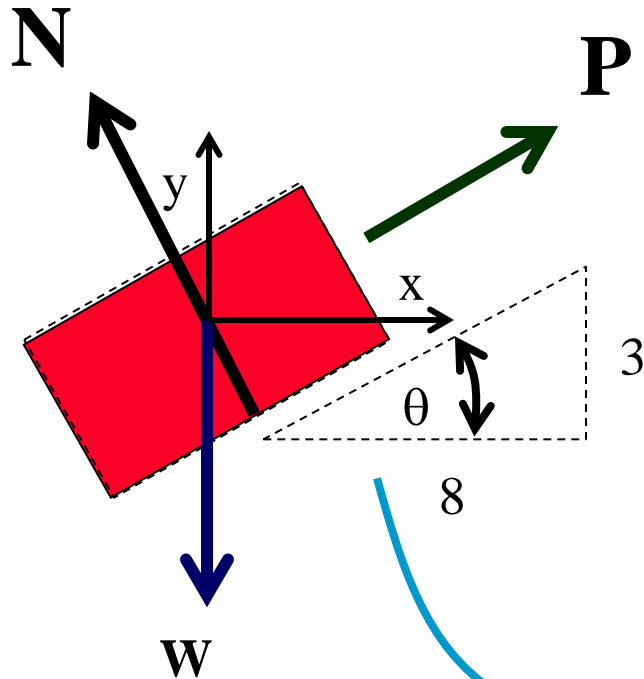


or

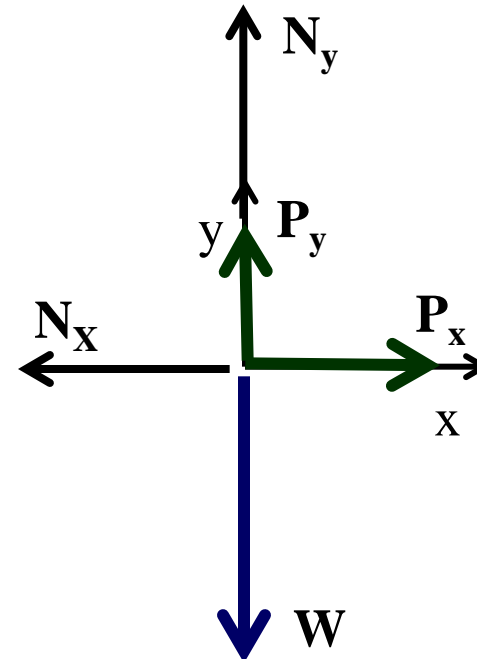




# Static analysis



- ◆ Find a force system that is parallel to the selected coordinate system and equal to the original system ~ find vectorial components of the forces in the selected system



$$\begin{aligned} P_x &= P \cos \theta & P_y &= P \sin \theta \\ N_x &= N \sin \theta & N_y &= N \cos \theta \end{aligned}$$

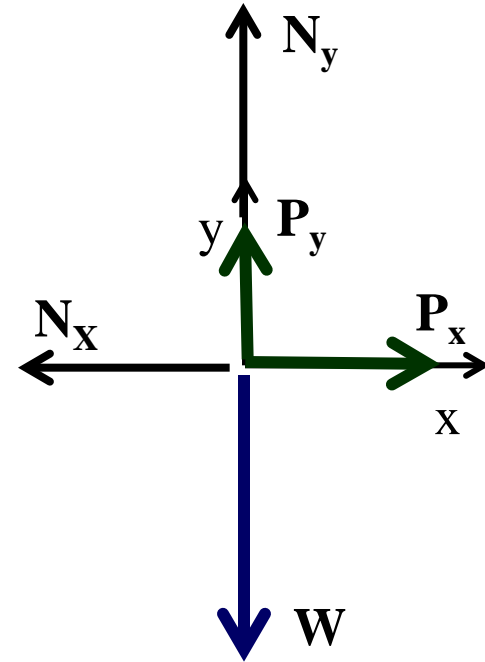
# Static analysis

---

- ◆ Sum the forces (according to their directions) and equate each of them to zero

$$\sum F_x = P_x - N_x = 0$$

$$\sum F_y = P_y + N_y - W = 0$$



# Static analysis

---

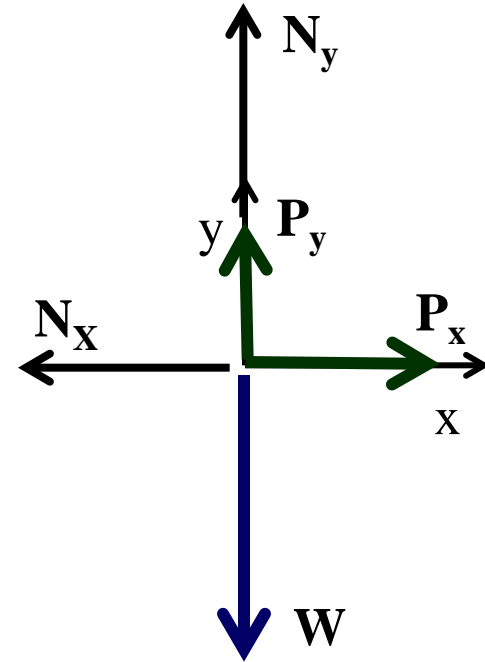
- ◆ For this problem,
- ◆  $W = 25$  lbs
- ◆  $P = ?$  and  $N = ?$
- ◆ 2 unknowns
- ◆ 2 equations

$$\sum F_x = P_x - N_x = 0$$

$$\sum F_y = P_y + N_y - W = 0$$

$$P_x = P \cos \theta \quad P_y = P \sin \theta$$

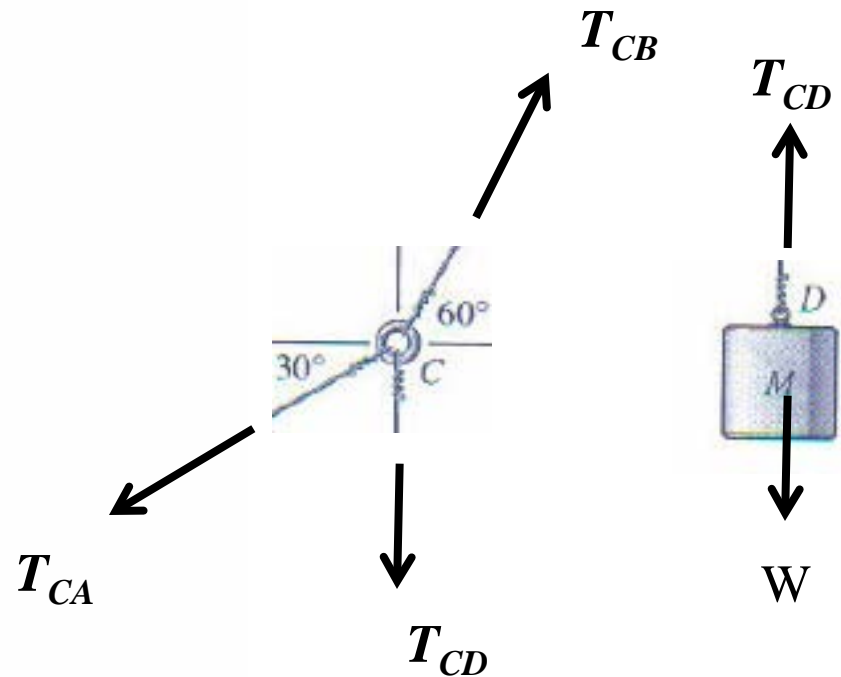
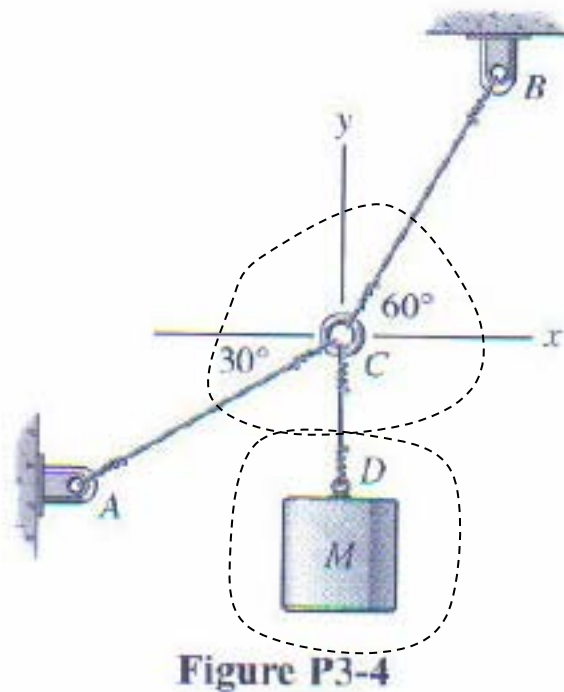
$$N_x = N \sin \theta \quad N_y = N \cos \theta$$



# Example: cable tension

A 750-kg body is supported by the flexible cable system shown in Fig. P3-4. Determine the tensions in cables AC, BC, and CD.

Note the direction of  $T_{CD}$  in the two FBDs



$T$  = Cable tension  $\rightarrow$  outward from the FBD  
Department of Mechanical Engineering

# Example: cable tension

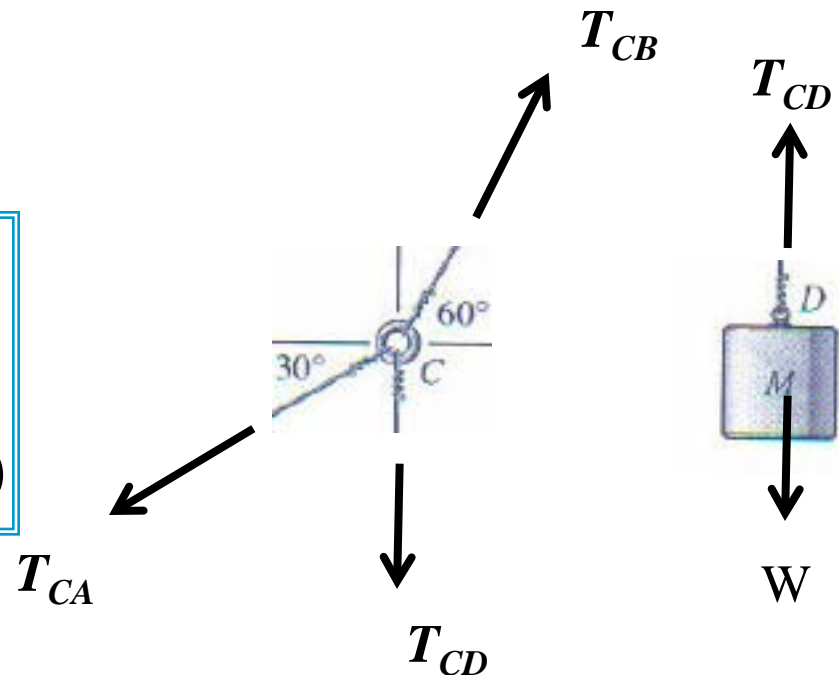
- ◆ The equilibrium equations
- ◆ 3 unknowns
- ◆ 3 equations

Note the direction of  $T_{CD}$  in the two FBDs

$$T_{CD} = W$$

$$T_{CB} \cos 60 = T_{CA} \cos 30$$

$$T_{CB} \sin 60 = T_{CD} + T_{CA} \sin 30$$



$T$  = Cable tension  $\rightarrow$  outward from the FBD

# Exempl

3-20 A mass  $m$  is to be supported by two cables ( $A$  and  $B$ ) as shown Fig. P3-20. If the maximum force that the cables can withstand is 15 kN, determine the maximum mass  $m$  that can be supported.

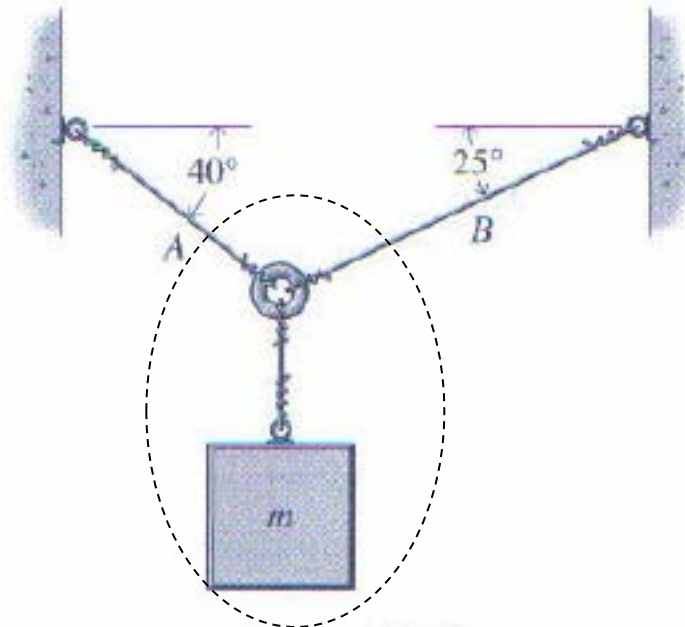
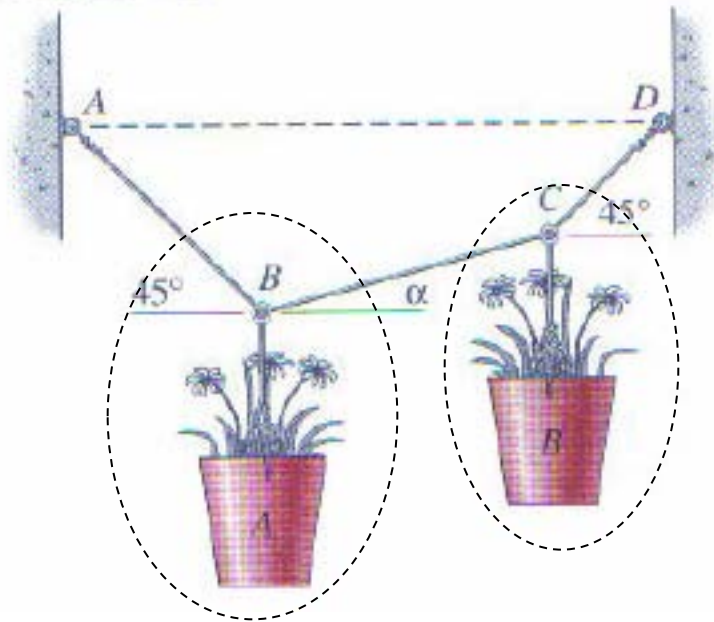


Figure P3-20

- ◆ Note: Cables  $A$  and  $B$  do not experience the same tension

# Exam

3-9\* Two flower pots are supported with cables as shown in Fig. P3-9. If pot  $A$  weighs 10 lb and pot  $B$  weighs 8 lb, determine the tension in each of the cables and the slope of cable  $BC$ .



Note:

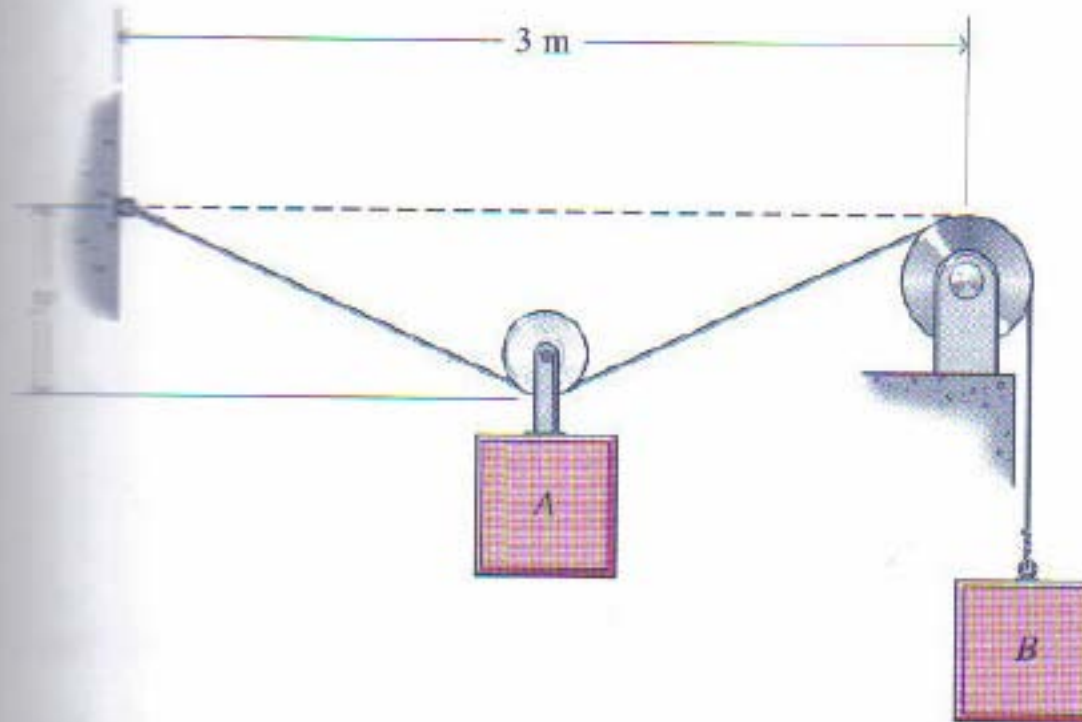
4 unknowns = 3 tensions + 1 angle

4 equations = 2 x 2 equations

Remember: tension direction is always leaving the FBD

# Exam

3-16 A continuous cable is used to support two blocks as shown in Fig. P3-16. Block  $A$  is supported by a small wheel that is free to roll on the cable. Determine the displacement  $y$  of block  $A$  for equilibrium if the masses of blocks  $A$  and  $B$  are 22 kg and 34 kg, respectively.

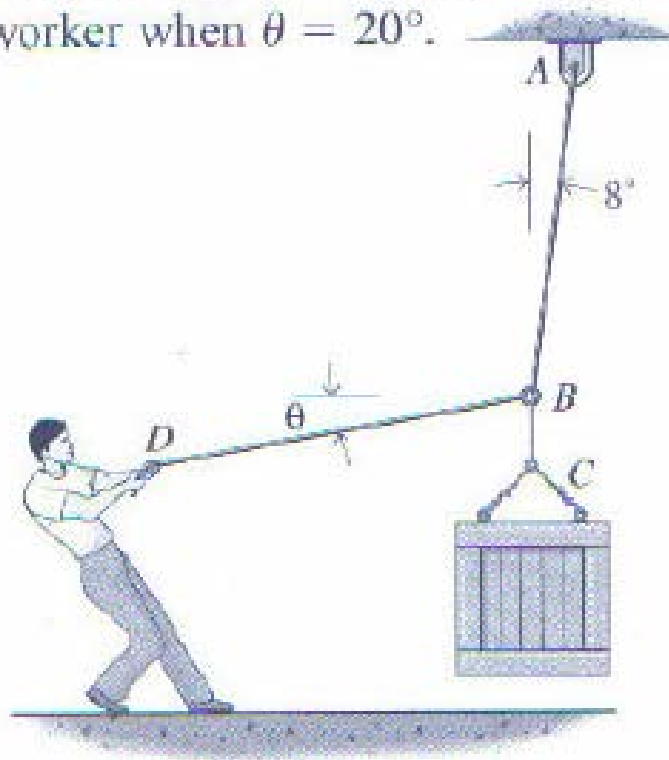




# Example:

---

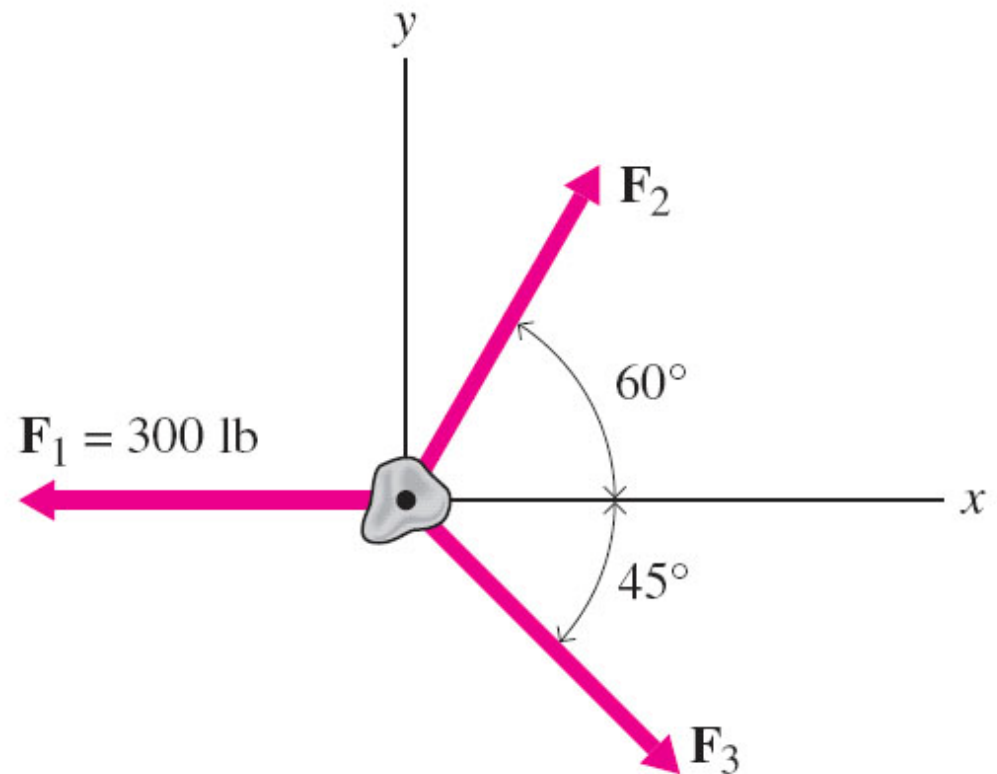
3-14\* In order to hold a 130-kg crate in a stationary position, a worker exerts a force  $\mathbf{P}$  at an angle  $\theta$  on a rope as shown in Fig. P3-14. Determine the force exerted by the worker when  $\theta = 20^\circ$ .



# Example

---

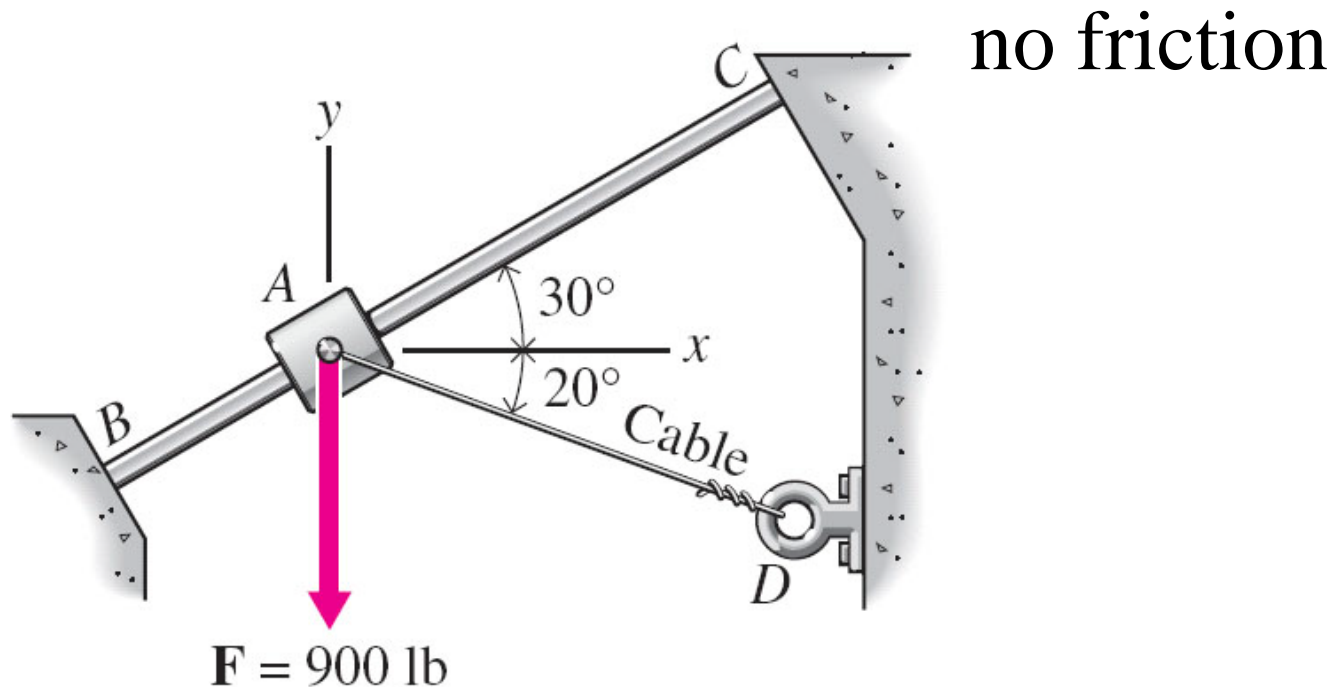
- ◆ Find  $\underline{\mathbf{F}}_2$  and  $\underline{\mathbf{F}}_3$  so that  $\underline{\mathbf{R}} = 0$



# Example

---

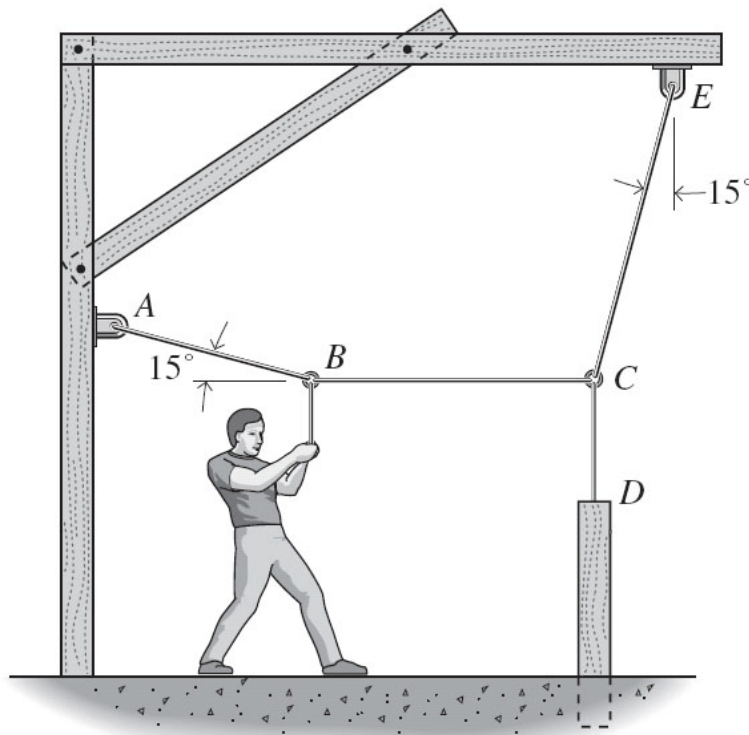
- ◆ Find cable tension and normal force exerted by the rod BC to the collar



# Example

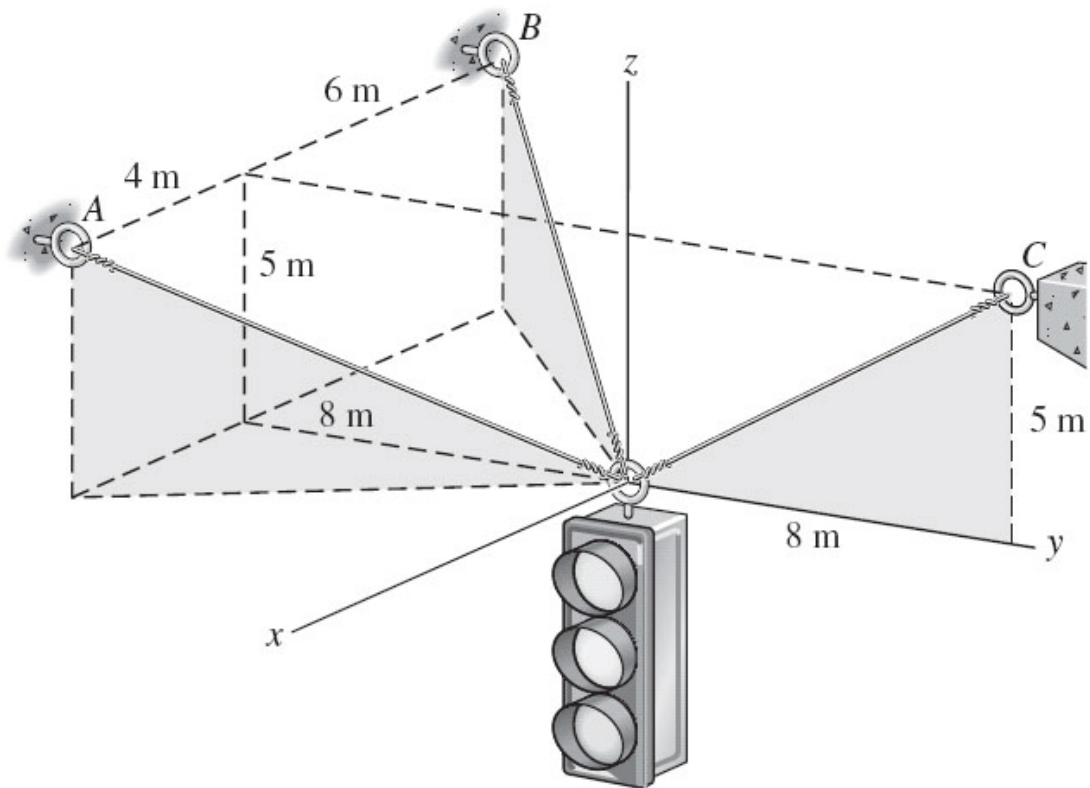
---

- ◆ How much (vertical) force the man must apply if  $T_{CD} = 2000 \text{ lb}$ ?



# Example

- ◆ If the light is 1000 kg, what are the cable tensions?



# Summary

---

The particle is in *equilibrium* when the resultant of all forces acting on it is zero.

To solve a problem involving a particle in equilibrium, draw a *free-body diagram* showing all the forces acting on the particle. The conditions which must be satisfied for particle equilibrium are

$$R_x = \sum F_x = 0 \quad R_y = \sum F_y = 0 \quad R_z = \sum F_z = 0$$

In *two-dimensions*, only two of these equations are needed

$$\sum F_x = 0 \quad \sum F_y = 0$$