Module-2

Lecture-6

Cruise Flight - Power required, Velocity for Minimum Power required

Cruise flight: power required & velocity for minimum power required

$$P_{req} = T_{req}V = DV = \frac{WV}{\frac{C_L}{C_D}} = \sqrt{\frac{2W^3C_D^2}{S\rho C_L^3}} \tag{1}$$

$$\Rightarrow P_{req} \propto \frac{1}{\frac{C_L^{\frac{3}{2}}}{C_D}}$$

Thus, following conclusions can be made from the above expression:

- For power required to be minimum, $C_L^{3/2}/C_D$ should be maximum.
- The airplane needs to be flown at C_L such that $C_L^{3/2}/C_D$ is maximum.
- The airplane needs to have sufficient speed so that the lift produced by the aircraft at C_L corresponding to C_L for $\left[C_L^{3/2}/C_D\right]_{max}$ is able to balance the weight of the aircraft.

Now an interesting question arises that how the power required is dependent on the velocity of an aircraft?

- In order to find an answer to this question let us write Equation 1 in another way

$$P_{req} = DV = \bar{q}SVC_D = \left(C_{D_o} + \frac{C_L^2}{\pi ARe}\right)\bar{q}SV$$

$$\Rightarrow P_{req} = \left\{C_{D_o} + \frac{\left(\frac{W}{\bar{q}S}\right)^2}{\pi ARe}\right\}\bar{q}SV$$

$$P_{req} = \frac{1}{2}\rho V^3 SC_{D_o} + \frac{\frac{W^2}{\frac{1}{2}\rho VS}}{\pi ARe} \tag{2}$$

A typical variation of power required, P_{req} with velocity is presented in Figure 1. Referring to the figure it can easily be interpretted that there is a particular speed at which power required to maintain level flight is minimum.

- For minimum power required,

$$\frac{\partial P_{req}}{\partial V} = 0$$

$$\frac{\partial^2 P_{req}}{\partial V^2} > 0$$

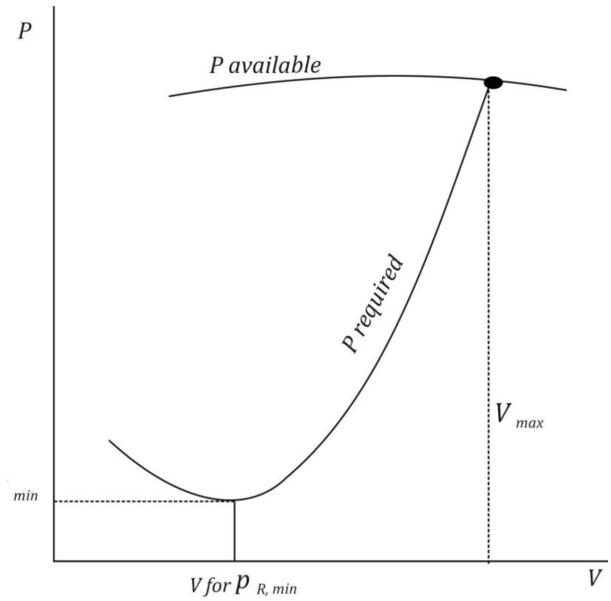

Figure 1: Power available and power required as a function of speed

- So differentiating Equation 1 and equating it to zero, we will get:

$$\frac{\partial P_{req}}{\partial V} = \frac{3}{2}\rho V^2 S C_{D_o} - \frac{\frac{W^2}{\frac{1}{2}\rho V^2 S}}{\pi A R e} = 0$$

$$\Rightarrow \frac{3}{2}\rho V^2 S \left(C_{D_o} - \frac{\frac{W^2}{\frac{3}{4}\rho^2 V^4 S^2}}{\pi A R e} \right) = 0$$

$$\Rightarrow \frac{3}{2}\rho V^2 S \left(C_{D_o} - \frac{\frac{1}{3} C_L^2}{\pi A R e} \right) = 0$$

$$\Rightarrow C_{D_o} - \frac{1}{3} C_{D_i} = 0$$

where C_{D_i} is the induced drag.

- So the aerodynamic condition for minimum power required is

$$C_{D_i} = 3 C_{D_o} \tag{3}$$

- Now to calculate CL for minimum power required we know that, induced drag coefficient,

$$C_{D_i} = K C_L^2$$

Using Equation 2 we can write:

$$K C_L^2 = 3 C_{D_o}$$

$$C_{L_{minPreq}} = \sqrt{\frac{3 C_{D_o}}{K}} \tag{4}$$

Once we get $C_{L_{minPreq}}$, then what is the velocity for cruise at which $C_L^{3/2}/C_D$ value is maximum (for minimum power required)?

- As it is well known that, lift needs to balance weight in cruise. So,

$$L = \frac{1}{2}\rho V^2 S C_L = W$$

- This could also be written as

$$W = \frac{1}{2}\rho V^2 S C_{L_{minPreq}}$$

$$\Rightarrow V = \sqrt{\frac{2\left(\frac{W}{S}\right)}{\rho C_{L_{minPreq}}}}$$

- So by using the result obtained in Equation 3 the velocity for minimum power required can be shown to be:

$$V_{minP_{req}} = \sqrt{\frac{2\left(\frac{W}{S}\right)}{\rho\sqrt{\frac{3C_{D_o}}{K}}}} \tag{5}$$

- Hence once the altitude is decided, the pilot should be instructed to trim the airplane at $V_{minP_{req}}$ to satisfy the minimum power required condition.

- This speed at a given altitude could easily be obtained by substituting the values of W/S, C_{D_o}, K in Equation 5.