

A decorative frame consisting of two thick black L-shaped lines. One L-shape is on the left, with a vertical line extending downwards and a horizontal line extending to the right. The other L-shape is on the right, with a vertical line extending upwards and a horizontal line extending to the left. They meet at the top and bottom corners, framing the central text.

NORMAL DISTRIBUTION

Dr Bibhunandini Das

Normal Distribution

- Normal distribution- Normal probability distribution
- Most useful theoretical distribution for continuous variables
- First described by Abraham Demoivre
- The normal distribution is an approximation to binomial distribution
- The normal curve is represented in several forms

Normal Distribution

- The basic form relating to the curve with mean μ and standard deviation σ :

- $$P(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- X = Values of the continuous random variable
- μ = Mean of the normal random variable
- e = Mathematical constant approximated by 2.7183
- π = Mathematical constant approximated by 3.1416
 - $(\sqrt{2\pi} = 2.5066)$

Graphs of Normal Distribution

- The normal distribution can have different shapes depending on different values of μ and σ but there is one and only one normal distribution for any given pair of values for μ and σ
- Normal distribution is a limiting case of binomial distribution when (
 - $n \rightarrow \infty$
 - Neither p nor q is very small)

Graphs of Normal Distribution

- Normal distribution is a limiting case of Poisson distribution when its mean m is large.
- The mean of a normally distributed population lies at the centre of its normal curve
- The mean of a normally distributed population lies at the centre of its normal curve
- The two tails of the normal probability distribution extend infinitely and never touch the horizontal axis

Properties of Normal Distribution

- The normal curve is bell-shaped and symmetrical in its appearance
- The height of the normal curve is at its maximum at the mean. The mean and mode of the normal distribution coincides
- There is one maximum point of the normal curve which occurs at the mean
- One maximum point- so the normal curve is unimodal (it has only one mode)
- The point of inflexion, i.e., the points where the change in curvature occurs are $\bar{X} \pm \sigma$

Properties of Normal Distribution

- The variable distributed according to the normal curve is a continuous one
- The first and third quartiles are equidistant from the median
- The mean deviation is 4th or more precisely 0.7979 of the standard deviation
- The area under the normal curve distributed as:
 - Mean $\pm 1\sigma$ covers 68.27% area (34.135% area will lie on either side of the mean)
 - Mean $\pm 2\sigma$ covers 95.45% area
 - Mean $\pm 3\sigma$ covers 99.73% area

Constants of the Normal Distribution

- The mean of the normal distribution is \bar{X}
- The standard deviation of the normal distribution is σ

- $\mu_2 = \sigma^2, \mu_3 = 0, \mu_4 = 3\sigma^2$

- β_1 or coefficient of skewness

- $\beta_1 = \frac{\mu_3^2}{\mu_2^2} = 0$

- β_2 or moment coefficient of kurtosis

- $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\sigma^4}{\sigma^4} = 3$

Central Limit Theorem

What is Central Limit Theorem?

- The sampling distribution of the sample means approaches a normal distribution as the sample size gets larger
- Especially true for sample sizes over 30
- More samples, especially large ones, the graph of sample means look like a normal distribution

The Central Limit Theorem and Means

- The average of sample means will be the population mean
- Add up the means of all the samples- find the average
- That average will be the actual population mean
- Average of all the standard deviation of all the samples- actual standard deviation of the population