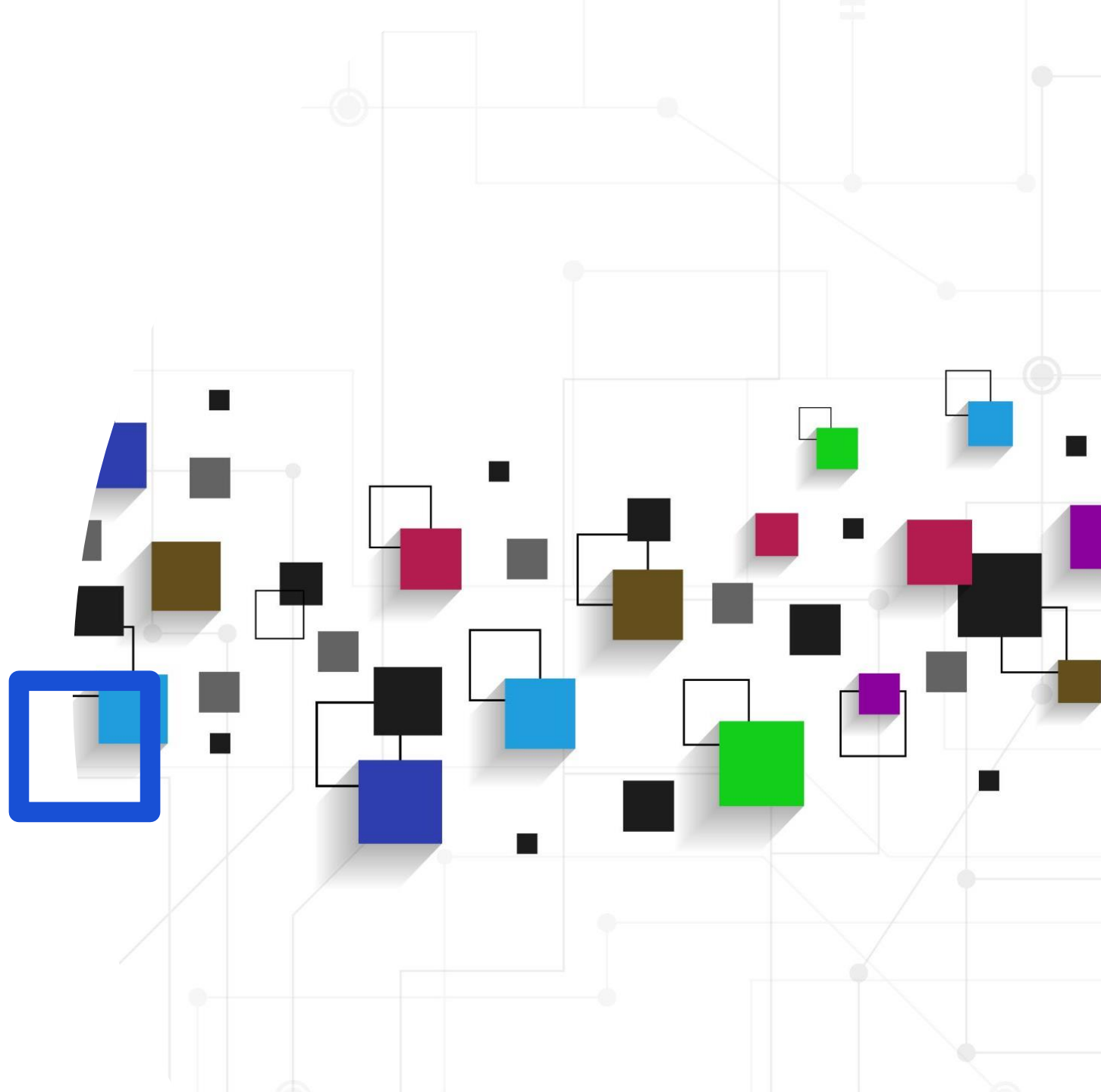


Poisson Distribution

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Poisson Probability Distribution

- Poison- named after the French mathematician S. Poison
- Widely used for a discrete random variable
- The Poison process measures the number of occurrences of an outcome of a discrete random variable in a predetermined **time interval, space or volume**, for which the **average number** of occurrences is known or can be determined

Poisson Probability Distribution

- The Poisson probability distribution is an approximation to a binomial distribution when the probability of success, p is very small and n is large
- $\mu = np$ is small, preferably $np > 7$
- Often called the *law of improbable events*

Poisson Probability Distribution

- Implies that the probability, p , of a particular outcomes occurrence is very small
- Poisson distribution occurs in a business situations-there are only few successes in an interval of time against a large number of failures and vice versa
- Has single independent outcomes that are mutually exclusive

Conditions for Poisson Process

- The outcomes in any interval of time occur randomly and independently of one another
- The probability of occurrence of an outcome in a small interval of time is proportional to the length of the interval but is independent of the specific time interval

Conditions for Poisson Process

- The probability of occurrence of more than one outcome in a small interval of time is proportional to the length of the interval but is independent of the specific time interval
- The average number of occurrence of outcomes is constant in equal intervals of time

Poisson Distribution

- $P(r) = \frac{e^{-m} m^r}{r!}$

- Where $r = 0, 1, 2, 3, 4, \dots$
- $e = 2.7183$ (the base of natural logarithms)
- $m =$ the mean of the Poisson distribution (np) or the average number of occurrences of an event

Poisson Distribution

- The Poisson probability distribution is concerned with certain processes that can be described by a discrete random variable
- The probabilities of 0, 1, 2.....successes are given by the successive terms of the expansion

$$\bullet e^{-m} \left(1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots + \frac{m^r}{r!} + \dots \right)$$

Poisson Distribution

Number of Successes (X)	Probability P(X)	Number of Successes (X)	Probability P(X)
0	e^{-m}	4	$\frac{m^4 e^{-m}}{4!}$
1	me^{-m}	:	:
2	$\frac{m^2 e^{-m}}{2!}$	r	$\frac{m^r e^{-m}}{r!}$
3	$\frac{m^3 e^{-m}}{3!}$:	:

Constants of the Poisson Distribution

- Since p is very small in case of Poisson distribution, the value of e^{-np} is almost equal to 1
- The constants of the Poisson distribution can be easily obtained by putting 1 in place of q in the constants of the binomial distribution

Constants of the Poisson Distribution

- The various constants of the Poisson distribution are:
 - The mean of the Poisson distribution = m , and
 - The standard deviations is \sqrt{m} or $\mu_2 = m$

Poisson Distribution

- A discrete distribution with a single parameter m
- As m increases, the distribution shifts to the right.
- All poisson probability distributions are skewed to the right
- Probability of rare events (the probabilities tend to be high for small number of occurrences)