Module-4

Lecture-19

Maneuvering Flight: Steady Pull up, Relationship between stick fixed Neutral and Maneuvering point.

Steady pull-up

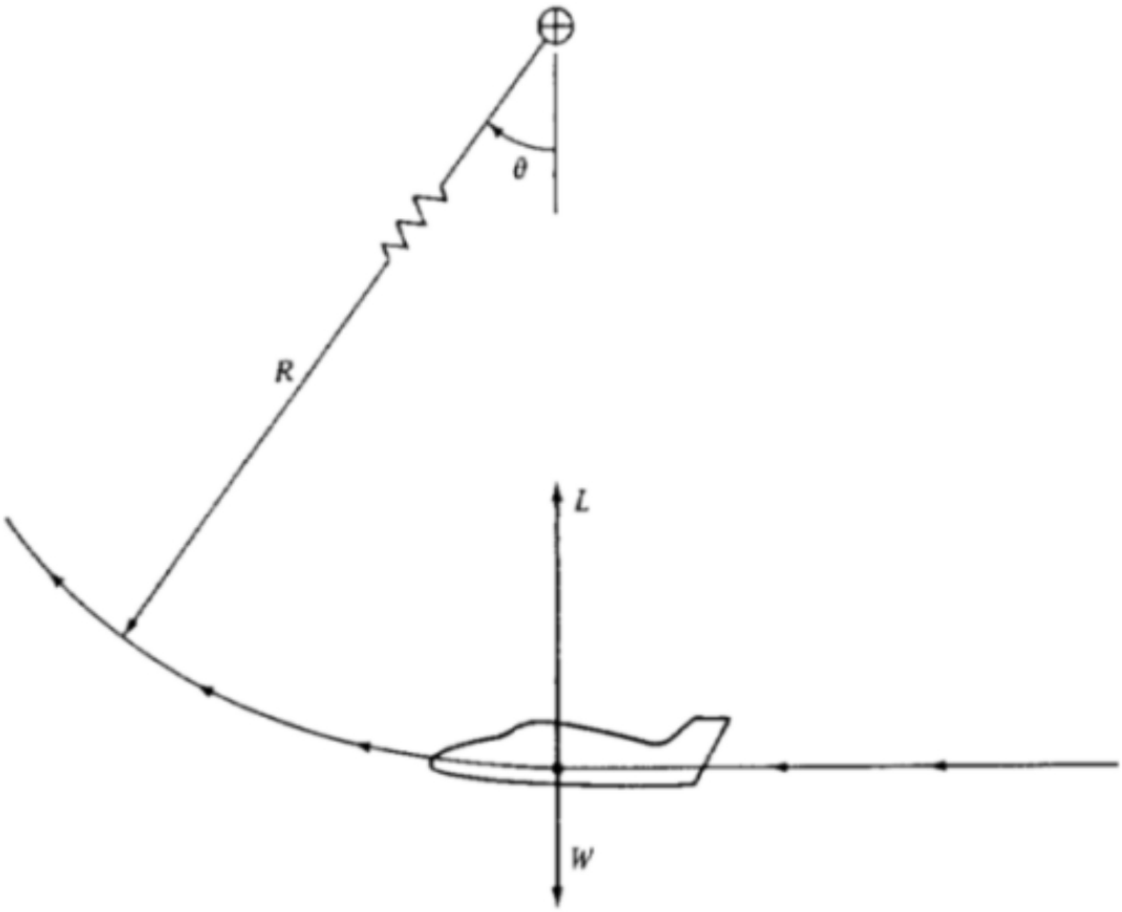

Figure 1: Figure representing free body diagram of an aircraft during pull up maneuver

$$L - w = \frac{mV^2}{R}$$

$$L = nW$$

$$(n-1)W = \frac{mV^2}{R}$$

As, $W = mg$ and $q = V/R$

$$q = \frac{g}{V}(n-1)$$

Hence,

$$R = \frac{V^2}{g}\frac{1}{(n-1)}$$

Note:

- Pull-up:

$$q = \frac{g}{V}(n-1)$$

- Steady coordinated turn:

$$q = \left(n - \frac{1}{n}\right)\frac{g}{V}$$

- Thus, in compact form we can write the following:

$$q = \frac{g}{V}\left(n - \frac{k}{n}\right)$$

  $k = n$ for pull up, $k = 1$ for steady coordinated turn

Because of the pitch rate q tail will experience additional angle of attack

$$\Delta\alpha_t = q\frac{l_t}{V}$$

Because of this $\Delta\alpha_t$ there will be additional lift on the tail during ϕ maneuver. Therefore, elevator needs to be deflected up to nullify this additional nose down moment (due to additional lift at tail).

How much δe *is required?*

- Recall, $\tau = d\alpha_t/d\delta e$; i.e. how much tail angle changes per unit elevator deflection. δe should be such that $\tau\delta e$ compensates additional angle ql_t/V. So we can write:

$$\tau\Delta\delta e + q\frac{l_t}{V} = 0$$

$$\Delta\delta e = \frac{ql_t}{\tau V}$$

- A factor of 1.1 is generally used to account for the contribution to stability due to pitch rate from the fuselage position ahead of wing. So:

$$\Delta\delta e = 1.1\frac{ql_t}{\tau V}$$

  will be used for future derivation. Using,

$$q = \frac{g}{V}\left(n - \frac{k}{n}\right)$$

  where $k = n$ for pull-up and $k = 1$ for steady coordinated turn. We have:

$$\Delta\delta e = \frac{1.1gl_t}{\tau V^2}\left(n - \frac{k}{n}\right)$$

- Thus, during maneuver (with pitch rate q), the elevator required to trim a/c at $C_L = nW/(\rho V^2 S/2)$ pitching at the rate q will be given by

$$\delta e = \delta_{e_o} + \left(\frac{d\delta e}{dC_L}\right) C_L - \frac{1.1 g l_t}{\tau V^2}\left(n - \frac{k}{n}\right)$$

$$\delta e = \delta_{e_o} + \left(\frac{d\delta e}{dC_L}\right) \frac{nW}{\frac{1}{2}\rho V^2 S} - \frac{1.1 g l_t}{\tau V^2}\left(n - \frac{k}{n}\right)$$

- By differentiating the above expression with respect to 'n' one can show that,

$$\frac{d\delta e}{dn} = -\frac{1}{V^2}\left[\frac{1.1 g l_t}{\tau}\left(1 + \frac{k'}{n^2}\right) + \left\{\frac{\frac{2W}{S}}{\rho C_{m_{\delta e}}}\left[\frac{dC_m}{dC_L}\right]_{fix}\right\}\right]$$

where,

$$k' = 0 \text{ for pull-up}$$

$$k' = 1 \text{ for steady coordinated turn}$$

Relationship between Stick fixed Neutral and Maneuvering Point (Pull Up)

- Stick fixed maneuvering point is the $c.g.$ location at which $d\delta e/dn = 0$

- Using above equation, we get:

$$\left[\frac{dC_m}{dC_L}\right]_{fix} = -\frac{1.1 g l_t}{\tau}\frac{\rho C_m \delta e}{2\frac{W}{S}}$$

$$\bar{x}_{cg} - \bar{n}_o = -\frac{1.1 g l_t}{\tau}\frac{\rho C_m \delta e}{2\frac{W}{S}}$$

$$\bar{x}_{cg} = \bar{n}_o - \frac{1.1 g l_t}{\tau}\frac{\rho C_m \delta e}{2\frac{W}{S}} = \bar{n}_m$$

$$\bar{n}_m = \bar{n}_o - \frac{1.1 g l_t}{\tau}\frac{\rho C_m \delta e}{2\frac{W}{S}}$$

- So, $\bar{n}_m > \bar{n}_o$ i.e. stick fixed maneuvering point is aft of stick fixed neutral point. This is consistent as the pitch rate provides additional stability through tail.