

# Risk and Return

- What is Return?????
- Quantification of Return????
- What is the Measurement Unit????
- What is Risk?????
- Quantification of Risk??????
- What is the Measurement Unit?????

# Risk and Return

- Risk → the chance that realized returns differ significantly from expected returns
- We assume investors like returns and dislike risk
- Historically, there has been a tradeoff between risk and return
  - To achieve a higher return, an investor should be willing to accept more risk
  - Lower Risk is obtained only through lower returns
  - **Can we measure Risk?**

# Measures of Risk

- Standard Deviation/Variance – measures a portfolio's total volatility and its total risk (that is, systematic and unsystematic risk)
- The total risk is divided into two parts.
- Total Risk = Systematic Risk + Unsystematic Risk

# Variance and Standard Deviation

Standard Deviation:- Extent of deviation of returns from the average value of returns. Square root of the average of square of deviation of the observed returns from their expected value of returns.

Variance in the average value of the squares of deviation of the observed returns from the expected value of return.

# Variance and Standard Deviation

- The lower the variance and standard deviation, the less risk is associated with that asset.
- The risk in question deals with the uncertainty that is present with possible return scenarios.
- For example, if there is no risk ( an expected return with absolute certainty), then there is no variance or standard deviation of the return.

# Investment Risks

- Systematic Risks/Non Diversifiable/Un controllable
  - Market risk
  - Interest rate risk
  - Purchasing power risk
  - Foreign currency (exchange rate) risk
  - Reinvestment risk
- Unsystematic Risks/Diversifiable/Controllable
  - Business risk
  - Financial risk
  - Default risk
  - Country (or regulation) risk

# Measure of Systematic Risk

- Beta – a commonly used measure of systematic risk that is derived from regression analysis
- Unsystematic risk is residual.

# MANAGING RISK

- A person is said to be risk averse if he/she exhibits a dislike of uncertainty.



# MANAGING RISK

- Individuals can reduce risk, choosing any of the following:
  - Buy insurance
  - Diversify
  - Accept a lower return on their investments

# The Markets for Insurance

- One way to deal with risk is to buy insurance.
- The general feature of insurance contracts is that a person facing a risk pays a fee to an insurance company, which in return agrees to accept all or part of the risk.

# Diversification of Unsystematic Risk

- *Diversification* refers to the reduction of risk achieved by replacing a single risk with a large number of smaller unrelated risks.

# Diversification of Unsystematic Risk

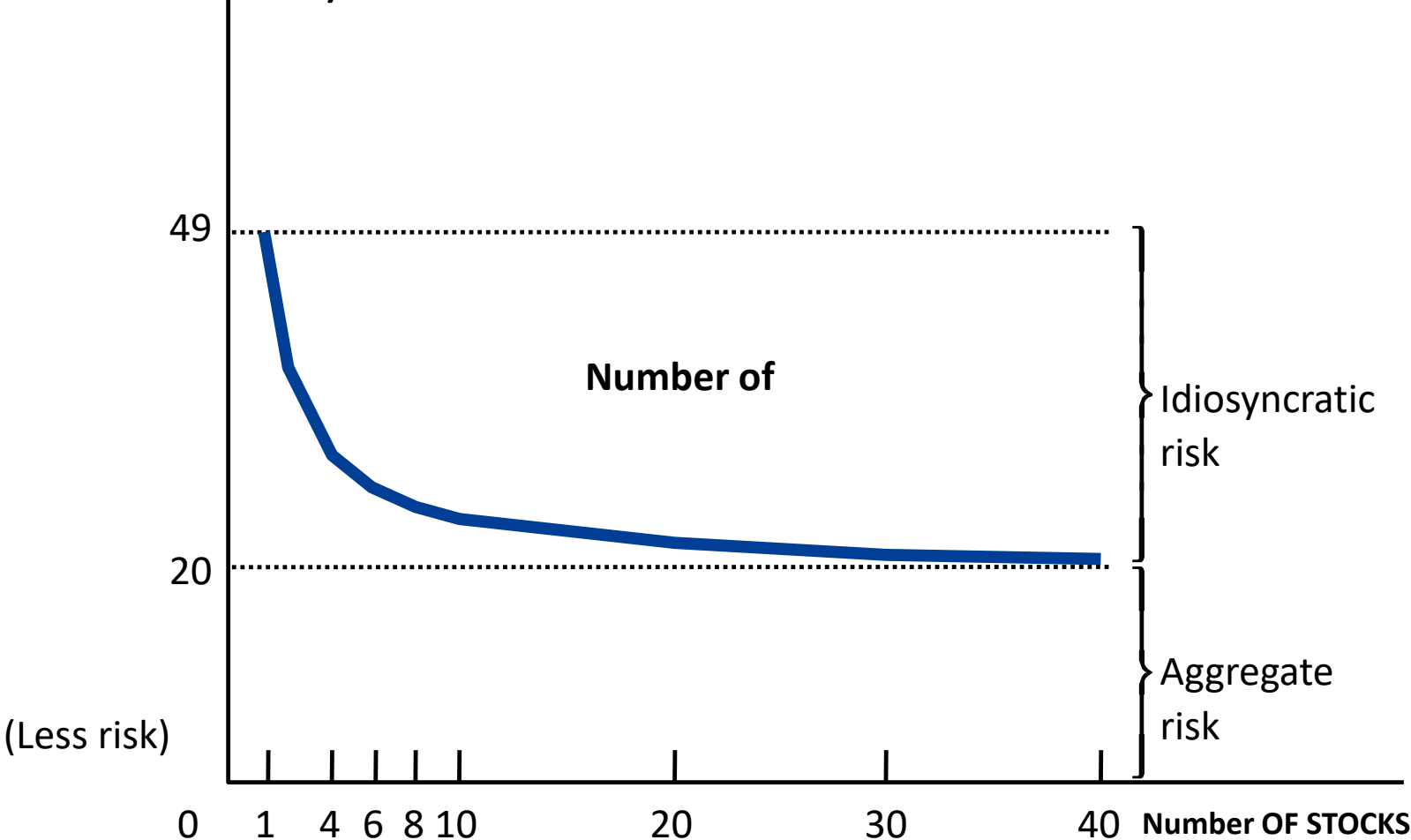
- Unsystematic risk is the risk that affects only a single person. The uncertainty associated with specific companies.

# Diversification of Unsystematic Risk

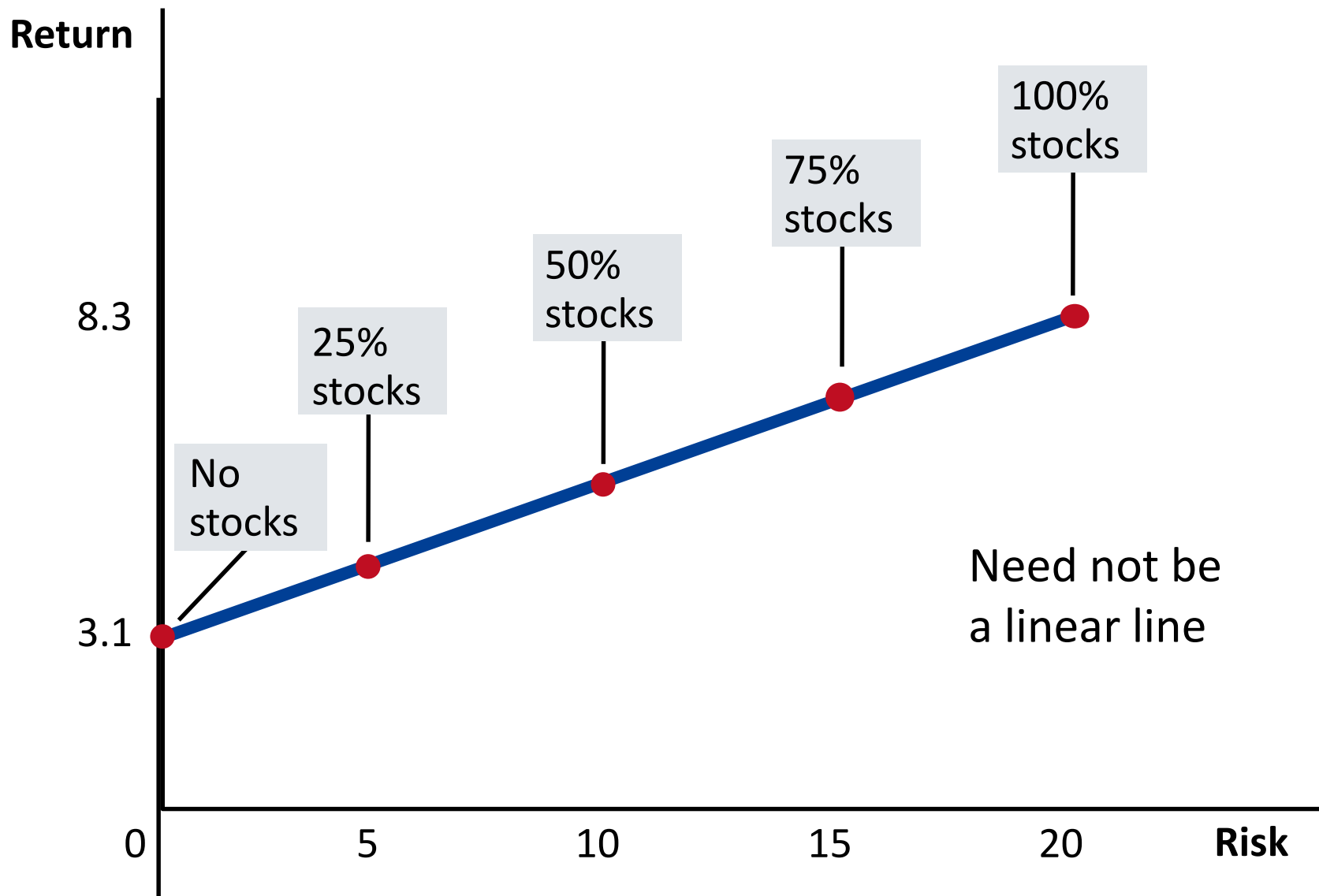
- Aggregate risk is the risk that affects all economic actors at once, the uncertainty associated with the entire economy.
- Diversification cannot remove aggregate risk.

# Diversification

Risk (standard Deviation)



# The Tradeoff between Risk and Return



# Type of Calculations in Risk and Return



# Ex Ante

## "before the event"

- A term that refers to future events, such as future returns or prospects of a company. Using ex-ante analysis helps to give an idea of future movements in price or the future impact of a newly implemented policy.
- An example of ex-ante analysis is when an investment company values a stock ex-ante and then compares the predicted results to the actual movement of the stock's price.
- **Limitation-Returns and risk can be calculated after-the-fact (ie. You use actual realized return data). This is known as an *ex post* calculation.**
- **Or you can use forecast data...this is an *ex ante* calculation.**

# Ex Post

- Ex-post translated from Latin means "after the fact". The use of historical returns has traditionally been the most common way to predict the probability of incurring a loss on any given day. Ex-post is the opposite of ex-ante, which means "before the event".
- Companies may try to obtain ex-post data to forecast future earnings. Another common use for ex-post data is in studies such as value at risk (VaR), a probability study used to estimate the maximum amount of loss a portfolio could incur on any given day.

# Ex Post Calculation of Risk and return

# Risk- Standard Deviation (ex post)

- The formula for the standard deviation when analyzing sample data (realized returns or ex post) is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (k_i - \bar{k}_i)^2}{n - 1}}$$

Where  $k$  is a realized return on the stock and  $n$  is the number of returns used in the calculation of the mean.

# Holding Period Return

(For investments that yield dividend cash flow returns)

$$HPR = \frac{\text{Ending Price} - \text{Beginning Price} + \text{Dividend}}{\text{Beginning Price}}$$

$$HPR = \frac{P_1 - P_0 + D_1}{P_0}$$

# Ex Post - Holding Period Return

Price of stock at the beginning of the year = 21.25

Price of the stock at the end of the year = 22.10

Dividend received each quarter = .10

$$HPR = \frac{P_1 - P_0 + D_1}{P_0} = \frac{(22.10 - 21.25) + (4)(0.10)}{21.25}$$

$$HPR = \frac{0.85 + 0.40}{21.25}$$

$$HPR = \frac{1.25}{21.25} = 0.0588 = 5.9\%$$

# The Bias Inherent in the Arithmetic Average

- Arithmetic averages can yield incorrect results because of the problems of bias inherent in its calculation.
  - Consider an investment that was purchased for 10, rose to 20 and then fell back to 10.
  - Let us calculate the HPR in both periods:

$$HPR_1 = \frac{20 - 10}{10} = \frac{10}{10} = 100\%$$

$$HPR_2 = \frac{10 - 20}{20} = \frac{-10}{20} = -50\%$$

- The arithmetic average return earned on this investment was:

$$Average = \frac{100\% - 50\%}{2} = \frac{50\%}{2} = 25\%$$

# The Bias Inherent in the Arithmetic Average

## Example Continued ...

- The answer is clearly incorrect since the investor started with 10 and ended with 10.
- The correct answer may be obtained through the use of the geometric average:

$$\begin{aligned} \textit{GeometricAverage} &= \sqrt[n]{\sum_{i=1}^n (1 + r_i)} - 1 \\ &= [(1 + 100\%)(1 + (-50\%))]^{1/2} - 1 \\ &= [(2)(.5)]^{1/2} - 1 \\ &= (1)^{1/2} - 1 = 1 - 1 = 0 \end{aligned}$$



# Geometric Versus Arithmetic Average Returns

Consider two investments with the following realized returns over the past few years:

Holding Period Returns		
	<b>IBM Stock</b>	<b>Government Bonds</b>
<b>Year</b>		
2000	12.0%	6.0%
2001	12.0%	6.0%
2002	12.0%	6.0%
2003	12.0%	6.0%
2004	12.0%	6.0%
2005	12.0%	6.0%

If the returns are equal over time, the arithmetic average return will equal the geometric average return.

# Geometric Versus Arithmetic Average Returns

Arithmetic Average Return :

$$\begin{aligned}\bar{R} &= \frac{HPR_1 + HPR_2 + HPR_3 + HPR_4 + HPR_5 + HPR_6}{N} \\ &= \frac{12\% + 12\% + 12\% + 12\% + 12\% + 12\%}{6} \\ &= \frac{72}{6} = 12\%\end{aligned}$$

Year	Holding Period Returns	
	IBM Stock	Government Bonds
2000	12.0%	6.0%
2001	12.0%	6.0%
2002	12.0%	6.0%
2003	12.0%	6.0%
2004	12.0%	6.0%
2005	12.0%	6.0%

**SAME  
ANSWER !**

Geometric Average Return :

$$\begin{aligned}\bar{G} &= (1 + HPR_1)(1 + HPR_2)(1 + HPR_3)(1 + HPR_4)(1 + HPR_5)(1 + HPR_6)]^{\frac{1}{6}} - 1 \\ &= (1.12)(1.12)(1.12)(1.12)(1.12)(1.12)^{\frac{1}{6}} - 1 \\ &= 1.973822685^{.16667} - 1 \\ &= 12\%\end{aligned}$$

# Geometric Versus Arithmetic Average Returns

***Now consider volatile returns:***

Arithmetic Average Return :

$$\begin{aligned}\bar{R} &= \frac{HPR_1 + HPR_2 + HPR_3 + HPR_4 + HPR_5 + HPR_6}{N} \\ &= \frac{40\% - 30\% + 33\% + 5\% + 32\% - 8\%}{6} \\ &= \frac{72}{6} = 12\%\end{aligned}$$

Geometric Average Return :

$$\begin{aligned}\bar{G} &= (1 + HPR_1)(1 + HPR_2)(1 + HPR_3)(1 + HPR_4)(1 + HPR_5)(1 + HPR_6)]^{\frac{1}{6}} - 1 \\ &= (1.40)(0.70)(1.33)(1.05)(1.32)(.92)^{\frac{1}{6}} - 1 \\ &= 1.661991408^{.16667} - 1 \\ &= 8.84\%\end{aligned}$$

Year	Holding Period Returns	
	IBM Stock	Government Bonds
2000	40.0%	11.0%
2001	-30.0%	4.0%
2002	33.0%	8.0%
2003	5.0%	3.0%
2004	32.0%	6.0%
2005	-8.0%	4.0%

Arithmetic Average = 12.0%      6.0%  
Standard Deviation = 27.71%      3.03%

**NOT THE  
SAME  
ANSWER !**

**Volatility of returns over time eats away at your realized returns!!!** *The greater the volatility the greater the difference between the arithmetic and geometric average. Arithmetic average OVERSTATES the return!!!*

# Measuring Returns

- When you are trying to find average returns, especially when those returns rise and fall, always remember to use the geometric average.
- The greater the volatility of returns over time, the greater the difference you will observe between the geometric and arithmetic averages.

# Ex Ante Calculation of Risk and Return

# Ex Ante -Expected Return

- An investor might say that a given asset will be expected to yield a 10% return. This is however a *point estimate*.
- Pressed further, the investor will admit that the asset could possibly provide a return of -10% under certain conditions or as high as 25%.
- The uncertainty in the actual range of possible returns is indeed a form of risk.

# Expected Return of Security when Probability is Given

- Say that the investor believes that with a 30% probability, a given asset will have a 10% return. A -10% return is determined to happen with a 10% probability. The 25% return can occur with a 60% probability.

Probability of Return	Possible Return
30%	10%
10%	-10%
60%	25%

# Expected Return, when Probabilities are given

- In order to find the expected return the following formula is used:

$$\text{Expected Return} = \sum (\text{Probability of Return}) \times (\text{Possible Return})$$

- For our example:

$$\text{Expected Return} = (0.30)(10\%) + (0.10)(-10\%) + (0.60)(25\%) = 17\%$$



# So Why is Expected Return Important?

- Expected Return is the most basic form of risk analysis.
- An asset with perfect certainty of return will have only one possible return. This is rare.
- The challenge is to determine proper probability weights in order to calculate an expected return value that accurately captures the risk associated with an asset's returns.

# Variance and Standard Deviation, When Probability is given

- Used to Quantify the risks associated with possible returns.

Variance =  $\sum$  (Probability of Return) X ( Possible Return – Expected Return)<sup>2</sup>

$$\sigma = P_1 (X_1 - \bar{X})^2 + P_2 (X_2 - \bar{X})^2 \dots \dots \dots$$

$$\sigma = \sqrt{\sum_{i=1}^n (k_i - \bar{k}_i)^2 P_i}$$

- For our previous example the variance was 130 and the standard deviation was 11.4%

# Coefficient of Variation

- Sometimes Variance and Standard Deviation can be misleading.
- If conditions for two or more investment alternatives are not similar then a measure of relative variability is needed.
- A widely used measure of relative variability is the Coefficient of Variation (CV)

# Coefficient of Variation

- The Coefficient of Variation is usually calculated with the following formula:

$$CV = \text{Standard Deviation} / \text{Expected Rate of Return}$$

- or -

$$CV = \text{Standard Deviation} / \text{Mean}$$

# An Example of CV

- Assume Stock A and Stock B have widely differing rates of return and standard deviations of return.
- Using standard deviation analysis, Stock A seems to be less risky than Stock B.

	Stock A	Stock B
Expected Return	7%	12%
Standard Deviation	5%	7%

# However....

- Using CV analysis, the results are different.
- CV of Stock A =  $5\% / 7\% = 0.714$
- CV of Stock B =  $7\% / 12\% = 0.583$
  
- The CV figure shows that Stock B has less relative variability or lower risk per unit of expected return.

# Conclusion

- Use Standard Deviation to compare different assets and choose the one with the least amount of risk and the highest possible return.
- Historical Standard Deviation can be used to look at the past performance of an asset. Can also be used to compare two or more assets.
- The Coefficient of Variation should be used to compare assets in different industries or widely differing expected returns.

# Calculations

- Calculation of Beta
- Calculation of total Risk=  $\sigma^2$
- **Systematic Risk=  $B^2 \cdot \sigma_m^2$**

**$\sigma^2 =$  Systematic risk + unsystematic risk**

$$B^2 \cdot \sigma_m^2 + \sigma_{ex}^2$$

$$\beta = \text{Cov}_{12} / \sigma_m^2 = \Sigma dx dy / \Sigma dx^2$$



# Beta

– A security's beta is

$$\beta_i = \frac{COV(\tilde{R}_i, \tilde{R}_m)}{\sigma_m^2}$$

where  $\tilde{R}_m$  = return on the market index

$\sigma_m^2$  = variance of the market returns

$\tilde{R}_i$  = return on Security  $i$

# Risk and Return for Portfolio

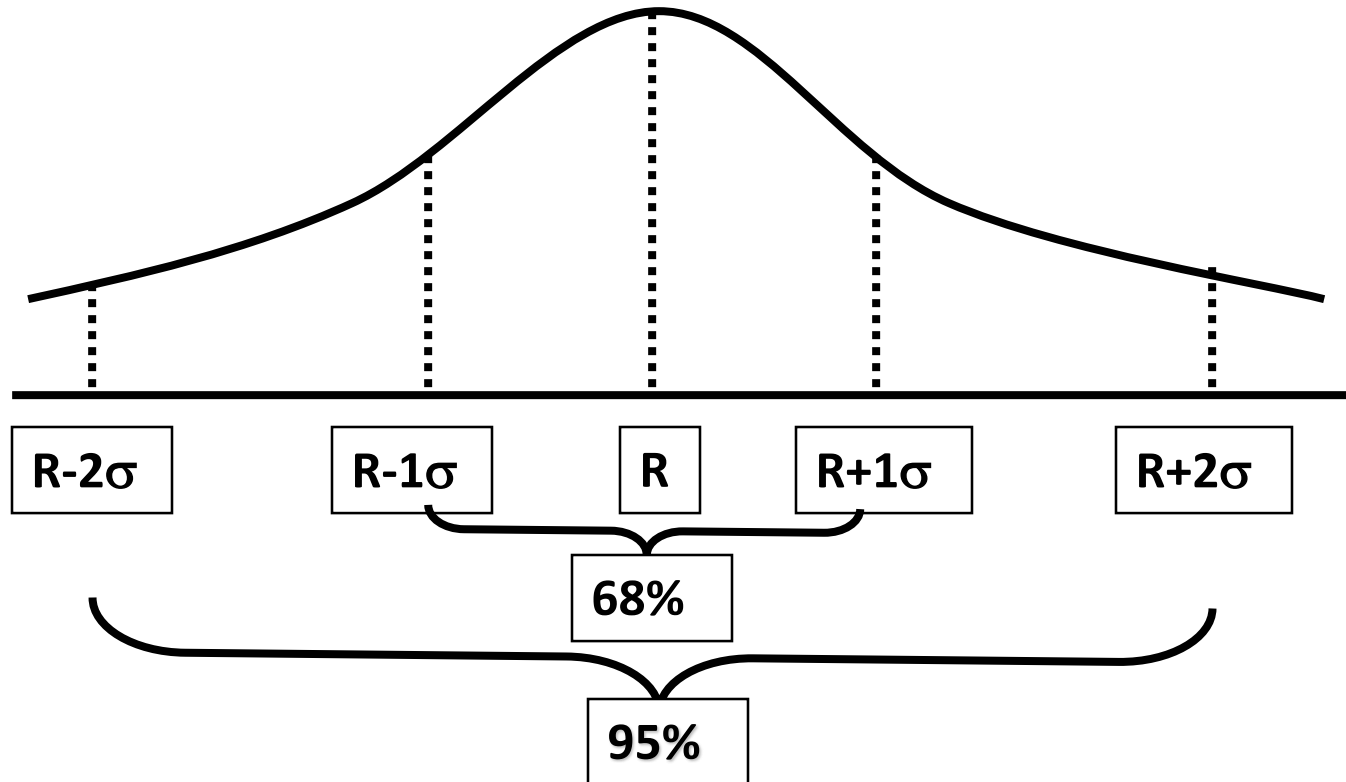
# Markowitz Portfolio Theory and Normality

- If returns are normally distributed they are completely described by their mean and variance
  - So investors can chose *portfolios* based solely on the mean and variance
- Investors will prefer portfolios with high means and low variances

# Single Financial Assets

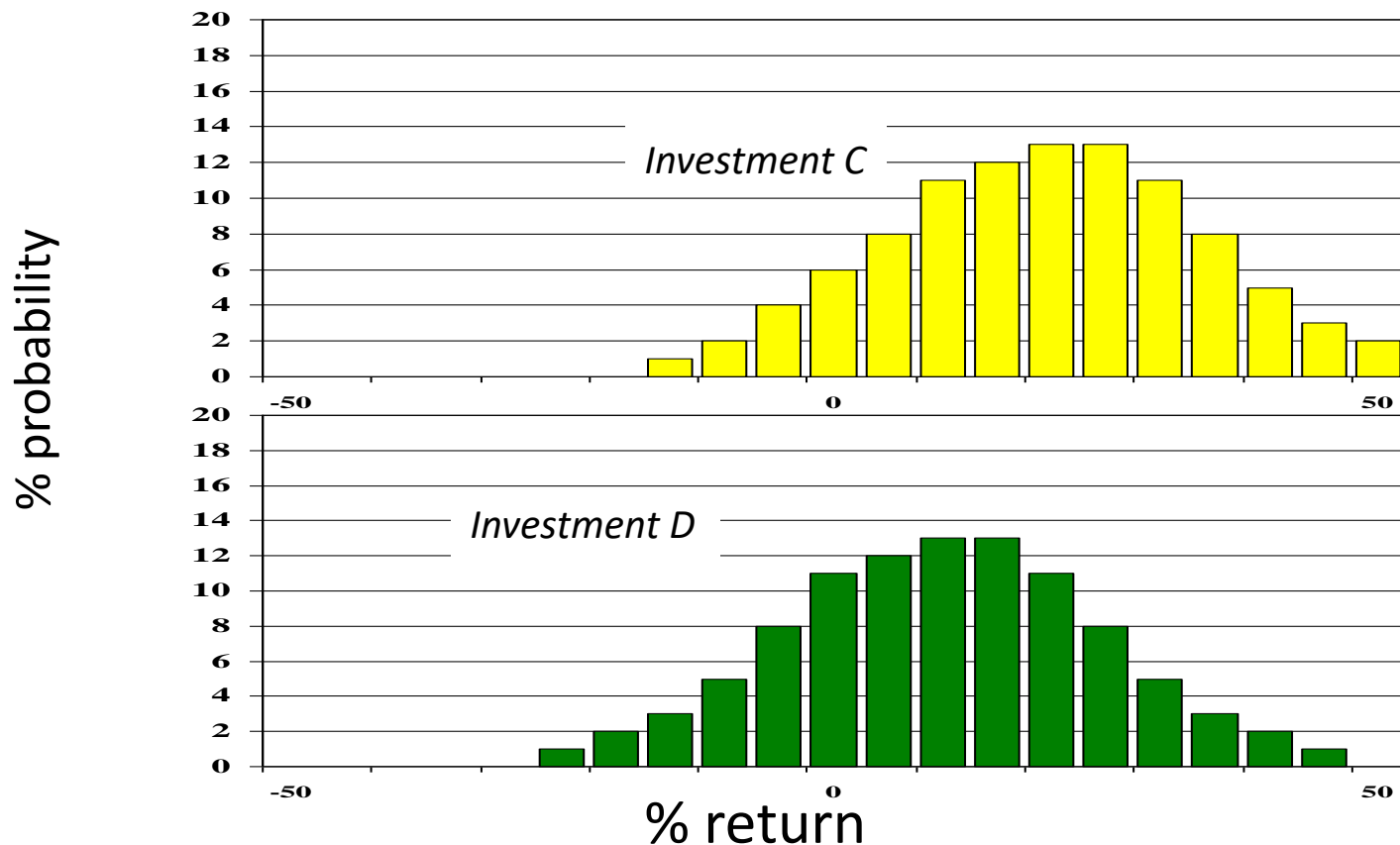
**Historical Risk**

Normal Distribution



# Markowitz Portfolio Theory

C is preferred to D, standard deviations are the same, but C has a higher mean. If C & D had the same mean, the one with the lower variance would be preferred.



# Risk and Return - MPT

- Prior to the establishment of Modern Portfolio Theory, most people only focused upon investment returns...they ignored risk.
- With MPT, investors had a tool that they could use to dramatically reduce the risk of the portfolio without a significant reduction in the expected return of the portfolio.
- **Harry Markowitz: Founder of Portfolio Theory**

# Risk and Return - MPT

- Harry Markowitz's "Portfolio Selection" *Journal of Finance* article (1952) set the stage for modern portfolio theory
  - The first major publication indicating the importance of security return correlation in the construction of stock portfolios
  - Markowitz showed that for a given level of expected return and for a given security universe, knowledge of the covariance and correlation matrices are required

# Risk and Return - MPT

- Harry Markowitz's ***efficient portfolios***:
  - Those portfolios providing the maximum return for their level of risk
  - Those portfolios providing the minimum risk for a certain level of return



# Risk and Return - MPT

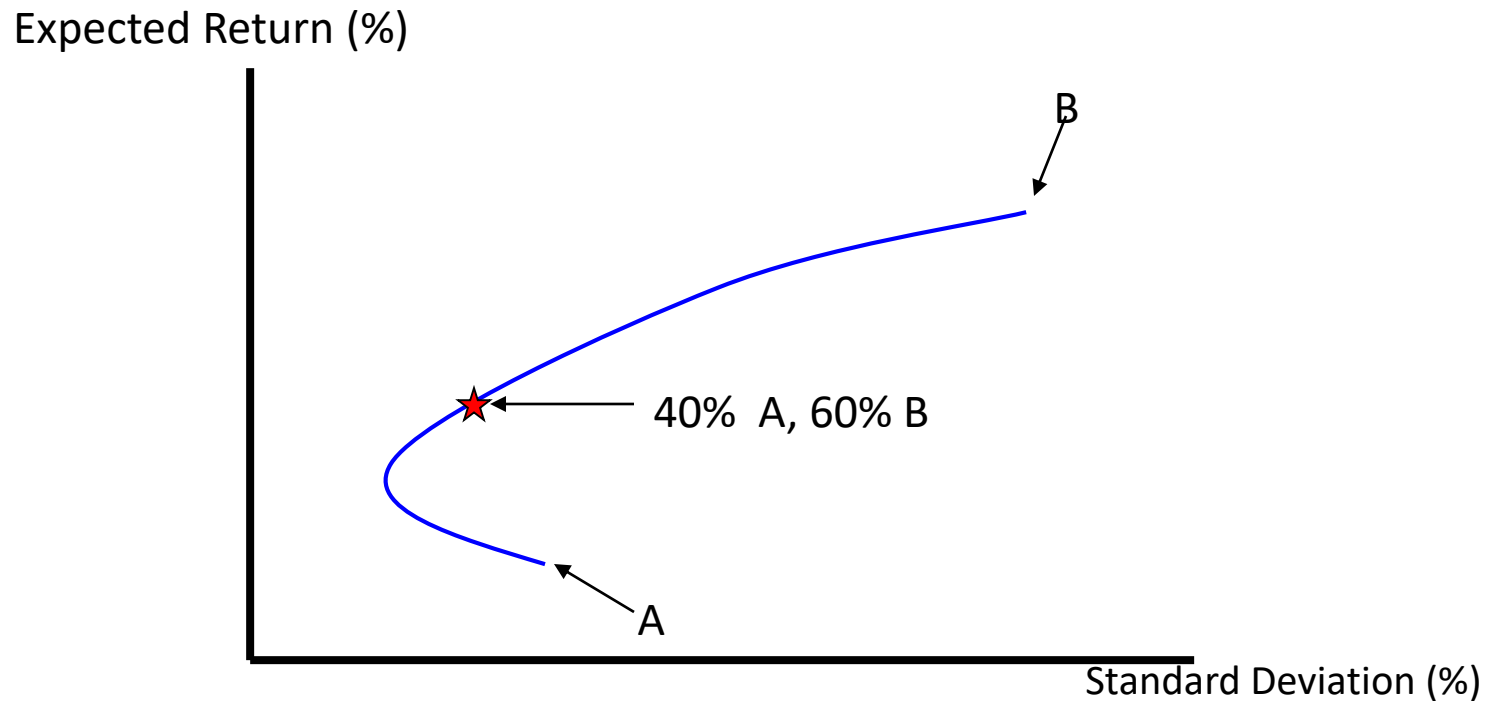
- A portfolio's performance is the result of the performance of its components
  - The return realized on a portfolio is a linear combination of the returns on the individual investments
  - The variance of the portfolio is *not* a linear combination of component variances

# Risk and Return - MPT

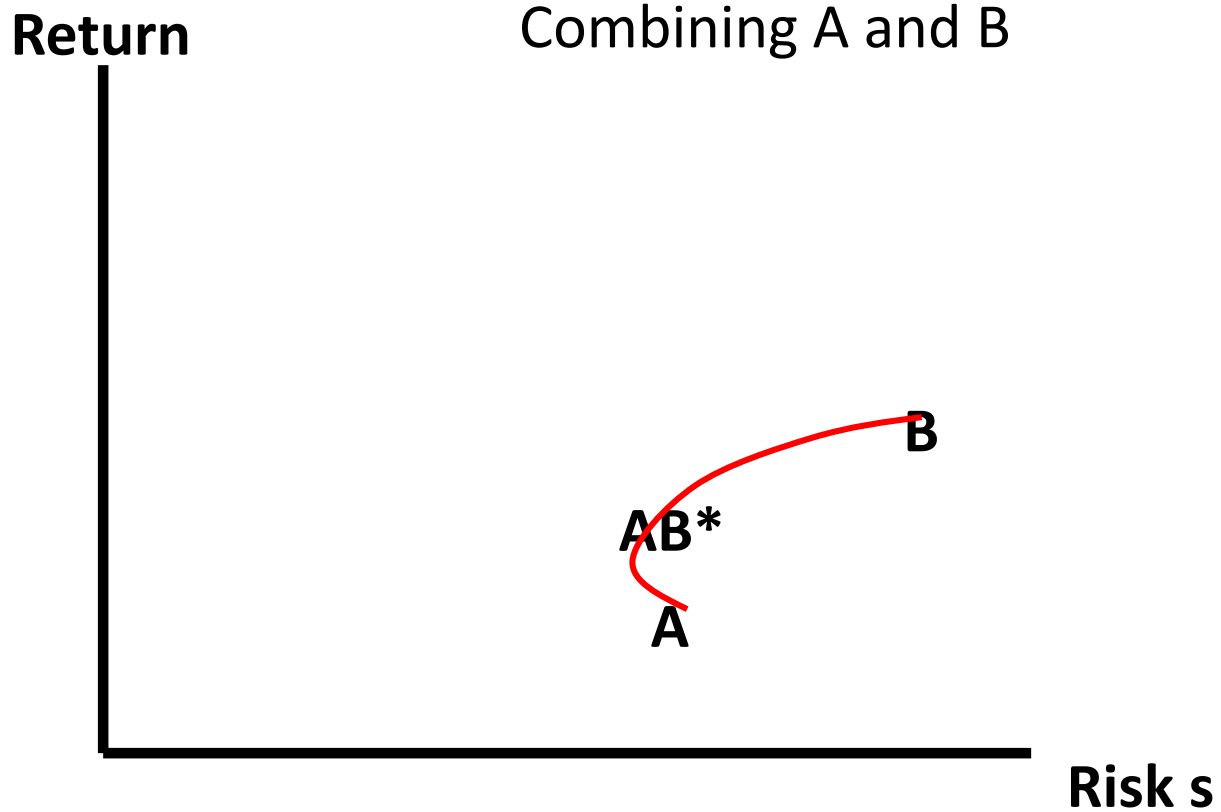
- The degree to which the returns of two stocks co-move is measured by the correlation coefficient.
- The correlation coefficient between the returns on two securities will lie in the range of +1 through - 1.
- +1 is perfect positive correlation.
- -1 is perfect negative correlation.

# Markowitz Portfolio Theory

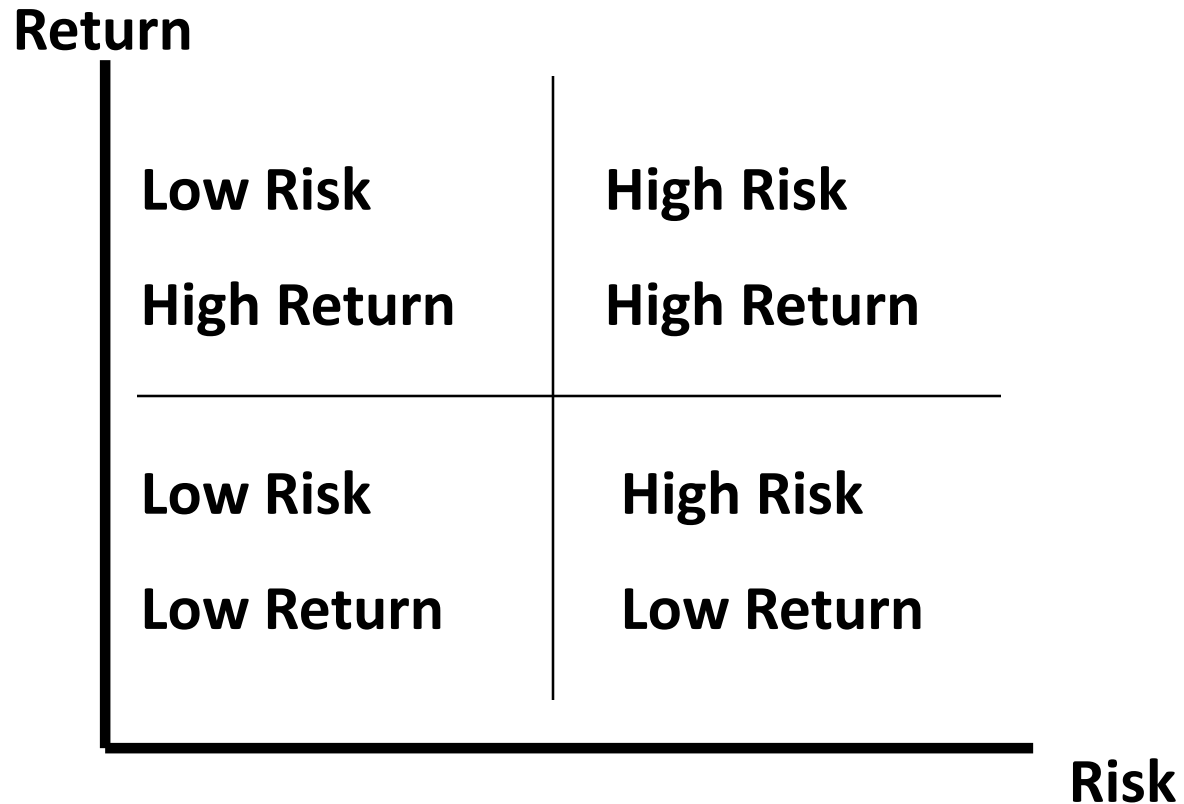
Expected Returns and Standard Deviations vary given different weights for shares in the portfolio.



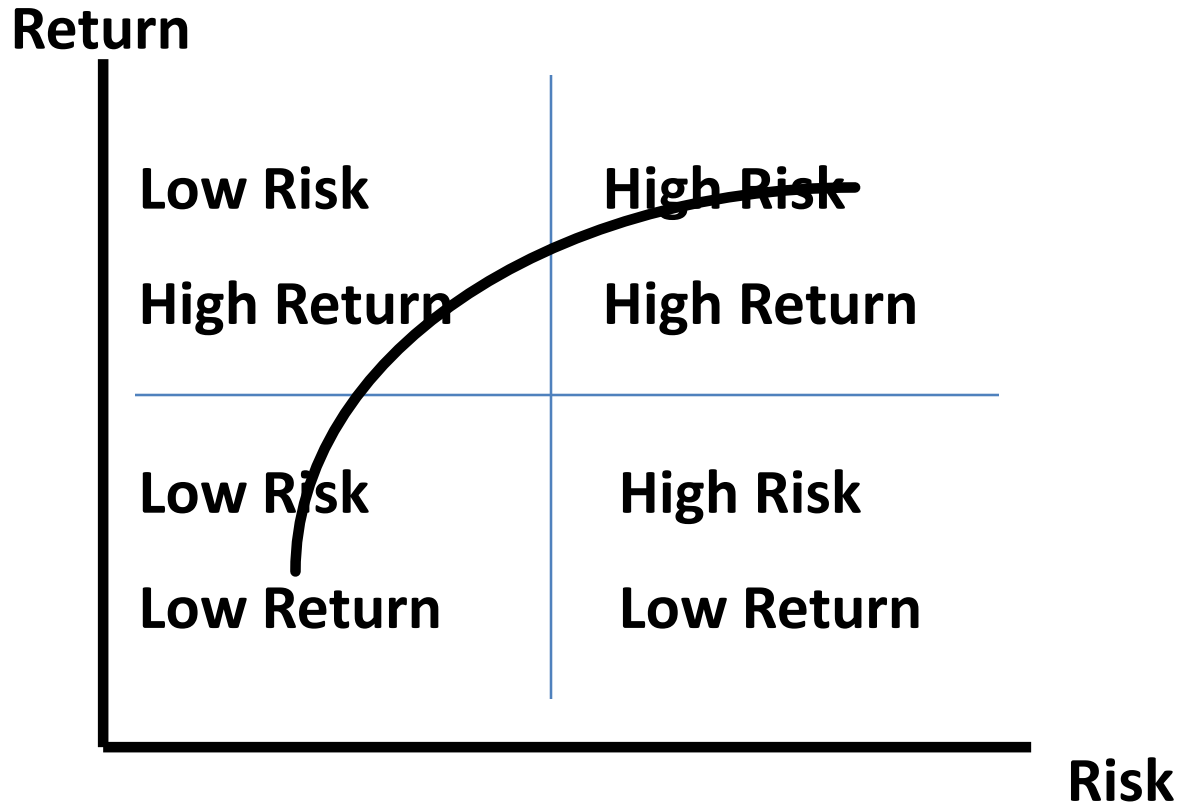
# Efficient Frontier



# Efficient Frontier

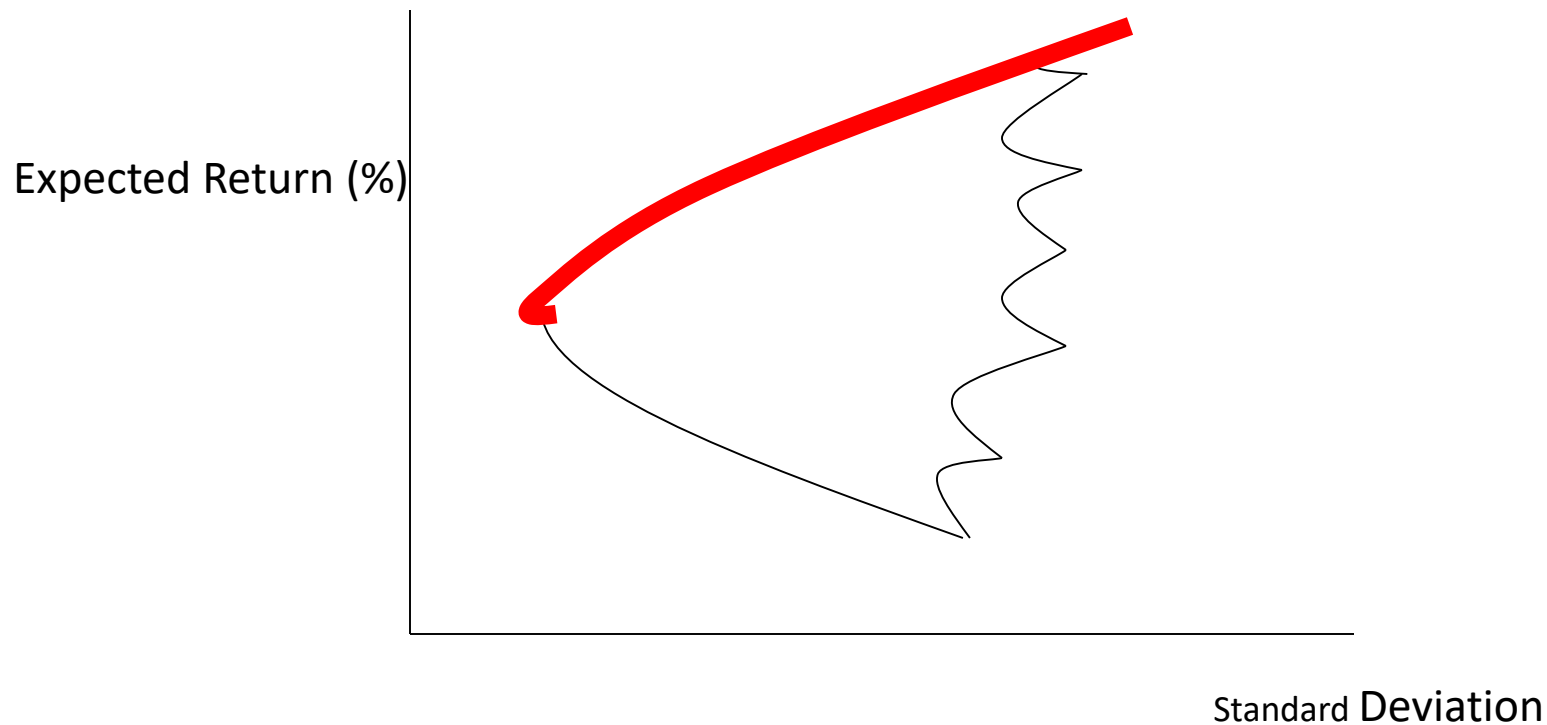


# Efficient Frontier



# Efficient Frontier

- The jelly fish shape contains all possible combinations of risk and return: The feasible set
- The red line constitutes the efficient frontier: Highest return for given risk



# Portfolio Risk and Return- Ex Ante



# Portfolio Return

- The expected return of a portfolio is a weighted average of the expected returns of the components:

$$E(\tilde{R}_p) = \sum_{i=1}^n [x_i E(\tilde{R}_i)]$$

where  $x_i$  = proportion of portfolio  
invested in security  $i$  and

$$\sum_{i=1}^n x_i = 1$$

# Portfolio variance

- Understanding portfolio variance is the essence of understanding the mathematics of diversification
  - The variance of a linear combination of random variables is *not* a weighted average of the component variances

# Grouping Individual Assets into Portfolios

- The riskiness of a portfolio that is made of different risky assets is a function of three different factors:
  - the riskiness of the individual assets that make up the portfolio
  - the relative weights of the assets in the portfolio
  - the degree of comovement of returns of the assets making up the portfolio
- The standard deviation of a two-asset portfolio may be measured using the Markowitz model:

$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

# Variance of A Linear Combination

- Return variance is a security's *total risk*

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \rho_{AB} \sigma_A \sigma_B$$

Total Risk    Risk from A    Risk from B    Interactive Risk

- Most investors want portfolio variance to be as low as possible without having to give up any return

# Variance of A Linear Combination

- If two securities have low correlation, the interactive risk will be small
- If two securities are uncorrelated, the interactive risk drops out
- If two securities are negatively correlated, interactive risk would be negative and would reduce total risk

# Efficient Frontier

## Example

Correlation Coefficient = .4

<u>Shares</u>	<u>s</u>	<u>% of Portfolio</u>	<u>Avg Return</u>
ABC Corp	28	60%	15%
Big Corp	42	40%	21%

Weighted Average Standard Deviation = 33.6

**Standard Deviation Portfolio = 28.1**

**Return = weighted avg = Portfolio = 17.4%**

} Gain from  
Diversification

Let's Add share New Co' to the portfolio

# Efficient Frontier

## Example

Correlation Coefficient = .3

<u>Shares</u>	<u>s</u>	<u>% of Portfolio</u>	<u>Avg Return</u>
Portfolio	28.1	50%	17.4%
<b><i>New Corp</i></b>	<b><i>30</i></b>	<b><i>50%</i></b>	<b><i>19%</i></b>

NEW Standard Deviation = weighted avg = 31.80

**NEW Standard Deviation = Portfolio = 23.43**

**NEW Return = weighted avg = Portfolio = 18.20%**

NOTE: Higher return & Lower risk

# Formulas

$$\text{Correlation } r = \mathbf{Cov}_{12} / \sigma_1 \sigma_2$$

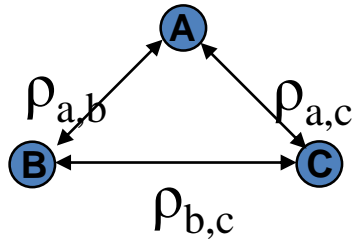
$\mathbf{Cov}_{12} = \Sigma dx dy / n-1$  where  $dx$  is deviation from mean

$$r = \frac{\Sigma dx dy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$$



# Risk of a Three-asset Portfolio

We need 3 (three) correlation coefficients between A and B; A and C; and B and C.

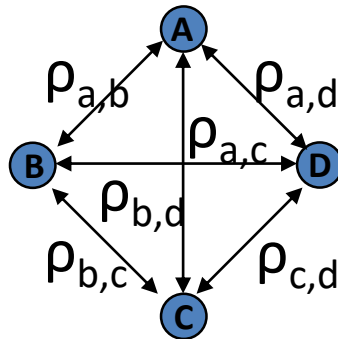


$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + \sigma_C^2 w_C^2 + 2w_A w_B \rho_{A,B} \sigma_A \sigma_B + 2w_B w_C \rho_{B,C} \sigma_B \sigma_C + 2w_A w_C \rho_{A,C} \sigma_A \sigma_C}$$

# Risk of a Four-asset Portfolio

The data requirements for a four-asset portfolio grows dramatically if we are using Markowitz Portfolio selection formulae.

We need 6 correlation coefficients between A and B; A and C; A and D; B and C; C and D; and B and D.



# The n-Security Case

- A ***covariance matrix*** is a tabular presentation of the pair wise combinations of all portfolio components
  - The required number of covariance's to compute a portfolio variance is  $(n^2 - n)/2$
  - Any portfolio construction technique using the full covariance matrix is called a ***Markowitz model***

# Diversification Potential

- The potential of an asset to diversify a portfolio is dependent upon the degree of co-movement of returns of the asset with those other assets that make up the portfolio.
- In a simple, two-asset case, if the returns of the two assets are perfectly negatively correlated it is possible (depending on the relative weighting) to eliminate all portfolio risk.
- This is demonstrated through the following chart.

# Example of Portfolio Combinations and Correlation

Perfect Positive  
Correlation – no  
diversification

Asset	Expected Return	Standard Deviation	Correlation Coefficient
<b>A</b>	5.0%	15.0%	1
<b>B</b>	14.0%	40.0%	

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	17.5%
80.00%	20.00%	6.80%	20.0%
70.00%	30.00%	7.70%	22.5%
60.00%	40.00%	8.60%	25.0%
50.00%	50.00%	9.50%	27.5%
40.00%	60.00%	10.40%	30.0%
30.00%	70.00%	11.30%	32.5%
20.00%	80.00%	12.20%	35.0%
10.00%	90.00%	13.10%	37.5%
0.00%	100.00%	14.00%	40.0%

# Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
<b>A</b>	5.0%	15.0%	0.5
<b>B</b>	14.0%	40.0%	

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	15.9%
80.00%	20.00%	6.80%	17.4%
70.00%	30.00%	7.70%	19.5%
60.00%	40.00%	8.60%	21.9%
50.00%	50.00%	9.50%	24.6%
40.00%	60.00%	10.40%	27.5%
30.00%	70.00%	11.30%	30.5%
20.00%	80.00%	12.20%	33.6%
10.00%	90.00%	13.10%	36.8%
0.00%	100.00%	14.00%	40.0%

Positive Correlation  
– weak diversification potential

# Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0
B	14.0%	40.0%	

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	14.1%
80.00%	20.00%	6.80%	14.4%
70.00%	30.00%	7.70%	15.9%
60.00%	40.00%	8.60%	18.4%
50.00%	50.00%	9.50%	21.4%
40.00%	60.00%	10.40%	24.7%
30.00%	70.00%	11.30%	28.4%
20.00%	80.00%	12.20%	32.1%
10.00%	90.00%	13.10%	36.0%
0.00%	100.00%	14.00%	40.0%

No Correlation – some diversification potential

} Lower risk than asset A

# Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
<b>A</b>	5.0%	15.0%	-0.5
<b>B</b>	14.0%	40.0%	

Negative Correlation – greater diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	12.0%
80.00%	20.00%	6.80%	10.6%
70.00%	30.00%	7.70%	11.3%
60.00%	40.00%	8.60%	13.9%
50.00%	50.00%	9.50%	17.5%
40.00%	60.00%	10.40%	21.6%
30.00%	70.00%	11.30%	26.0%
20.00%	80.00%	12.20%	30.6%
10.00%	90.00%	13.10%	35.3%
0.00%	100.00%	14.00%	40.0%

}



# Example of Portfolio Combinations and Correlation

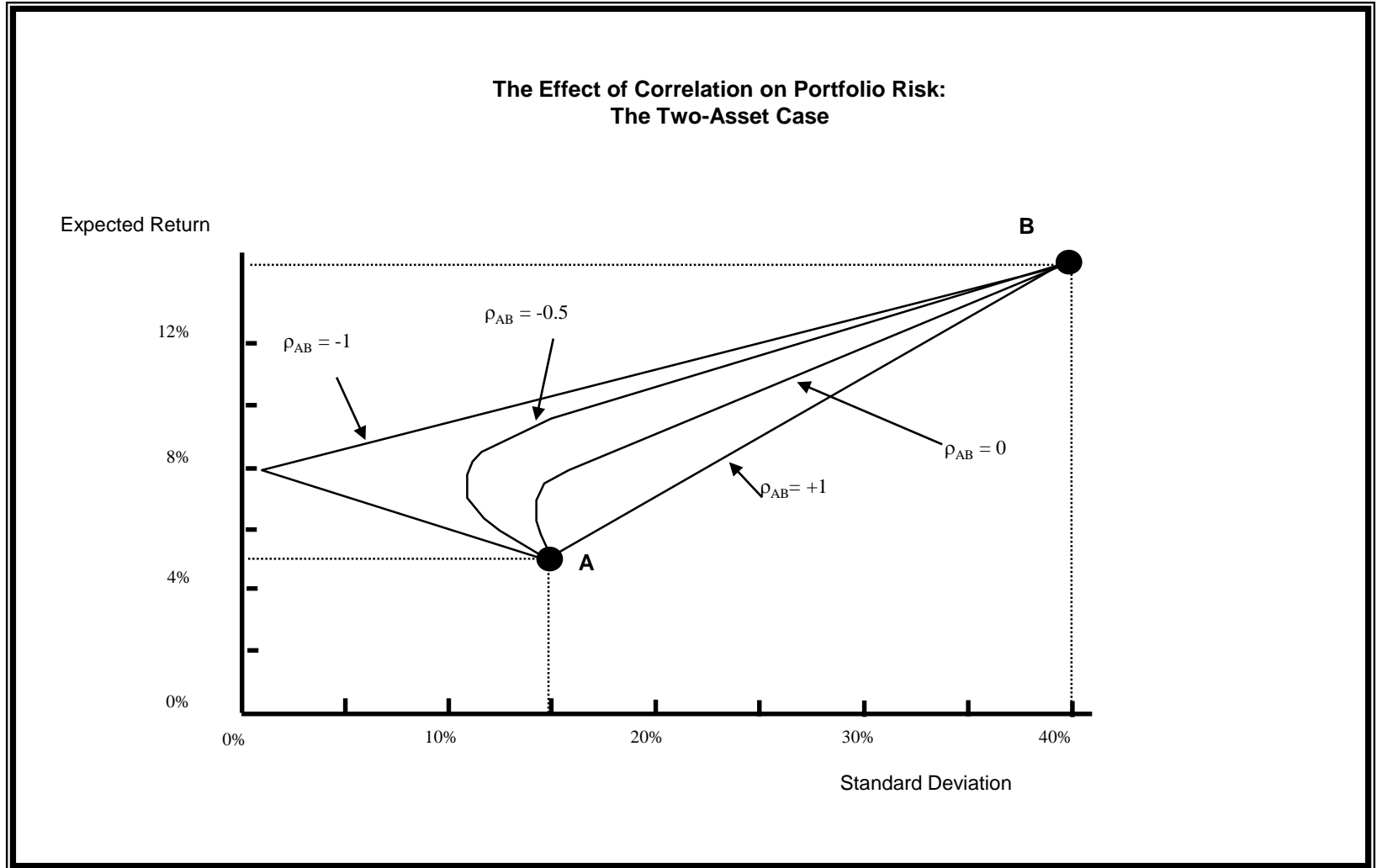
Asset	Expected Return	Standard Deviation	Correlation Coefficient
<b>A</b>	5.0%	15.0%	-1
<b>B</b>	14.0%	40.0%	

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	9.5%
80.00%	20.00%	6.80%	4.0%
70.00%	30.00%	7.70%	1.5%
60.00%	40.00%	8.60%	7.0%
50.00%	50.00%	9.50%	12.5%
40.00%	60.00%	10.40%	18.0%
30.00%	70.00%	11.30%	23.5%
20.00%	80.00%	12.20%	29.0%
10.00%	90.00%	13.10%	34.5%
0.00%	100.00%	14.00%	40.0%

Perfect Negative Correlation – greatest diversification potential

Risk of the portfolio is almost eliminated at 70% asset A

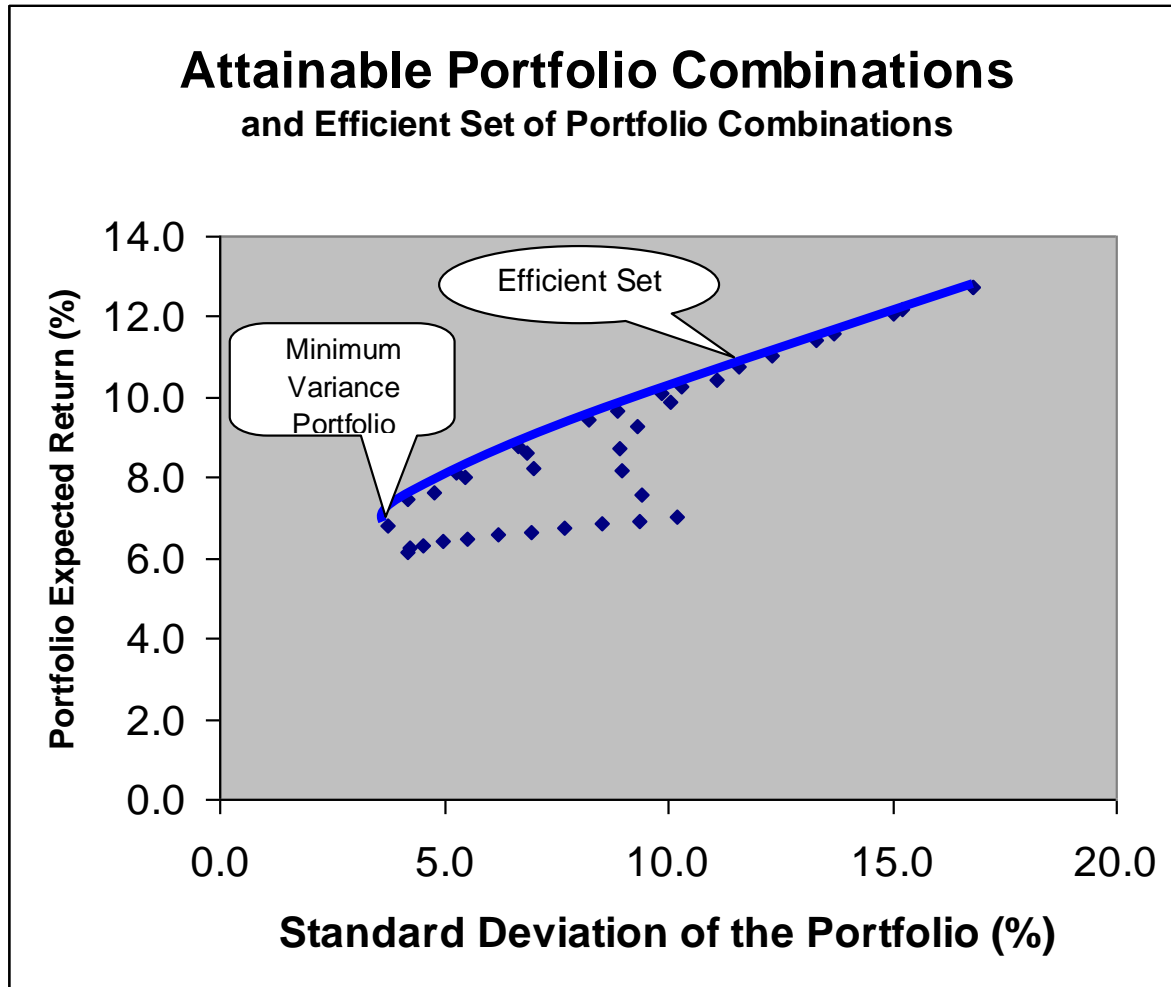
# Diversification of a Two Asset Portfolio Demonstrated Graphically



# An Exercise using T-bills, Stocks and Bonds

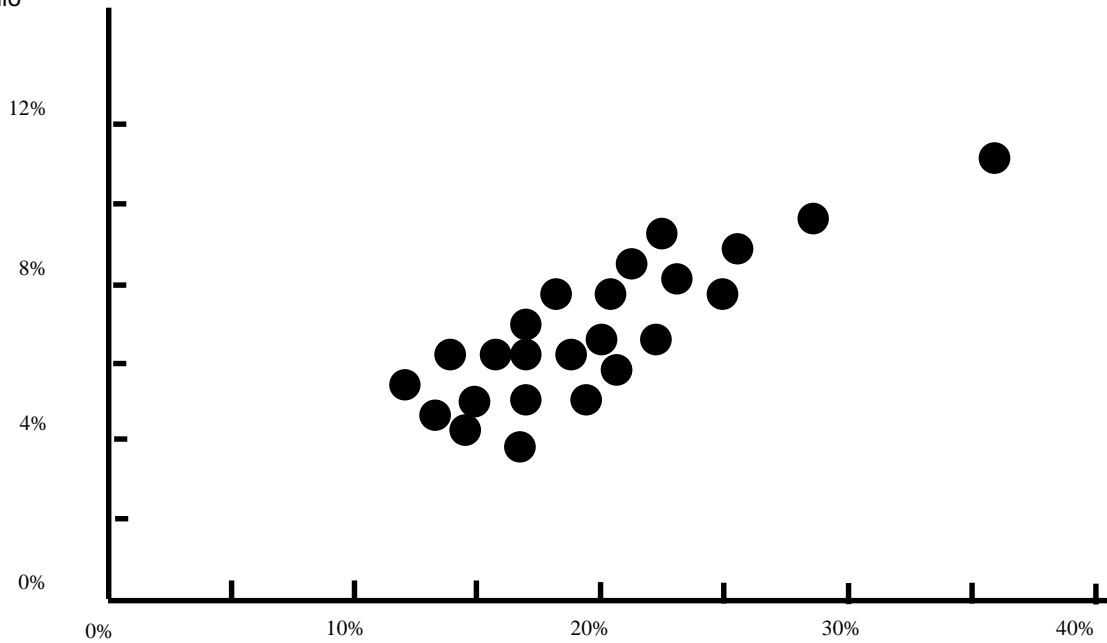
Base Data:			<b>Stocks</b>	<b>T-bills</b>	<b>Bonds</b>	
	Expected Return		12.73383	6.151702	7.007872	
	Standard Deviation		0.168	0.042	0.102	
Correlation Coefficient Matrix:						
		<b>Stocks</b>	1	-0.216	0.048	
		<b>T-bills</b>	-0.216	1.000	0.380	
		<b>Bonds</b>	0.048	0.380	1.000	
Portfolio Combinations:						
Weights				Portfolio		
Combination	Stocks	T-bills	Bonds	Expected Return	Variance	Standard Deviation
1	80.0%	10.0%	10.0%	11.5	0.0181	13.5%
2	90.0%	10.0%	0.0%	12.1	0.0226	15.0%
3	80.0%	20.0%	0.0%	11.4	0.0177	13.3%
4	70.0%	20.0%	10.0%	10.8	0.0138	11.7%
5	60.0%	20.0%	20.0%	10.3	0.0106	10.3%
6	50.0%	25.0%	25.0%	9.7	0.0079	8.9%
7	40.0%	20.0%	40.0%	9.1	0.0065	8.1%
8	30.0%	0.0%	70.0%	8.7	0.0080	8.9%
9	20.0%	80.0%	0.0%	7.5	0.0018	4.2%
10	10.0%	70.0%	20.0%	7.0	0.0018	4.3%
11	0.0%	100.0%	0.0%	6.2	0.0017	4.2%

# Results Using only Three Asset Classes



### Plotting Achievable Portfolio Combinations

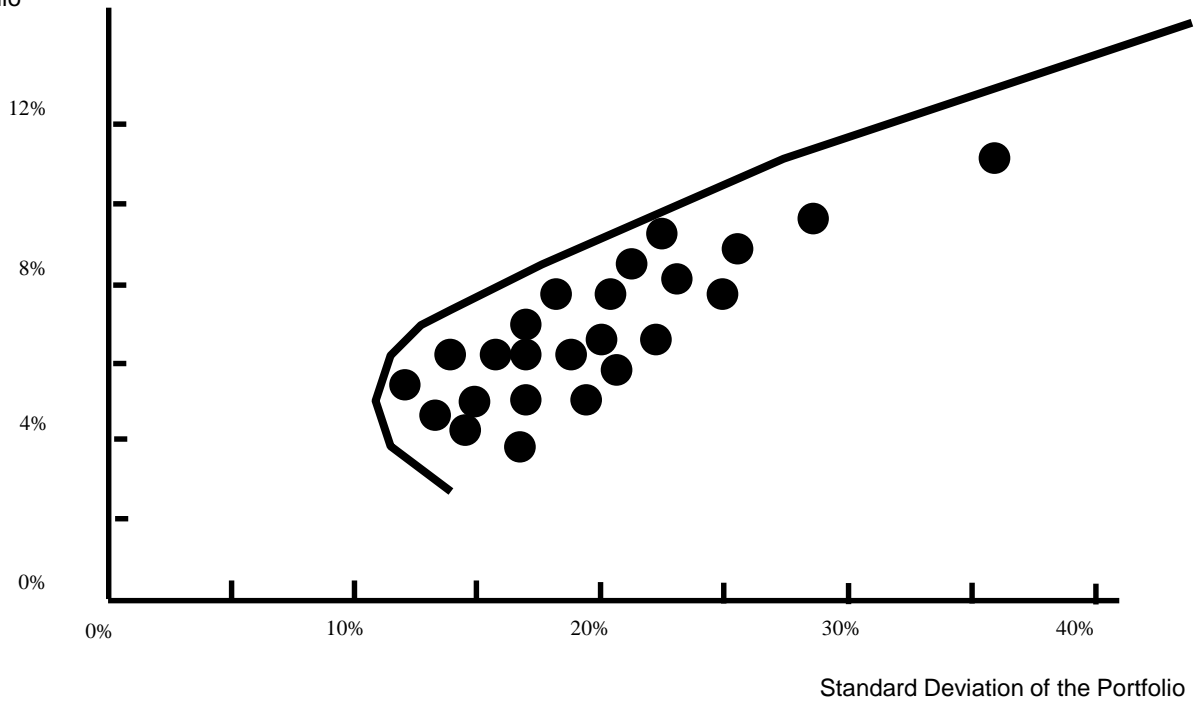
Expected Return on the Portfolio



Standard Deviation of the Portfolio

### The Efficient Frontier

Expected Return on the Portfolio



# Data Limitations

- Because of the need for so much data, MPT was a theoretical idea for many years.
- Later, a student of Markowitz, named William Sharpe worked out a way around that...creating the Beta Coefficient as a measure of volatility and then later developing the CAPM.

# Question

Assume the following statistics for Stocks A, B, and C:

	<i>Stock A</i>	<i>Stock B</i>	<i>Stock C</i>
Expected return	.20	.14	.10
Standard deviation	.232	.136	.195



# Question

The correlation coefficients between the three stocks are:

	<i>Stock A</i>	<i>Stock B</i>	<i>Stock C</i>
<i>Stock A</i>	1.000		
<i>Stock B</i>	0.286	1.000	
<i>Stock C</i>	0.132	-0.605	1.000

# Question

**An investor seeks a portfolio return of 12%.**

**Which combinations of the stocks accomplish this objective?  
Which of those combinations achieves the least amount of risk?**

# Question

**Solution: Two combinations achieve a 12% return:**

**1) 50% in B, 50% in C:  $(.5)(14\%) + (.5)(10\%) = 12\%$**

**2) 20% in A, 80% in C:  $(.2)(20\%) + (.8)(10\%) = 12\%$**

**3) There may be many other combinations???**

# Question

**Calculate the variance of the B/C combination:**

$$\begin{aligned}\sigma_p^2 &= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \rho_{AB} \sigma_A \sigma_B \\ &= (.50)^2 (.0185) + (.50)^2 (.0380) \\ &\quad + 2(.50)(.50)(-.605)(.136)(.195) \\ &= .0046 + .0095 - .0080 \\ &= .0061\end{aligned}$$

# Question

**Calculate the variance of the A/C combination:**

$$\begin{aligned}\sigma_p^2 &= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \rho_{AB} \sigma_A \sigma_B \\ &= (.20)^2 (.0538) + (.80)^2 (.0380) \\ &\quad + 2(.20)(.80)(.132)(.232)(.195) \\ &= .0022 + .0243 + .0019 \\ &= .0284\end{aligned}$$

# Question

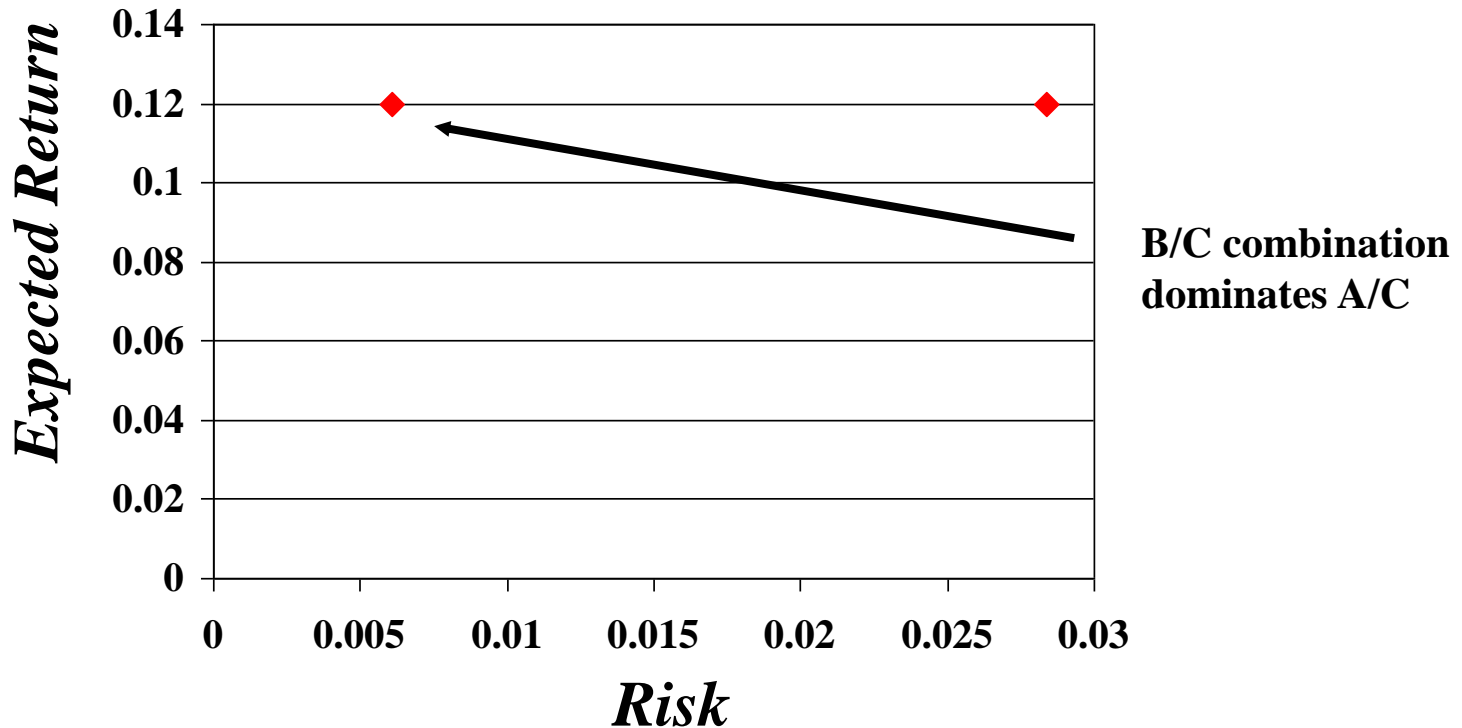
**Investing 50% in Stock B and 50% in Stock C achieves an expected return of 12% with the lower portfolio variance. Thus, the investor will likely prefer this combination to the alternative of investing 20% in Stock A and 80% in Stock C.**

# Concept of Dominance

- A portfolio dominates all others if:
  - For its level of expected return, there is no other portfolio with less risk
  - For its level of risk, there is no other portfolio with a higher expected return

# Concept of Dominance

In the previous example, the B/C combination dominates the A/C combination:





# Minimum Variance Portfolio

- The minimum variance portfolio is the particular combination of securities that will result in the least possible variance
- Solving for the minimum variance portfolio requires basic calculus

# Minimum Variance Portfolio

- For a two-security minimum variance portfolio, the proportions invested in stocks A and B are:

$$x_A = \frac{\sigma_B^2 - \sigma_A \sigma_B \rho_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A \sigma_B \rho_{AB}}$$

$$x_B = 1 - x_A$$

# Minimum Variance Portfolio

**Assume the same statistics for Stocks A and B as in the previous example. What are the weights of the minimum variance portfolio in this case?**

# Minimum Variance Portfolio

**Solution:** The weights of the minimum variance portfolios in this case are:

$$x_A = \frac{\sigma_B^2 - \sigma_A \sigma_B \rho_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A \sigma_B \rho_{AB}} = \frac{.06 - (.224)(.245)(.5)}{.05 + .06 - 2(.224)(.245)(.5)} = 59.07\%$$

$$x_B = 1 - x_A = 1 - .5907 = 40.93\%$$

# Minimum Variance Portfolio

