Module-2

Lecture-9

Cruise Flight - Range and Endurance of Jet driven Aircraft.

Range & Endurance - Jet powered aircraft

- In case of jet powered aircrafts specific fuel consumption is given as Thrust Specific Fuel Consumption and it is defined as weight of fuel consumed per unit thrust per unit time.
- Here thrust is used in contrast to power in case of propeller driven aircraft.

$$TSFC = \frac{N(fuel)}{(N(thrust))(s)} \tag{1}$$

$$\Rightarrow \frac{N(fuel)}{s} = (TSFC)T_A \tag{2}$$

Here T_A is thrust available.

Endurance

- For level, un-accelerated flight, the pilot adjusts the throttle such that thrust available, T_A equals the thrust required, T_R. Therefore, weight of fuel consumed per hour will be minimum when thrust required is minimum.
- We know that for minimum thrust required, C_L/C_D should be maximum.
- Therefore for a jet aircraft, maximum endurance occurs when the airplane is flying at a velocity such that T_R is minimum.

Let us calculate the expression for endurance of a jet aircraft.

- Let dW be the very small change in the weight of the airplane due to fuel consumption over a time increment dt. Then

$$dW = -c_t T_A dt$$

$$\Rightarrow dt = -\frac{dW}{c_t T_A} \tag{3}$$

- Then, the overall endurance can be computed from Equation 3 as:

$$E = -\int_{W_o}^{W_1} \frac{dW}{c_t T_A}$$

$$E = \int_{W_o}^{W_1} \frac{1}{c_t} \frac{L}{D} \frac{dW}{W}$$

thus,

$$E = \frac{1}{c_t} \frac{C_L}{C_D} \ln \frac{W_o}{W_1} \tag{4}$$

Range

- Similarily to obtain the expression for range, we know from earlier discussions that in order to cover the longest distance, the aircraft should only consume minimum weight of fuel per unit distance. For a jet aircraft:

$$\frac{N(fuel)}{(m)} = \frac{(TSFC)T_A}{V}$$

- For steady, level flight, $T_A = T_R$, minimum weight of fuel per unit distance corresponds to a minimum T_A/V. Kindly note that, since $T_A = T_R$, hence range of aircraft (with jet engine) will be maximum if $N(fuel)/m$ is minimum or T_R/V is minimum.

- $[T_R/V]_{min}$ corresponds to tangent point shown in Figure 1

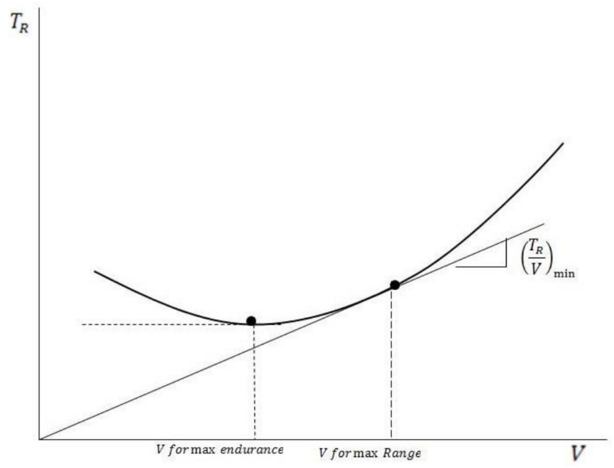

Figure 1: Thrust required curve to determine $\left[\frac{T_R}{V}\right]_{min}$

- For steady flight, $T_R = D$. Thus,

$$\frac{T_R}{V} = \frac{D}{V} = \frac{\frac{1}{2}\rho V^2 S C_D}{V} = \frac{1}{2}\rho V S C_D = \frac{1}{2}\rho S\sqrt{\frac{2\left(\frac{W}{S}\right)}{\rho C_L}}$$

$$\frac{T_R}{V} \propto \frac{1}{\frac{C_L^{\frac{1}{2}}}{C_D}} \tag{5}$$

- Hence maximum range for a jet aircraft occurs when the aircraft is flying at a velocity such that $C_L^{1/2}/C_D$ is maximum. It can be shown that such a requirement corresponds to:

$$C_L = \sqrt{\frac{3C_{D_o}}{K}}$$

- The expression for overall range for a Jet aircraft can be derived as:

$$ds = V dt = -\frac{V dW}{c_t T_A}$$

$$\Rightarrow R = \int_0^R ds = -\int_{W_o}^{W_1} \frac{V_\infty}{d} W c_t T_A$$

- For steady, level flight, the pilot adjusts the throttle such that $T_R = T_A$ and recalling for steady, level flight $L = W$ and $T = D$, we get:

$$R = \int_{W_1}^{W_o} \frac{V_\infty}{c_t} \frac{C_L}{C_D} \frac{dW}{W}$$

since,

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

- R becomes,

$$R = \int_{W_1}^{W_o} \sqrt{\frac{2}{\rho_\infty S}} \frac{\frac{C_L^{\frac{1}{2}}}{C_D}}{c_t} \frac{dW}{W^{\frac{1}{2}}}$$

thus,

$$\Rightarrow R = 2\sqrt{\frac{2}{\rho_\infty S}} \frac{1}{c_t} \frac{C_L^{\frac{1}{2}}}{C_D} \left(W_o^{\frac{1}{2}} - W_1^{\frac{1}{2}}\right) \tag{6}$$