Module-2

Lecture-8

Cruise Flight - Range and Endurance of Propeller Driven Aircraft

Range and Endurance

Range is defined as the total distance (measured with respect to ground) traversed by the airplane on a full tank of fuel.

Endurance is defined as the total time that an airplane stays in the air on a full tank of fuel.

For different applications, it may be desirable to maximize one or other, or both characteristics. The parameters that maximize range are different from those that maximizes the endurance. Additionally, these parameters are also different for propeller and jet powered aircrafts.

Range & Endurance : Propeller driven aircraft

For a propeller driven aircraft, the most important factor that influences range and endurance is the specific fuel consumption of the reciprocating engine.

Specific Fuel Consumption *(SFC), is defined as the weight of the fuel consumed by the reciprocating engine per unit power per unit time.*

$$SFC = \frac{N(fuel)}{(J/s)(s)} \tag{1}$$

Endurance

In order to stay airborne for the longest duration, i.e. for maximum endurance the engine must use minimum Newton's of fuel per unit time. From the Equation 1, we can see that:

$$\frac{N(fuel)}{(s)} \propto SFC(P_R) \tag{2}$$

So from Equation 2 depicting the proportionality, we quickly conclude that for maximum endurance, the power required by the airplane should be minimum. We have already shown in our previous discussions, that for an aircraft to fly at the minimum power required, $C_L^{3/2}/C_D$ should be maximum. Designating SFC as c and considering the product $c.P.dt$, where P is engine power and dt is a small increment in time, we have:

$$cPdt = \frac{N(fuel)}{(J/s)(s)} \times \frac{J}{s} \times s = N(fuel) \tag{3}$$

Thus, $cPdt$ represents the differential change in the weight of fuel over a small interval of time, dt. Let,

W_o - gross weight of the airplane

W_1 - weight of the airplance without fuel

W_f - weight of the fuel

Then, we have:

$$W_1 = W_o - W_f \tag{4}$$

and

$$dW_f = dW = -cPdt \tag{5}$$

$$\Rightarrow dt = -\frac{dW}{cP} \tag{6}$$

Denoting endurance as E

$$\int_0^E dt = -\int_{W_o}^{W_1} \frac{dW}{cP}$$

$$\Rightarrow E = \int_{W_o}^{W_1} \frac{dW}{cP} \tag{7}$$

Range

Now considering range; in order to cover the longest distance, we must ensure minimum weight of fuel consumed per unit distance. From the relations discussed above, we can get the proportionality:

$$\frac{N(fuel)}{(m)} \propto \frac{SFC(P_R)}{V}$$

Thus for obtaining maximum range for any flight, the ratio P_R/V should be minimum. $[P_R/V]_{min}$ for cruise flight implies that thrust required is minimum and for T_R to be minimum, C_L/C_D should be maximum. Minimum value of P_R/V precisely corresponds to the tangent point in Figure 1, which also corresponds to $[L/D]_{max}$ or $[C_L/C_D]_{max}$. Now

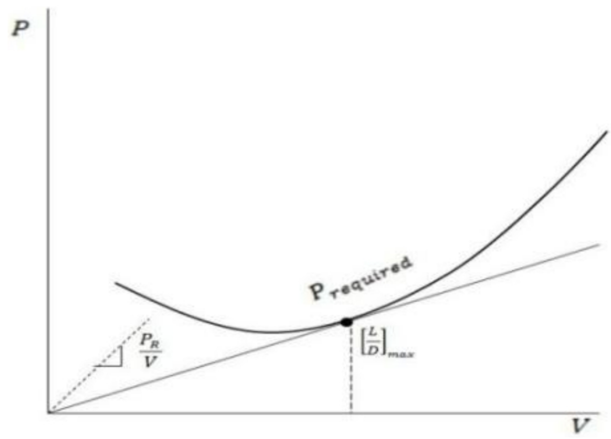

Figure 1: Power required curve to determine $\left[\frac{L}{D}\right]_{max}$

to calculate the range, from Equation 7:

$$ds = V dt = -\int_{W_o}^{W_1} \frac{V dW}{cP}$$

$$\Rightarrow \int_0^R ds = \int_{W_1}^{W_o} \frac{V dW}{cP}$$

$$\Rightarrow R = \int_{W_1}^{W_o} \frac{V dW}{cP} \tag{8}$$

Breguet Formula

For a propeller driven aircraft, we know that:

$$P_A = \eta P$$

thus,

$$P = \frac{P_A}{\eta} = \frac{DV}{\eta} \tag{9}$$

Substitute Equation 9 in Equation 8, we get:

$$R = \int_{W_1}^{W_o} \frac{V dW}{cP} = \int_{W_1}^{W_o} \frac{V \eta dW}{cDV} = \int_{W_1}^{W_o} \frac{\eta dW}{cD} \tag{10}$$

Multiplying Equation 10 by W/W and noting that for steady, level flight, $W = L$, we get:

$$R = \int_{W_1}^{W_o} \frac{\eta}{cD} \frac{W}{W} dW = \int_{W_1}^{W_o} \frac{\eta}{c} \frac{L}{D} \frac{dW}{W}$$

$$\Rightarrow \frac{\eta}{c} \frac{C_L}{C_D} \int_{W_1}^{W_o} \frac{dW}{W}$$

thus,

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_o}{W_1} \tag{11}$$

Similarly by using Equation 7 and Equation 9 and by applying steady, level flight condition, $L = W$, we get:

$$E = \int_{W_1}^{W_o}\frac{dW}{cP} = \int_{W_1}^{W_o}\frac{\eta dW}{cDV} = \int_{W_1}^{W_o}\frac{\eta}{c}\frac{L}{DV}\frac{dW}{W}$$

Substituting,

$$L = W = \frac{1}{2}\rho V^2 S C_L \text{ and then } V = \sqrt{\frac{2\left(\frac{W}{S}\right)}{\rho C_L}}$$

we get:

$$E = \int_{W_1}^{W_o}\frac{\eta}{c}\frac{C_L}{C_D}\sqrt{\frac{\rho S C_L}{2}}\frac{dW}{W^{\frac{3}{2}}} \tag{12}$$

Assuming C_L, C_D, η, c and ρ(constant altitude) are all constant, Equation 12 becomes:

$$E = -2\frac{\eta}{c}\frac{C_L^{\frac{3}{2}}}{C_D}\left[\frac{\rho S}{2}\right]^{\frac{1}{2}}\left[W^{-\frac{1}{2}}\right]_{W_1}^{W_o}$$

$$E = \frac{\eta}{c}\frac{C_L^{\frac{3}{2}}}{C_D}(2\rho S)^{\frac{1}{2}}\left(W_1^{-\frac{1}{2}} - W_o^{-\frac{1}{2}}\right) \tag{13}$$