LECTURE 10

Permeability Of Soil:

Pressure, Elevation and Total Heads

In soils, the interconnected pores provide passage for water. A large number of such flow paths act together, and the average rate of flow is termed the coefficient of permeability, or just permeability. It is a measure of the ease that the soil provides to the flow of water through its pores.



At point **A**, the pore water pressure (**u**) can be measured from the height of water in a standpipe located at that point.

The height of the water column is the **pressure head** (h_w). $h_w = u/\Box_w$

To identify any difference in pore water pressure at different points, it is necessary to eliminate the effect of the points of measurement. With this in view, a datum is required from which locations are measured.

The elevation head (h_z) of any point is its height above the datum line. The height of water level in the standpipe above the datum is the **piezometric head** (h).

$\mathbf{h} = \mathbf{h}_{\mathbf{z}} + \mathbf{h}_{\mathbf{w}}$

Total head consists of *three components:* elevation head, pressure head, and velocity head. As seepage velocity in soils is normally low, velocity head is ignored, and total head becomes equal to the piezometric head. Due to the low seepage velocity and small size of pores, the flow of water in the pores is steady and laminar in most cases. Water flow takes place between two points in soil due to the difference in total heads.

Darcy's law states that there is a linear relationship between flow velocity (\mathbf{v}) and hydraulic gradient (\mathbf{i}) for any given saturated soil under steady laminar flow conditions.



If the rate of flow is **q** (volume/time) through cross-sectional area (**A**) of the soil mass, Darcy's Law can be expressed as

 $\mathbf{v} = \mathbf{q}/\mathbf{A} = \mathbf{k}.\mathbf{i}$

where $\mathbf{k} =$ permeability of the soil $\mathbf{i} = \Delta \mathbf{h}/\mathbf{L}$ $\Delta \mathbf{h} =$ difference in total heads $\mathbf{L} =$ length of the soil mass

The flow velocity (\mathbf{v}) is also called the Darcian velocity or the **superficial velocity**. It is different from the actual velocity inside the soil pores, which is known as the **seepage velocity**, \mathbf{v}_{s} . At the particulate level, the water follows a tortuous path through the pores. Seepage velocity is always greater than the superficial velocity, and it is expressed as:

$$v_{s} = \frac{q}{A_{v}} = \frac{q}{A_{v}} \cdot \frac{A}{A} \approx \frac{v}{n}$$

where A_V = Area of voids on a cross section normal to the direction of flow n = porosity of the soil

Permeability of Different soils:

Permeability (k) is an engineering property of soils and is a function of the soil type. Its value depends on the average size of the pores and is related to the distribution of particle sizes, particle shape and soil structure. The ratio of permeabilities of typical sands/gravels to those of typical clays is of the order of 10^6 . A small proportion of fine material in a coarse-grained soil can lead to a significant reduction in permeability.

For different soil types as per grain size, the orders of magnitude for permeability are as follows:

Soil	k (cm/sec)
Gravel	10 ⁰
Coarse sand	10° to 10^{-1}
Medium sand	10^{-1} to 10^{-2}

Fine sand	10^{-2} to 10^{-3}
Silty sand	10^{-3} to 10^{-4}
Silt	1 x 10 ⁻⁵
Clay	10^{-7} to 10^{-9}

In soils, the permeant or pore fluid is mostly water whose variation in property is generally very less. Permeability of all soils is strongly influenced by the density of packing of the soil particles, which can be represented by void ratio (\mathbf{e}) or porosity (\mathbf{n}).

For Sands

In sands, permeability can be empirically related to the square of some representative grain size from its grain-size distribution. For filter sands, Allen Hazen in 1911 found that $\mathbf{k} \square 100 (\mathbf{D}_{10})^2$ cm/s where \mathbf{D}_{10} = effective grain size in cm.

Different relationships have been attempted relating void ratio and permeability, such as $\mathbf{k} \square \mathbf{e}^3/(1+\mathbf{e})$, and $\mathbf{k} \square \square \mathbf{e}^2$. They have been obtained from the Kozeny-Carman equation for laminar flow in saturated soils.

 $k = \frac{1}{k_0 k_T S_s^2} \cdot \frac{e^3}{1+e} \cdot \frac{\gamma_w}{\eta}$

where \mathbf{k}_0 and \mathbf{k}_T are factors depending on the shape and tortuosity of the pores respectively, \mathbf{S}_S is the surface area of the solid particles per unit volume of solid material, and $\Box \Box_w$ and $\Box \Box$ are unit weight and viscosity of the pore water. The equation can be reduced to a simpler form as

$$k = C.\frac{e^3}{1+e} \approx C.e^2$$

For Silts and Clays

For silts and clays, the Kozeny-Carman equation does not work well, and $\log k$ versus e plot has been found to indicate a linear relationship.

For clays, it is typically found that

$$\log_{10} k = \frac{e - e_k}{C_k}$$

where C_k is the permeability change index and e_k is a reference void ratio.

LECTURE 11

Laboratory Measurement of Permeability:

Constant Head Flow

Constant head permeameter is recommended for coarse-grained soils only since for such soils, flow rate is measurable with adequate precision. As water flows through a sample of cross-section area A, steady total head drop h is measured across length L.



Permeability **k** is obtained from:

$$k = \frac{qL}{Ah}$$

Falling Head Flow:

Falling head permeameter is recommended for fine-grained soils.



Total head **h** in standpipe of area **a** is allowed to fall. Hydraulic gradient varies with time. Heads h_1 and h_2 are measured at times t_1 and t_2 . At any time t, flow through the soil sample of cross-sectional area **A** is

$$q = k.h.\frac{A}{L} \qquad (1)$$

Flow in unit time through the standpipe of cross-sectional area **a** is

Equating (1) and (2),

$$-a \cdot \frac{dh}{dt} = k \cdot h \cdot \frac{A}{L}$$

or
$$-\frac{dh}{h} = \left(\frac{kA}{La}\right) dt$$

Integrating between the limits,

$$\begin{split} \log_e \left(\frac{h_1}{h_2} \right) &= \frac{k \cdot A}{L \cdot a} (t_2 - t_1) \\ k &= \frac{L \cdot a \cdot \log_e \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)} \\ &= \frac{2 \cdot 3L \cdot a \log_{10} \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)} \end{split}$$

Seepage in Soils:



A rectangular soil element is shown with dimensions dx and dz in the plane, and thickness dy perpendicuar to this plane. Consider planar flow into the rectangular soil element.

In the x-direction, the net amount of the water entering and leaving the element is

$$\frac{\delta V_{\chi}}{\delta x}$$
.dx.dy.dz

Similarly in the z-direction, the difference between the water inflow and outflow is

$$\frac{\delta V_z}{\delta z}$$
.dz.dx.dy

For a two-dimensional steady flow of pore water, any imbalance in flows into and out of an element in the z-direction must be compensated by a corresponding opposite imbalance in the x-direction. Combining the above, and dividing by dx.dy.dz, the **continuity equation** is expressed as

$$\frac{\delta V_x}{\delta x} + \frac{\delta V_z}{\delta z} = 0$$

From Darcy's law, $V_x = k_x \cdot \frac{\delta h}{\delta x}$, $V_z = k_z \cdot \frac{\delta h}{\delta z}$, where **h** is the head causing flow.

When the continuity equation is combined with Darcy's law, the equation for flow is expressed as:

$$\mathbf{k}_{\mathbf{x}} \cdot \frac{\delta^2 h}{\delta x^2} + \mathbf{k}_{\mathbf{z}} \cdot \frac{\delta^2 h}{\delta z^2} = 0$$

For an isotropic material in which the permeability is the same in all directions (i.e. $k_x = k_z$), the **flow equation** is

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

This is the **Laplace equation** governing two-dimensional steady state flow. It can be solved *graphically*, *analytically*, *numerically*, *or analogically*.

For the more general situation involving *three-dimensional* steady flow, Laplace equation becomes:

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta y^2} + \frac{\delta^2 h}{\delta z^2} = 0$$