

Session_7

Cauchy's First Integral Theorems

If $f(z)$ be single valued and analytic throughout a simply connected bounded domain D , then for every simple closed curve (or closed contour) C in D

$$\oint_C f(z) dz = 0$$



Proof: Let $f(z) = u(x, y) + iv(x, y)$ where $z = x + iy$

$$\text{Now } \oint_C f(z) dz = \oint_C (u dx - v dy) + i \oint_C (u dy + v dx) \quad (1)$$

Since $f(z)$ is analytic $\Rightarrow f'(z) = u_x + iv_x$ exists in D

Let $f'(z)$ be continuous $\Rightarrow u_x, u_y, v_x$ & v_y are continuous in D

Applying Green's theorem from the equation (1) :

$$\begin{aligned} \oint_C f(z) dz &= \iint_R \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx dy + i \iint_R \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) dx dy \\ &= 0 + i0 \quad (\text{by C-R equations}) \end{aligned}$$

(where R = Region bounded by C)

$$\therefore \oint_C f(z) dz = 0$$