

## System Modeling & Control

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Lesson - 10

### Signal Flow Graph Introduction

- Alternative method to block diagram representation, developed by Samuel Jefferson Mason.
- Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.
- A signal-flow graph consists of a network in which nodes are connected by directed branches.
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.



## Fundamentals of Signal Flow Graphs

• Consider a simple equation below and draw its signal flow graph:

y = ax

• The signal flow graph of the equation is shown below;



- Every variable in a signal flow graph is designed by a **Node**.
- Every transmission function in a signal flow graph is designed by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.



## **Signal-Flow Graph Models**

$$Y_1(s) = G_{11}(s) \cdot R_1(s) + G_{12}(s) \cdot R_2(s)$$

$$Y_2(s) = G_{21}(s) \cdot R_1(s) + G_{22}(s) \cdot R_2(s)$$





### **Signal-Flow Graph Models**

are inputs and  $x_1$  and  $x_2$  are outputs

 $a_{11} \cdot x_1 + a_{12} \cdot x_2 + r_1 = x_1$ 

 $a_{21} \cdot x_1 + a_{22} \cdot x_2 + r_2 = x_2$ 





## Signal-Flow Graph Models

 $x_o$  is input and  $x_4$  is output





# Construct the signal flow graph for the following set of simultaneous equations.

$$x_2 = A_{21}x_1 + A_{23}x_3 \qquad x_3 = A_{31}x_1 + A_{32}x_2 + A_{33}x_3 \qquad x_4 = A_{42}x_2 + A_{43}x_3$$

- There are four variables in the equations (i.e., x<sub>1</sub>,x<sub>2</sub>,x<sub>3</sub>,and x<sub>4</sub>) therefore four nodes are required to construct the signal flow graph.
- Arrange these four nodes from left to right and connect them with the associated branches.



## Terminologies

- An input node or source contain only the outgoing branches. i.e., X<sub>1</sub>
- An output node or sink contain only the incoming branches. i.e., X<sub>4</sub>
- A path is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,
- $X_1$  to  $X_2$  to  $X_3$  to  $X_4$  $X_1$  to  $X_2$  to  $X_4$  $X_2$  to  $X_3$  to  $X_4$ A forward path is a path from the input node to the output node. i.e., $X_1$  to  $X_2$  to  $X_3$  to  $X_4$ , and  $X_1$  to  $X_2$  to  $X_4$ , are forward paths. $X_1$  to  $X_2$  to  $X_3$  to  $X_4$ , and  $X_1$  to  $X_2$  to  $X_4$ , are forward paths.
- A feedback path or feedback loop is a path which originates and terminates on the same node. i.e.;

X<sub>2</sub> to X<sub>3</sub> and back to X<sub>2</sub> is a feedback path.



## Terminologies

- A self-loop is a feedback loop consisting of a single branch. i.e.; A<sub>33</sub> is a self loop.
- The gain of a branch is the transmission function of that branch.
- The path gain is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X<sub>1</sub> to X<sub>2</sub> to X<sub>3</sub> to X<sub>4</sub> is A<sub>21</sub>A<sub>32</sub>A<sub>43</sub>
- The loop gain is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X<sub>2</sub> to X<sub>3</sub> and back to X<sub>2</sub> is A<sub>32</sub>A<sub>23.</sub>
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.





- a) Input node.
- b) Output node.
- c) Forward paths.
- d) Feedback paths (loops).
- e) Determine the loop gains of the feedback loops.
- f) Determine the path gains of the forward paths.
- g) Non-touching loops



**1.**  $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ 

**2.**  $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$ 





Nontouching loop gains;

**1.**  $[G_2(s)H_1(s)][G_4(s)H_2(s)]$  **2.**  $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$ **3.**  $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$ 



- a) Input node.
- b) Output node.
- c) Forward paths.
- d) Feedback paths.
- e) Self loop.
  - Determine the loop gains of the feedback loops.
  - ) Determine the path gains of the forward paths.





(c) Forward Paths



 $X_1$  to  $X_2$  to  $X_3$  to  $X_4$  to  $X_5$  to  $X_6$  to  $X_7$  to  $X_8$ 

 $X_1$  to  $X_2$  to  $X_7$  to  $X_8$ 

 $X_1$  to  $X_2$  to  $X_4$  to  $X_5$  to  $X_6$  to  $X_7$  to  $X_8$ 











(e) Self Loop(s)



### (f) Loop Gains of the Feedback Loops



| $A_{32}A_{23}$ | $A_{76}A_{67};$      |
|----------------|----------------------|
| $A_{43}A_{34}$ | A 65 A 76 A 57       |
| $A_{54}A_{45}$ | A <sub>73</sub>      |
| A65 A56        | $A_{42}A_{34}A_{23}$ |

 $A_{72}A_{57}A_{45}A_{34}A_{23}$ 

 $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$ 





 $A_{42}A_{54}A_{65}A_{76}$ 

A 72

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#### Lesson - 11

# Mason's Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.



## Mason's Rule:

The transfer function, *C(s)/R(s)*, of a system represented by a signal-flow graph is;



Where

- n = number of forward paths.
- $P_i$  = the *i*<sup>th</sup> forward-path gain.
- $\Delta$  = Determinant of the system
- $\Delta_i$  = Determinant of the *I*<sup>th</sup> forward path
- Δ is called the signal flow graph determinant or characteristic function. Since Δ=0 is the system characteristic equation.



## Mason's Rule:



∆ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

 $\Delta_i$  = value of  $\Delta$  for the part of the block diagram that does not touch the i-th forward path ( $\Delta_i$  = 1 if there are no non-touching loops to the i-th path.)



#### Systematic approach

- 1. Calculate forward path gain *P<sub>i</sub>* for each forward path *i*.
- 2. Calculate all loop transfer functions
- 3. Consider non-touching loops 2 at a time
- 4. Consider non-touching loops 3 at a time
- 5. etc
- 6. Calculate  $\Delta$  from steps 2,3,4 and 5
- 7. Calculate  $\Delta_i$  as portion of  $\Delta$  not touching forward path *i*

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



$$L_1 = G_1 G_4 H_1, \quad L_2 = -G_1 G_2 G_4 H_2, \quad L_3 = -G_1 G_3 G_4 H_2$$

#### Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph

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#### There are no non-touching loops, therefore

 $\Delta$  = 1- (sum of all individual loop gains)

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - \left(G_1 G_4 H_1 - G_1 G_2 G_4 H_2 - G_1 G_3 G_4 H_2\right)$$



Eliminate forward path-1

 $\Delta_1 = 1$ - (sum of all individual loop gains)+...  $\Delta_1 = 1$ 

#### Eliminate forward path-2

 $\Delta_2 = 1$ - (sum of all individual loop gains)+...  $\Delta_2 = 1$ 



### Example#1: Continue

$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 - G_1 G_4 H_1 + G_1 G_2 G_4 H_2 + G_1 G_3 G_4 H_2}$$

$$=\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_2G_4H_2+G_1G_3G_4H_2}$$



# Find the transfer function, C(s)/R(s), for the signal-flow graph in figure below.



There is only one forward Path.



 $P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$ 

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### Non-touching loops taken two at a time.



L1 and L2:  $G_2(s)H_1(s)G_4(s)H_2(s)$  L2 and L3:  $G_4(s)H_2(s)G_7(s)H_4(s)$ L1 and L3:  $G_2(s)H_1(s)G_7(s)H_4(s)$ 



### Non-touching loops taken three at a time.



### L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$





#### Eliminate forward path-1

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## Example#2

$$\begin{split} \Delta &= 1 - [G_2(s)H_1(s) + G_4(s)H_2(s) \\ &+ G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &+ G_4(s)H_2(s)G_7(s)H_4(s)] \\ &- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)] \end{split}$$





### Example#3: Loop Gains of the Feedback Loops



 $L_{1} = A_{32}A_{23}$  $L_{2} = A_{43}A_{34}$  $L_{3} = A_{54}A_{45}$  $L_{4} = A_{65}A_{56}$ 

 $L_{5} = A_{76}A_{67}$   $L_{6} = A_{77}$   $L_{7} = A_{42}A_{34}A_{23}$   $L_{8} = A_{65}A_{76}A_{67}$   $L_{9} = A_{72}A_{57}A_{45}A_{34}A_{23}$   $L_{10} = A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$ 

### Example#3: two non-touching loops



### Example#3: Three non-touching loops











 $[G_{1} \cdot (-1)]; \quad [G_{2} \cdot (-1)]; \quad [G_{1} \cdot (-1) \cdot G_{2} \cdot 1]; \quad [(-1) \cdot G_{1} \cdot 1 \cdot (-1)]; \\ [(-1) \cdot G_{1} \cdot (-1) \cdot G_{2} \cdot 1 \cdot (-1)]; \quad [1 \cdot G_{2} \cdot 1 \cdot (-1)]; \quad [1 \cdot G_{2} \cdot 1 \cdot G_{1} \cdot 1 \cdot (-1)]. \\ \hline 3 \quad 2 \quad non-touching \ loops': \\ [G_{1} \cdot (-1)] \cdot [G_{2} \cdot (-1)]; \quad [(-1) \cdot G_{1} \cdot 1 \cdot (-1)] \cdot [G_{2} \cdot (-1)]; \\ \hline 1 \cdot G_{2} \cdot 1 \cdot (-1)] \cdot [G_{1} \cdot (-1)]. \\ \hline \end{bmatrix}$ 



Then:

$$\varDelta = 1 + 2G_2 + 4G_1G_2$$

4 forward paths:  

$$p_1 = (-1) \cdot G_1 \cdot 1$$
  $\Delta_1 = 1 + G_2$   
 $p_2 = (-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1$   $\Delta_2 = 1$   
 $p_3 = 1 \cdot G_2 \cdot 1$   $\Delta_3 = 1 + G_1$   
 $p_4 = 1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1$   $\Delta_4 = 1$ 



### We have

 $\frac{C(s)}{R(s)} = \frac{\sum p_k \Delta_k}{\Delta}$  $=\frac{G_2-G_1+2G_1G_2}{1+2G_2+4G_1G_2}$ 

Example-7: Determine the transfer function C/R for the block diagram below by signal flow graph techniques.





Since no loops touch the nodes of P2, 
$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_4 + G_2 G_4 H_1 - G_1 G_2 G_4 H_1 + G_2 G_3 G_4 H_2}{1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}$$
therefore  $\Delta_2 = \Delta$ .

Example-6: Find the control ratio C/R for the system given below.





### Feedback characteristics of control system

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In control system the reeclback is used to reduce the difference between the reference input and the controlled output. This difference is called an error En control system analyses. The feedback En control system not only reduces the error but also reduces the sensettenety of the system to parameter variations and unwanted Enternal and external desturbances. The parameter of the system may have a tendoncy to vary under the variety of changing conditions. Such parameter varéations affect the system performance adversig. Hence it is necessary to make the system Ensensettue to such parameter vouréations, as far as possible The beneficial effects of feedback in feedback system are:-1. The controlled variable accurately follows the descred value. I Effect on the controlled vareable of external desturbance other than those arrocéated with the reedback reneor are greatly reduced. III, Effect of varéation in controller and process parameters (the forward path) iv, Feedback En control system greatly emproves the speed of Etx response compare to the response speed capabéléty of the plant / components compositing the Ryptem (forward path)