

8. Spring design

Objectives

- Identify, describe, and understand principles of several types of springs including helical compression springs, helical extension springs, torsion tubes, and leaf spring systems.
- Design and analyze helical compression springs, including compatibility with allowable stresses.
- Develop necessary analytical tools for spring design.
- Understand the principles of operation of springs and how to analyze the energy being absorbed by springs.
- Review principles of design for other types of springs, such as extension springs and leaf springs.
- Select predesigned springs from manufacturers' catalogs and incorporate them in appropriate designs.

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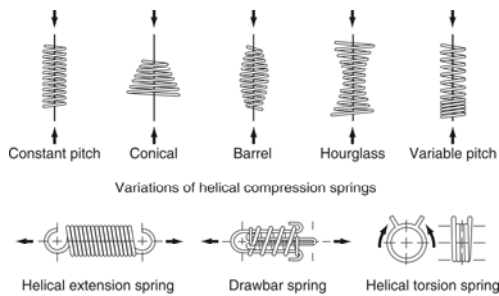
Introduction

- Springs give a relatively large elastic deflection.
- Application of springs
 - Control of motion in machines
 - Reduction of transmitted forces as a result of impact or shock loading
 - Storage of energy
 - Measurement of force

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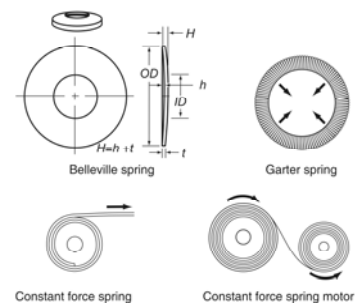
Fig 8.1 Common Types of Springs



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Fig 8.1 Common Types of Springs



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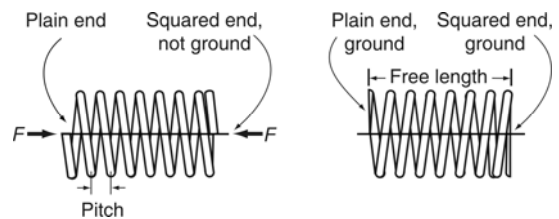
Helical Compression springs

- Made from round wire and wrapped in cylindrical form with a fixed pitch
- Plain end
 - Least expensive
 - Tends to bow sideways under load
- Plain and ground end
 - Better mating conditions being flat
 - Likely to get entangled in storage
- Squared end
- Squared and ground end

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Fig. 8.2 Types of compression springs



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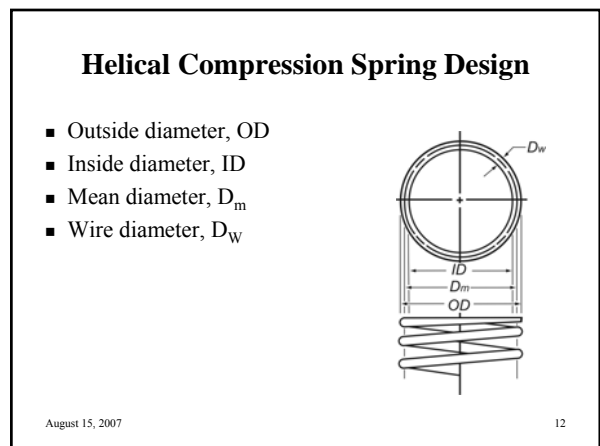
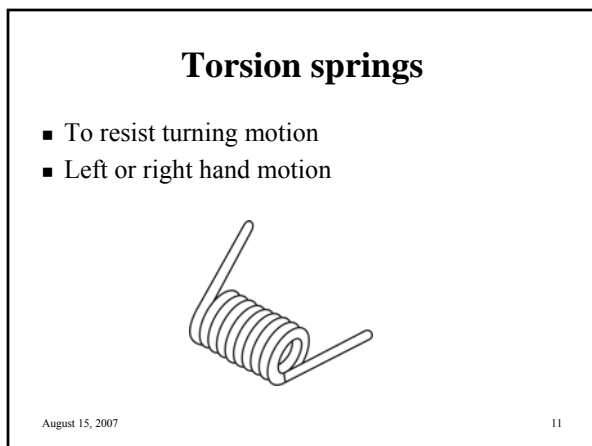
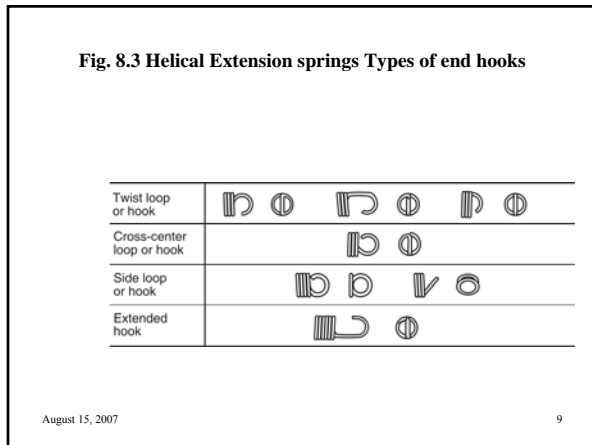
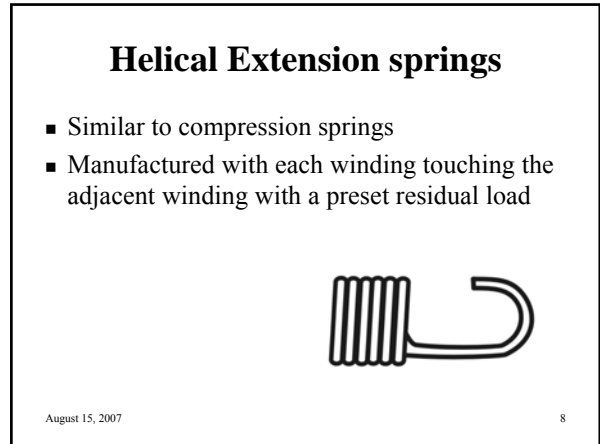
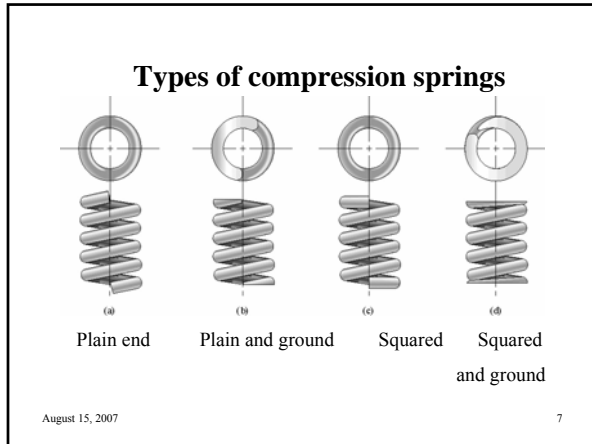
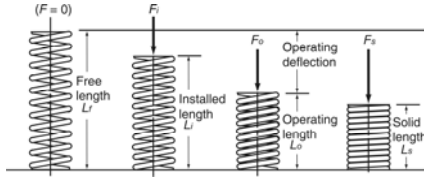


Fig. 8.6 Helical Compression Spring Design

- Free length, L_f
- Solid length, L_s
- Deflection, δ



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Spring Rate

- Spring rate (k) is ratio of change in force to the change in length

$$k = \frac{\Delta F}{\Delta L} \quad \text{Eq 8.1}$$

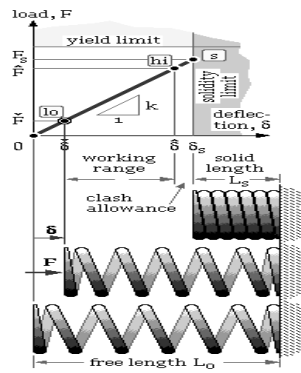
- Force (F) exerted by the spring is
- $F = k(L_f - L_o)$ Eq 8.2

- Appendix 12 Standard spring selection

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Spring animation



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Spring index

- Ratio of mean diameter of a spring to the wire from which the spring is constructed
- Spring index, C

$$C = \frac{D_m}{D_w} \quad \text{Eq 8.3}$$

- See Appendix 10 for wire sizes used

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Spring index

- Low indices result in difficulty with spring manufacture and in stress concentrations induced by curvature.
- Springs in the range $5 \leq C \leq 12$ are preferred, while indices less than 3 are generally impractical.

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Number of coils (Tab 8.1)

Type of ends	Free length	Total coils	Solid height
Plain	$PN_a + D_w$	N_a	$D_w N_a + D_w$
Plain ground	PN_a	N_a	$D_w N_a$
Squared	$PN_a + 3D_w$	$N_a + 2$	$D_w N_a + 3D_w$
Squared and ground	$PN_a + 2D_w$	$N_a + 2$	$D_w N_a + 2D_w$

N_a = Number of active coils; P = Pitch of coils; D_w = diameter of wire

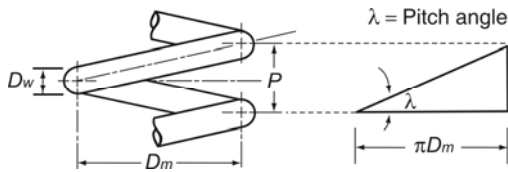
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Pitch

- Number of coils per inch of length

$$\lambda = \tan^{-1} \frac{P}{\pi D_m}$$



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Spring materials

- Steel
 - Most common
- Phosphor bronze
- Cold wound for small size (<5/16 in)
- Wound from hot rolled bar

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Types of Materials

- Hard drawn high carbon steel
- Oil tempered high carbon steel
- Stainless steel
- Light-duty springs
 - Copper or nickel based alloys

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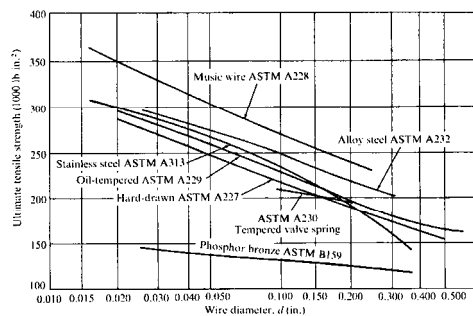
Spring materials

- Stress relieving
 - Heated to 400 – 800° F
 - Held for a period of time
- Shot peening

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Min tensile strength of Spring materials



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Allowable stress

- Allowable stress

$$S_u = \frac{(LF) Q}{(D_w)^x} \quad \text{Eq 8.6}$$
- Q = expected ultimate strength of a 1 inch bar
- x = factor (see Appendix 11)
- LF = loading factor

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Loading factor (LF)

- **0.405 – Light service:** static loads or upto 10,000 cycles of loading with a low rate of loading
- **0.324 – Average service:** Typical machine design situations; moderate –rate loading and upto one million cycles
- **0.263 – Severe service:** Rapid cycling above one million cycles; impact loading; possibility of shock

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Spring Stresses

- Torsional shear stress

$$S_s = \frac{T c}{J}$$

- A = wire cross sectional area
- c = radius of wire cross section
- J = polar moment of inertia
- D = diameter of wire

$$J = \frac{\pi D^4}{32}$$

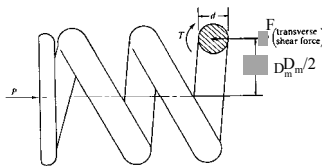
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Spring Stresses

- Torsional shear stress

$$S_s = \frac{16 T}{\pi D_w^3}$$



$$T = \frac{F D_m}{2}$$

FIGURE 12.6 FBD of compression coil spring

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Spring Stresses

- Torsional shear stress

$$S_s = \frac{8 F D_m}{\pi D_w^3} \quad \text{Eq 8.8}$$

- Peak stress is more because of the curvature effect

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Coil shear stress

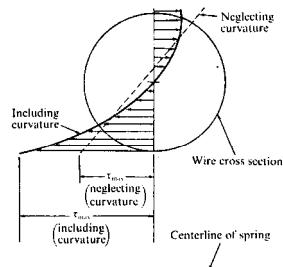


FIGURE 12.8 Effect of coil curvature on shear stress

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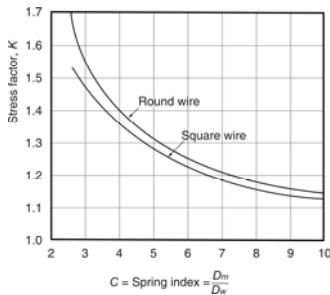
Wahl factor

- A stress factor is developed by Wahl to account for the the curvature.
- A plot of this factor against the spring index, C is shown in Fig. 8.8.

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Wahl factor



$$K = \frac{(4C - 1)}{(4C - 4)} + \frac{0.615}{C}$$

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Spring Stresses

- Spring sizing is based on

$$\frac{8KF D_m}{\pi D_w^3} \leq \frac{(LF) Q}{(D_w)^x} \quad \text{Eq 8.11}$$

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Example Problem 8-1: Spring Stresses

- Design a helical compression spring for the following conditions:
 - 3/4-inch maximum outside diameter
 - 2 inches free length
 - 35-pound load at a 1/2-inch deflection
- Assume severe service and S&G ends made from music wire.
- Try an outside diameter of .725-inch and solve for D_w .
- As many properties of the spring depend on the wire size, often a wire size is assumed and then verified.
- Assume $D_w = .1$ for this first trial.

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Example Problem 8-1: Spring Stresses

(Figure 8-8)

$$\frac{D_m}{D_w} = \frac{.725 - .1}{.1} = 6.25 \quad \text{Wahl} \approx 1.25$$

(Appendix 11)

$$S_u = \frac{Q}{D_w^x} = \frac{190 \text{ ksi}}{D_w^{.154}}$$

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Example Problem 8-1: Spring Stresses (cont'd.)

(From Text: Severe Service)

$$(LF) \frac{Q}{D_w^{.154}} = \frac{K 8F D_m}{\pi D_w^3} \quad (8-11)$$

$$\frac{D_w^3}{D_w^{.154}} = \frac{K 8F D_m}{(LF) \pi Q}$$

$$\frac{D_w^{2.85}}{D_w^{.154}} = \frac{1.25 (8) (35) (.625)}{.263 \pi (190,000)}$$

$$D_w = .100$$

- Use U.S. Steel 12-gage wire. $D_w = .105$ (Appendix 10)

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Deflection of Springs

- Deflection of a spring is given by

$$\delta = \frac{8 F D_m^3 N_a}{G D_w^4} = \frac{8 F C^3 N_a}{G D_w} \quad \text{Eq 8.12}$$

- G = shear modulus of wire material (Table 8.2)

- Spring rate, k is given by

$$k = \frac{G D_w^4}{8 D_m^3 N_a} \quad \text{Eq 8.13}$$

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Example Problem 8-2: Deflection of Springs and Spring Scale

- Determine the number of coils necessary to meet the design criteria using the spring from the previous problem.

$$\delta = \frac{8FD_o^3 N_a}{G D_w^4} \quad (8-12)$$

$$N_a = \frac{\delta G D_w^4}{8 F D_o^3}$$

$$N_a = \frac{5(11.85 \times 10^6)(.105)^4}{8(35)(.620)^3} \frac{\text{in} \cdot \text{lb/in}^2 \cdot \text{in}^4}{\text{lb} \cdot \text{in}^2}$$

$$N_a = 10.8 \quad (\text{active coils})$$

– Find total coils:

$$\text{Total Coils} = N_a + 2$$

$$10.8 + 2 = 12.8 \quad (\text{total coils with squared and ground ends})$$

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Example Problem 8-2: Deflection of Springs and Spring Scale (cont'd.)

– Find pitch:

$$L_f = PN_a + 2D_o$$

$$2 = P(10.8) + 2(.105)$$

$$P = .166 \text{ in}$$

– Find the spring rate for this spring:

$$k = \frac{G D_w^4}{8 D_o^3 N_a}$$

$$k = \frac{(11.85 \times 10^6)(.105)^4}{8(.620)^3(10.8)} \frac{\text{lb/in}^2 \cdot \text{in}^4}{\text{in}^3}$$

$$k = 70 \text{ lb/in}$$

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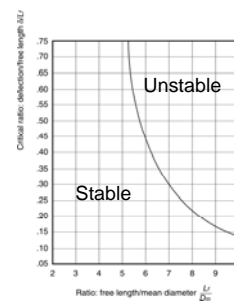
Spring Buckling

- When diameter is small compared to the length spring buckling can occur.

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Fig. 8.9 Spring buckling graph with square and ground ends on a flat surface



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Flat Springs

- Can be in the form of a cantilevered beam, simply supported beam or any other type of beam

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Leaf spring



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Example Problem 8-5: Flat Springs

- A diver deflects a diving board 4 inches.
- This diving board is made from an aluminum plate 1 inch thick by 12 inches wide and 72 inches long.
- What is the force at this point?

(Appendix 3)

$$I = \frac{bt^3}{12} = \frac{12 \text{ in} (1 \text{ in})^3}{12} = 1 \text{ in}^4$$

(Appendix 2)

$$\delta = -\frac{FL^3}{3EI}$$

$$F = \frac{3EI}{L^3} \delta$$

$$F = \frac{3 \text{ in} (3.10 \times 10^4 \text{ lb/in}^2) 1 \text{ in}^4}{(72 \text{ in})^3} = 322 \text{ lb}$$

– Determining stiffness

$$F = kx \quad \text{or} \quad F = k\delta$$

$$k = \frac{F}{\delta} = \frac{3EI}{L^3}$$

$$k = 3 \frac{10 \times 10^4 \text{ lb/in}^2 1 \text{ in}^4}{(72 \text{ in})^3}$$

$$k = 80 \text{ lb/in}$$

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Energy Absorption

- Elastic energy stored

$$U = \frac{F \delta}{2}$$

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Conclusions

- Springs produce a large deflection and used for a number of applications.
- Most springs are made of steel.
- Stress and deflection in coil springs was derived.
- Springs can be connected in series and parallel.
- There are a number of other spring configurations used in engineering.

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