



Theorems of Probability

Dr Bibhunandini Das

Two theorems

- The addition theorem
- The multiplication theorem



Addition Theorem

- ▶ If two events A and B are mutually exclusive the probability of the occurrence of either A or B is the sum of the individual probability of A and B.
- ▶ Symbolically
 - ▶ $P(A \text{ or } B) = P(A) + P(B)$

Proof of the theorem

- ▶ If an event A can happen in a_1 and B in a_2 ways, then the number of ways in which either event can happen is $a_1 + a_2$
- ▶ If the total number of possibilities is n , then by definition the probability of either the first or the second event happening is $\frac{a_1 + a_2}{n} = \frac{a_1}{n} + \frac{a_2}{n}$
- ▶ $\frac{a_1}{n} = P(A)$
- ▶ $\frac{a_2}{n} = P(B)$
- ▶ Hence $P(A \text{ or } B) = P(A) + P(B)$



When events are not mutually exclusive

- ▶ When events are not mutually exclusive/ it is possible for both events to occur- addition rule must be modified
- ▶ Example- What is the probability of drawing a king or a heart from a standard pack of cards
- ▶ King and heart are not mutually exclusive as king of heart can be drawn



Modified form of the addition theorem

- ▶ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
 - ▶ $P(A \text{ or } B)$ = probability of A or B happening when A or B are not mutually exclusive
 - ▶ $P(A)$ is probability of A happening
 - ▶ $P(B)$ is probability of B happening
 - ▶ $P(AB)$ is probability of A and B happening together



Multiplication Theorem

- ▶ States that if two events A and B are independent, the probability that both will occur is equal to the product of their individual probability
- ▶ Symbolically, if A and B are independent, then
 - ▶ $P(A \text{ and } B) = P(A) \times P(B)$
- ▶ The theorem can be extended to three or more independent events



Proof of the theorem

- ▶ If an event A can happen in n_1 ways of which a_1 are successful and the event B can happen in n_2 ways of which a_2 are successful, then we can combine each successful event in the first with each successful event in the second case
- ▶ Total number of successful happenings in both cases is $a_1 \times a_2$
- ▶ Total number of possible cases is $n_1 \times n_2$



Proof of the theorem

- ▶ The probability of the occurrence of both events is
- ▶ $\frac{a_1 \times a_2}{n_1 \times n_2} = \frac{a_1}{n_1} \times \frac{a_2}{n_2}$
- ▶ $\frac{a_1}{n_1} = P(A)$
- ▶ $\frac{a_2}{n_2} = P(B)$
- ▶ $P(A \text{ and } B) = P(A) \times P(B)$



Conditional Probability

- ▶ Multiplication theorem is not applicable in case of dependent events
- ▶ Two events A and B are said to be dependent when B can occur only when A is known to have occurred (or vice versa)
- ▶ The probability attached to such an event is called the conditional probability
- ▶ It is denoted by $P(A/B)$



Conditional Probability

➤ If two events A and B are dependent, then the conditional probability of B given A is

➤
$$P(B/A) = \frac{P(AB)}{P(A)}$$

Proof of Conditional Probability

► Suppose a_1 is the number of cases for the simultaneous happening of A and B out of $a_1 + a_2$ cases in which A can happen with or without happening of B

►
$$P(B/A) = \frac{a_1}{a_1 + a_2} = \frac{a_1/n}{a_1 + a_2/n} = \frac{P(AB)}{P(A)}$$

► Similarly, it can be shown that:

►
$$P(A/B) = \frac{P(AB)}{P(B)}$$



Modified forms of Multiplication Theorem

- ▶ Two events
 - ▶ $P(A \text{ and } B) = P(B) \times P(A/B)$
 - ▶ $P(A \text{ and } B) = P(A) \times P(B/A)$
- ▶ If three events are there
 - ▶ $P(ABC) = P(A) \times P(B/A) \times P(C/AB)$