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# System Modeling & Control

Presented by

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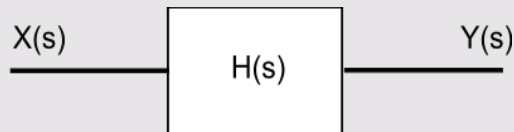


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## Lesson - 8

# Derivation of Transfer Function

A **Transfer Function** is the ratio of the output of a system to the input of a system, in the Laplace domain considering its initial conditions and equilibrium point to be zero. If we have an input function of  $X(s)$ , and an output function  $Y(s)$ , we define the transfer function  $H(s)$  to be:



$$H(s) = \frac{Y(s)}{X(s)}$$

**Where  $H(S) \equiv$  Transfer function**

# 1. Transfer Functions and Linear Systems

Linear engineering systems are those that can be modelled by linear differential

equations. We shall only consider those systems that can be modelled by constant coefficient ordinary differential equations.

Consider a system modelled by a second order differential equation

in which  $a, b, c$  are given constants and  $f(t)$  is a given function. In this case  $f(t)$  is often called the *input signal* or *forcing function* and the solution  $y(t)$  is often called the *output signal*. We shall assume that the initial conditions are zero (in this case  $y(0) = 0, \dot{y}(0) = 0$ ).

Now, taking the Laplace transform of the differential equation, gives

in which we have used  $y(0) = \dot{y}(0) = 0$  and where we have designated  $L\{y(t)\} = Y(s)$  and  $L\{f(t)\} = F(s)$ .

We define the transfer function of a system to be the ratio of the Laplace transform of the output signal to the input signal with the initial conditions as zero. The transfer function (a function of  $s$ ), is denoted by  $H(s)$ . In this case

$$H(s) \equiv \frac{Y(s)}{F(s)} = \frac{1}{as^2 + bs + c}$$

**Example:-** Find the impulse response function  $h(t)$  to a linear engineering system modelled by the differential equation

$$\frac{d^2y}{dt^2} + 4y = e^{-t} \quad y(0) = 0 \quad y'(0) = 0$$

**Solution**

and here

$$H(s) = \frac{1}{s^2 + 4} \quad (= \frac{1}{as^2 + bs + c} \quad \text{with } a = 1, b = 0, c = 4)$$

This is obtained by replacing the forcing function  $e^{-t}$  by the impulse function  $\delta(t)$  and then taking the Laplace transform. Using this:

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2 + 4}\right\} = \frac{1}{2} \sin 2t \cdot u(t)$$

Then the output  $y(t)$  corresponding to the input  $e^{-t}$  is given by the convolution of  $e^{-t}$  and  $h(t)$ . That is,

$$\begin{aligned} y(t) = (h * e^{-t})(t) &= \int_0^t \frac{1}{2} \sin 2(t-x) e^{-x} dx \\ &= \frac{1}{10} [\sin 2t - 2 \cos 2t + 2e^{-t}] \end{aligned}$$

(Note: the last integral can be determined by integrating by parts (twice). Alternatively, evaluate the integral using DERIVE.)