

Unit 15

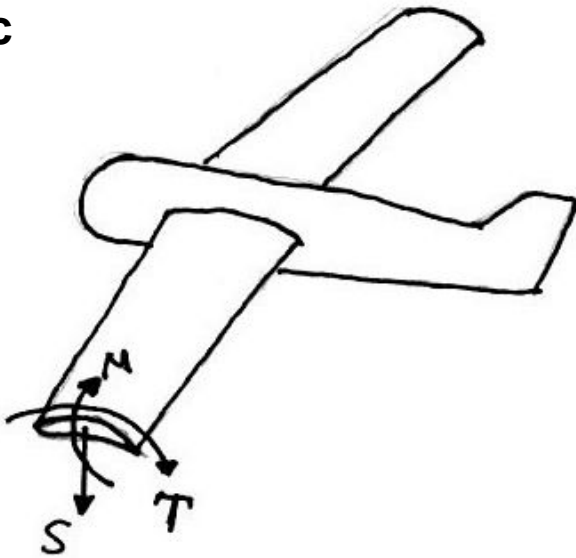
Shearing and Torsion (and Bending)

Thus far, we have concentrated on the bending of shell beams. However, in the general case a beam is subjected to:

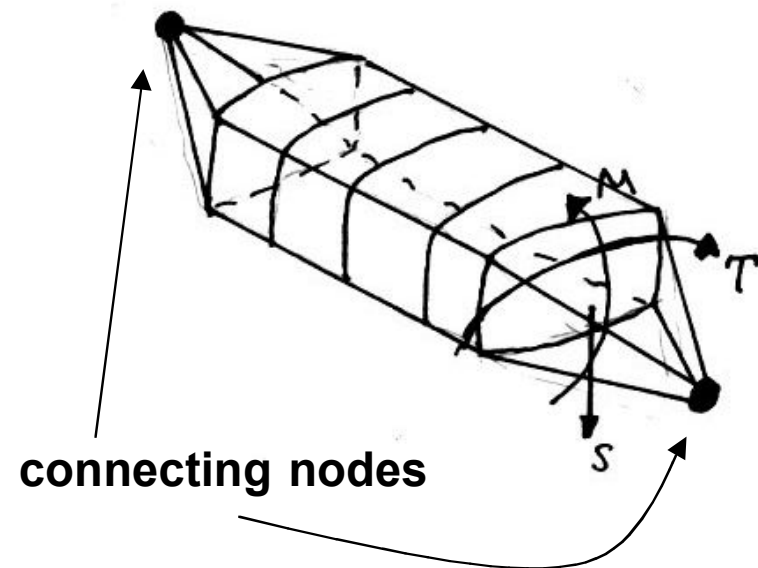
- axial load, F
- bending moments, M
- shear forces, S
- torque (torsional moments), T

Figure 15.1 Examples of general aerospace shell beam structures

Airc



ell



Idealize the cross-section of the shell beam into two parts:

- Parts that carry extensional stress, σ_{xx} (and thus the bending and axial loads)
- Parts that carry shear stress σ_{xs} (and thus the shear loads and torques)

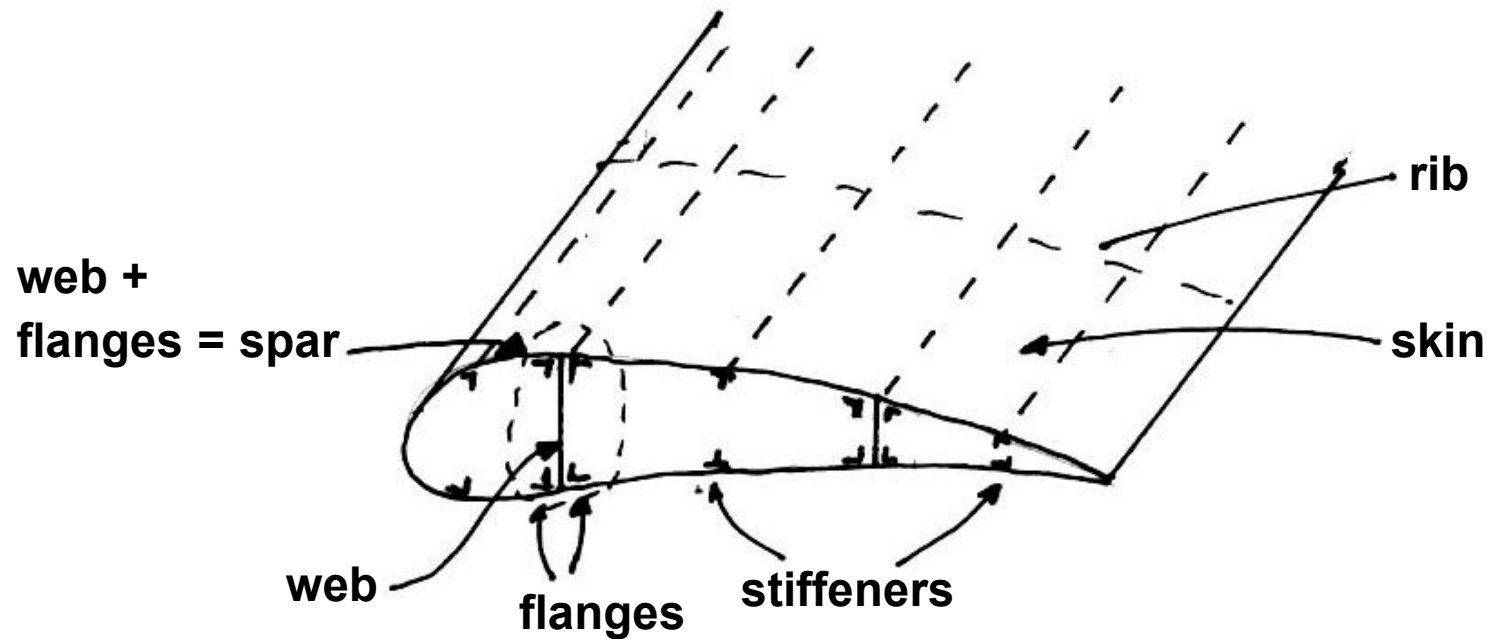
Two examples again...

- high aspect ratio wing with *semi-monocoque* construction

Notes:

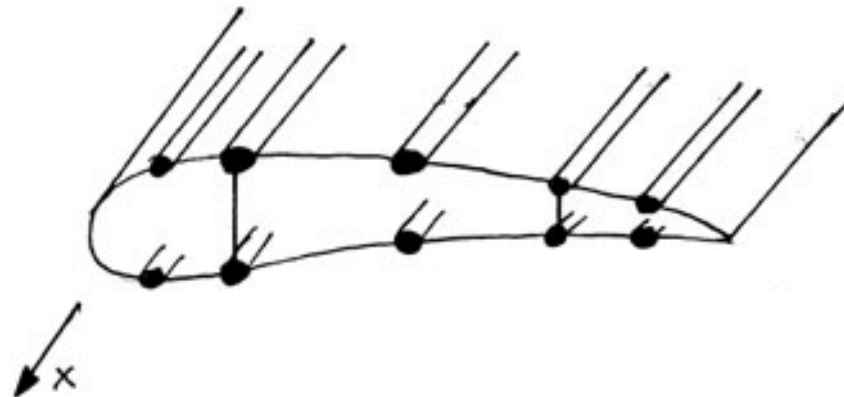
- *monocoque* construction
 - – all in one piece without internal framing
 - – from French “coque” meaning “eggshell”
 - – “mono” = one piece
- *semi-monocoque*
 - stressed skin construction with internal framework
 - still have “eggshell” to carry shear stresses, σ_{xs}
 - internal framework to carry axial stress, σ_{xx}

Figure 15.2 Representation of wing semi-monocoque construction



Idealize this section as:

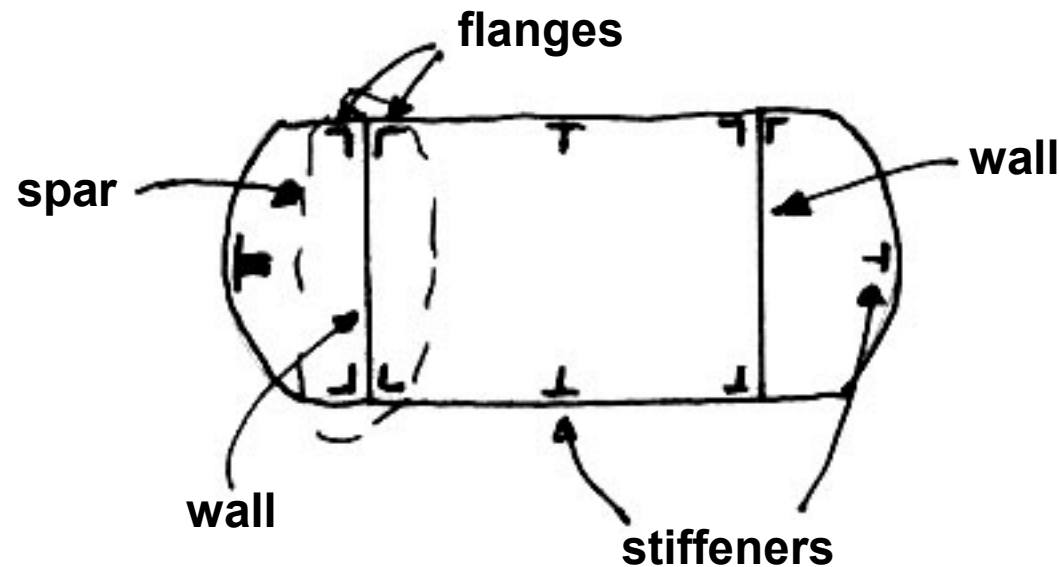
Figure 15.3 Idealization of wing semi-monocoque construction



- Skins and webs are assumed to carry only shear stress σ_{xs}
- Flanges and stringers are assumed to carry only axial stress σ_{xx}

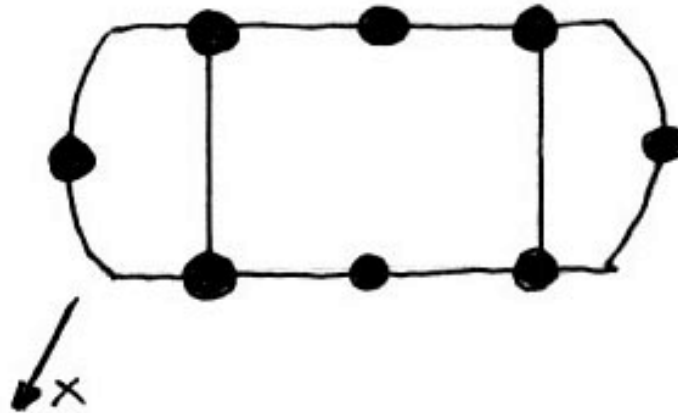
- Space habitat

Figure 15.4 Representation of space habitat semi-monocoque construction



Idealize as for wing:

Figure 15.5 Idealization of space habitat semi-monocoque construction



- Outer skin and walls are assumed to carry only shear stress σ_{xs}
- Flanges and stiffeners are assumed to carry only axial stress σ_{xx}

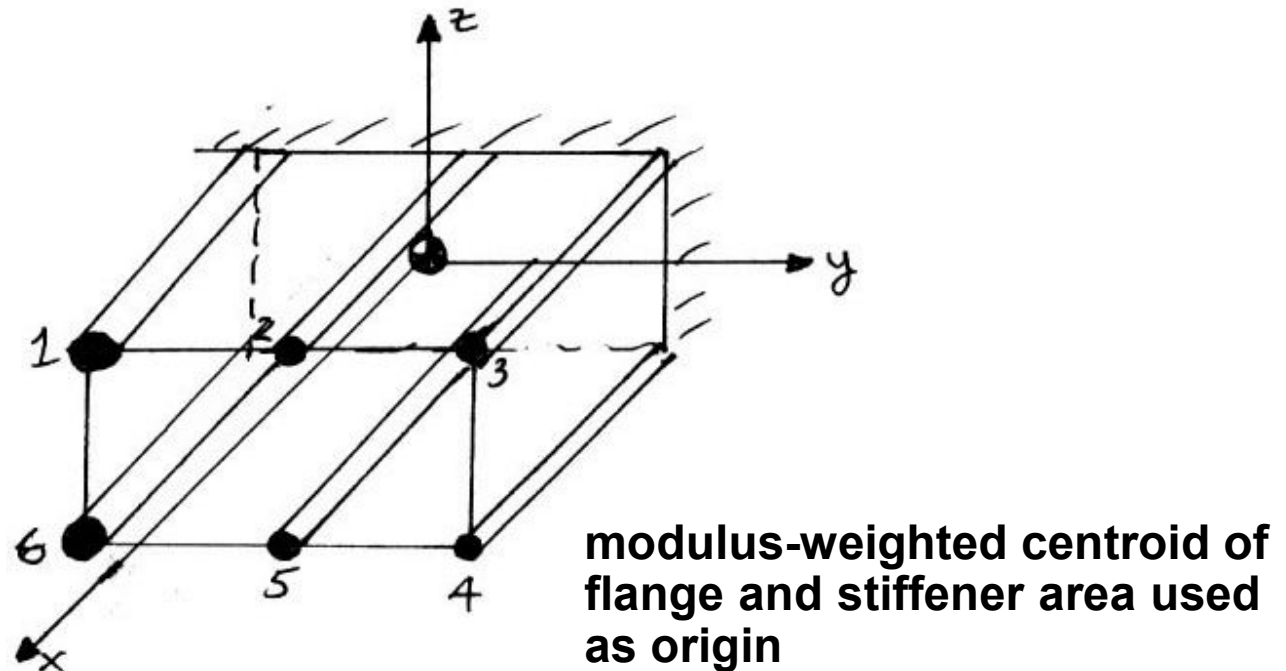
Analyze these cross-sections as a beam under combined bending, shear, and torsion. Utilize St. Venant assumptions:

1. There are enough closely spaced rigid ribs to preserve the shape of the cross-section (or enough stiffness in the internal bracing to do such)
2. The cross-sections are free to warp out-of-plane

Start to develop the basic equations by looking at the most basic case:

Single Cell "Box Beam"

Figure 15.6 Representation of geometry of single cell box beam



Breakdown the problem...

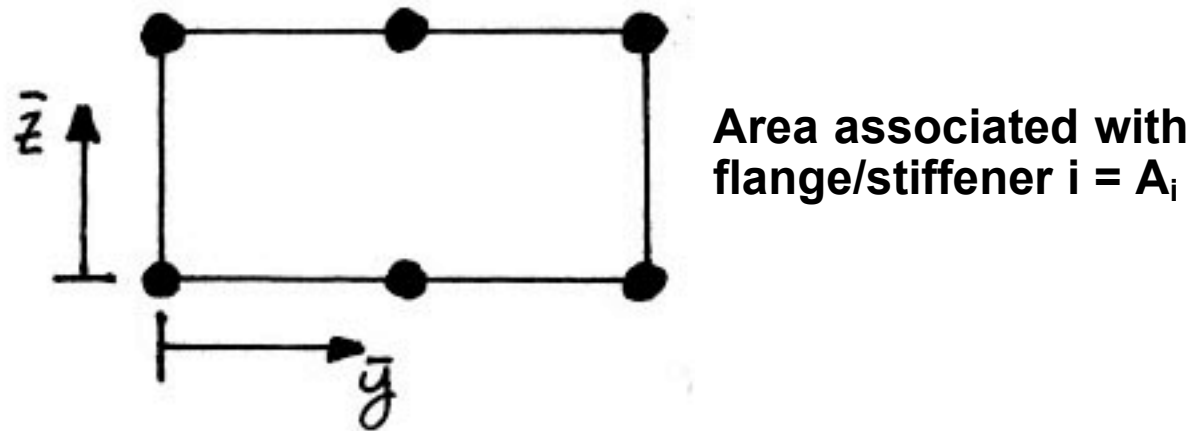
- (a) **Axial Bending Stresses**: Each flange/stiffener has some area associated with it and it carries axial stress only (assume σ_{xx} is constant within each flange/stiffener area)

The axial stress is due only to bending (and axial force if that exists -- *leave at zero for now*) and is therefore independent of the twisting since the wing is free to warp (except near root -- St. Venant assumptions)

* Find M , S , T from statics at any cross-section x of the beam

Consider the cross-section:

Figure 15.7 Representation of cross-section of box beam



Find the modulus-weighted centroid (Note: flange/stiffeners may be made from different materials)

- Choose some axis system \bar{y}, \bar{z} (convenience says one might use a “corner” of the beam)
- Find the modulus-weighted centroid location:

$$\bar{y}^* = \frac{\sum_i I_i^* y_i}{\sum_i A_i^*}$$

$$\bar{z}^* = \frac{\sum_i I_i^* z_i}{\sum_i A_i^*}$$

($\sum_{i=1}^n$ = sum over number of flanges/stiffeners)
number = n

(Note: If flanges/stiffeners are made of the same material, remove the asterisks)

- Find the moments of inertia with reference to the coordinate system with origin at the modulus-weighted centroid

$$I_y^* = \sum_i I_i^* z_i^{*2}$$

$$I_z^* = \sum_i I_i^* y_i^{*2}$$

$$I_{yz}^* = \sum_i I_i^* y_i^* z_i^*$$

- Find the stresses in each flange by using the equation previously developed:

$$\sigma_{xx} = \frac{E}{E_I} \underbrace{\frac{F^{TOT}}{A^*}}_0 - E_I f_2 y - E_I f_3 z - E_I \alpha \Delta T$$

0 for no axial force

(Will do an example of this in recitation)

- (b) **Shear stresses**: assume the skins and webs are thin such that the shear stress is constant through their thickness.

Use the concept of “shear flow” previously developed:

$$q = \sigma_{xs} t \quad [\text{Force/length}]$$

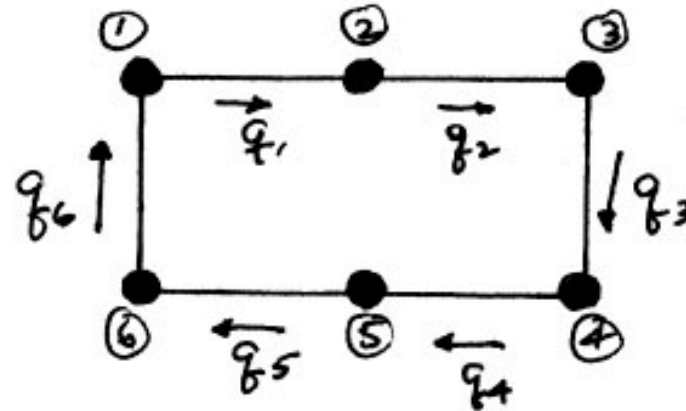
shear flow

shear stress (called this the shear resultant in the case of torsion)

thickness

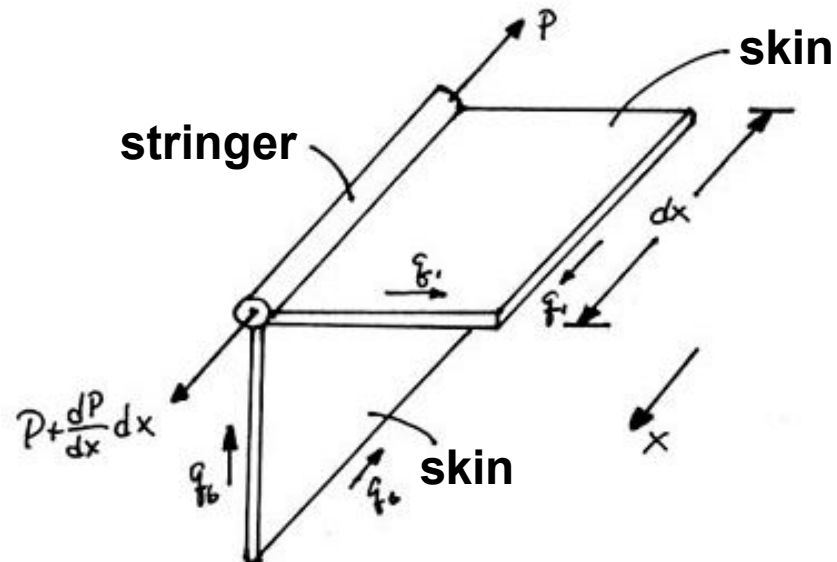
Look at the example cross-section and label the “joints” and “skins”

Figure 15.8 Representation of joints, skins, and shear flows in cross-section of box beam



Look at the equilibrium of joint ①:

Figure 15.9 Representation of skins and stringer and associated loads and shear flows at joint ①



Notes:

- The stringer only carries axial load
- The skin carries only shear flow
- The shear flow at the “end” of the skin (where it is “cut”) must be the same as at the edge (the cross-section cut). This is due to equilibrium ($\sigma_{xy} = \sigma_{yx}$)

- Apply equilibrium: $\sum F_x = 0$

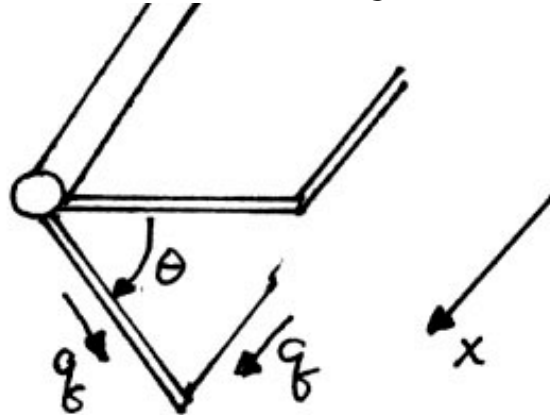
$$\Rightarrow -P + P + \frac{dP}{dx} dx + q_1 dx - q_6 dx = 0$$

$$\Rightarrow q_1 - q_6 = -\frac{dP}{dx}$$

More generally note that:

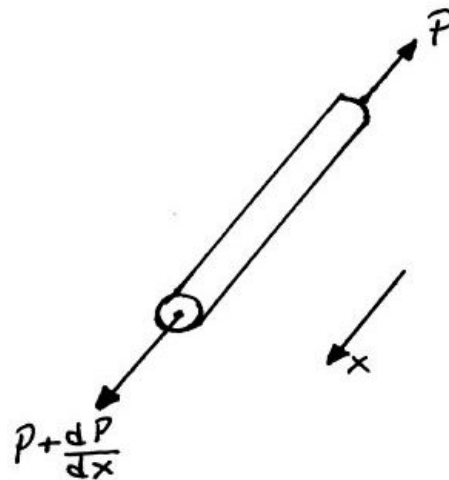
1. Angle that skin comes into joint doesn't matter since q along edge is always in x -direction

Figure 15.10 Representation of skins at joint coming in at any angle



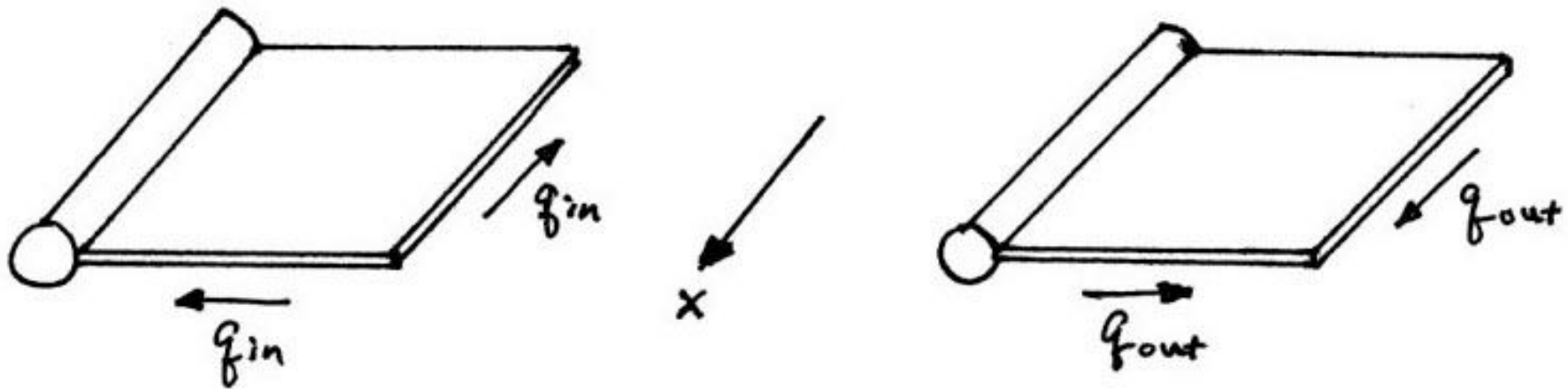
2. Stringer alone gives $\frac{dP}{dx}$ as its contribution

Figure 15.11 Representation of stringer isolated at joint



3. If shear flows “into” joint, its contribution is in the *negative* x-direction; if shear flows “out of” joint, its contribution is in the *positive* x-direction

Figure 15.12 Representation of shear flowing (left) *into* and (right) *out of* joint



Adding all this up:

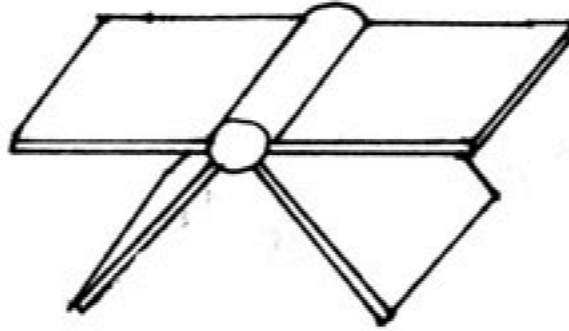
$$\frac{dP}{dx} - q_{in} + q_{out} = 0$$

$$\Rightarrow q_{out} - q_{in} = -\frac{dP}{dx}$$

Use this in general

For a more complicated joint, use superposition

Figure 15.13 Representation of joint with multiple skins



--> Need an expression for P -- start with:

$$P = A\sigma_{xx}$$

differentiating:

$$\Rightarrow \frac{dP}{dx} = A \frac{d\sigma_{xx}}{dx} + \sigma \underbrace{\frac{dA}{dx}}$$

= 0 since are considering stringers
with a uniform cross-section

Most general case:

$$q_{out} - q_{in} = -A \frac{d\sigma_{xx}}{dx}$$

Joint Equilibrium

Consider a simpler case:

- $I_{yz} = 0$ (symmetric section)
- $M_z = 0$

know that:

$$\sigma_{xx} = -\frac{M_y z}{I_y}$$

$$\Rightarrow \frac{dP}{dx} = A \frac{d}{dx} \left[-\frac{M_y z}{I_y} \right]$$

$$\frac{dP}{dx} = -\frac{Az}{I_y} \frac{dM_y}{dx}$$

Recall that:

$$\frac{dM_y}{dx} = S_z \quad (\text{shear resultant})$$

$$Az = Q_y \quad (\text{moment of area about } y)$$

So for this case, the joint equilibrium equation becomes:

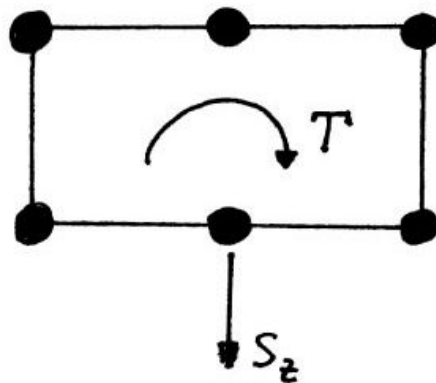
$$q_{out} - q_{in} = \frac{Q_y S_z}{I_y}$$

- Symmetric section
- $M_z = 0$

Now have an equation for the equilibrium of shear stresses at the joints. Shear stresses arise due to two reasons:

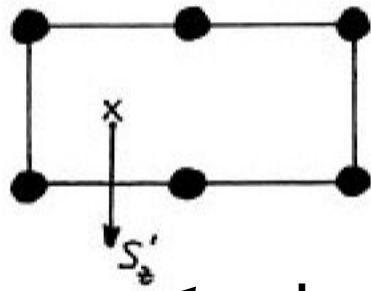
- Shear resultant
- Twisting

In general have at any cross-section:



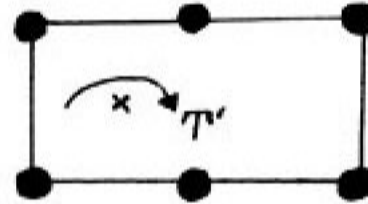
It is convenient to break up the problem into two separate problems:

(1) "Pure Shear"



+

(2) "Pure Twist"

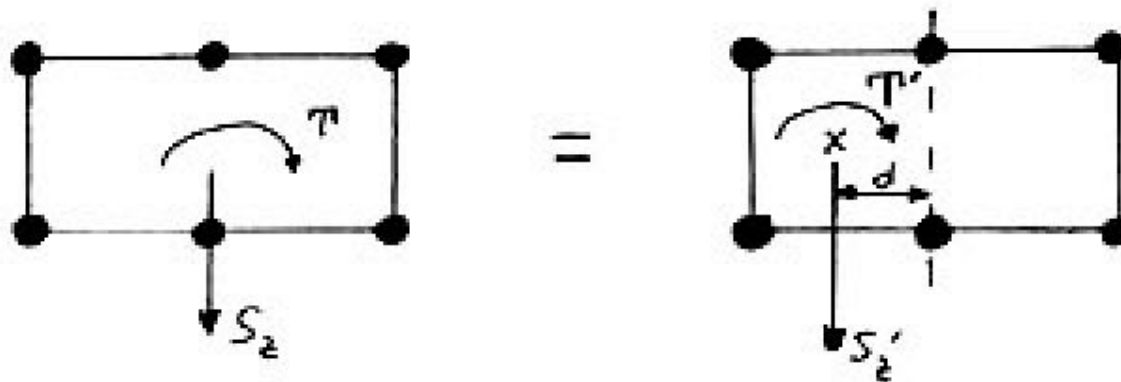


shear resultant acts
at shear center so
there is no twisting

- ◇ --> Solve each problem separately, then add the results (use superposition)

Condition: The two force systems (S_z, T and S_z', T') must be **equipollent**

Figure 15.14 Demonstration of equipollence of force systems



Define: d = distance from where shear resultant acts to shear center

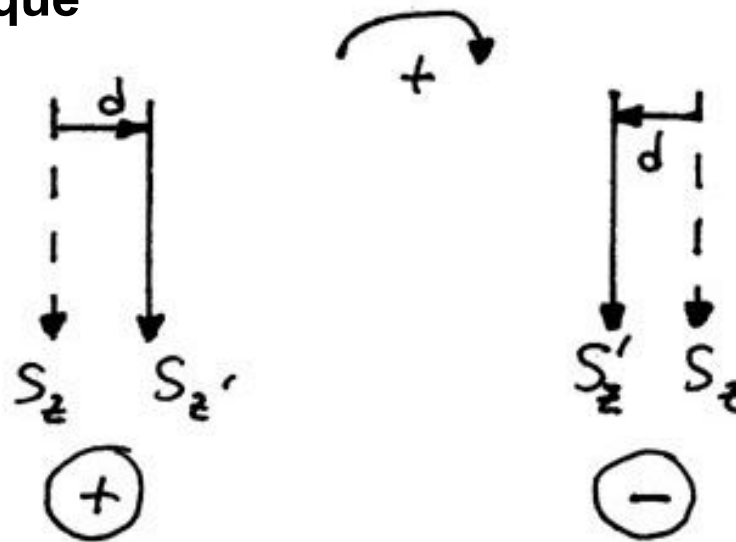
$$\sum F_z = \text{same} \Rightarrow S_z = S_z'$$

$$\sum T = \text{same} \Rightarrow T' - S_z d = T$$



careful: sign could be + or - depending upon direction S_z is moved!

Figure 15.15 Representation of positive and negative contribution of S_z to torque



Hint: Add up torques about line of S_z action in each case.
They must be the same!
(\Rightarrow d has magnitude and sign)

Solution procedure

Given:

- section properties
- loading $[T(x), S_z(x)]$

Find:

- shear stresses (flows) (n joints)
- shear center

⇒ (n + 1) variables

1. Consider “Pure Shear” case

a) Apply joint equilibrium equation at each joint

Note: n joints will yield n-1 **independent** equations. (one is dependent since the section is closed)

b) Use Torque Boundary Condition

$$\sum T_{\text{internal}} = T_{\text{applied}}$$

This is torque equivalence, **not** equilibrium

Do this about the line of action of S_z

Then:

$$T_{\text{applied}} = S_z d$$

$$\sum T_{\text{internal}} = \sum q_i (\text{moment arm})_i (\text{skin length})_i$$

- c) Specified no twist (Pure Shear Case), so apply the **No Twist Condition**

Recall from Torsion Theory:

$$\oint \tau ds = 2AG \frac{d\alpha}{dx}$$

$$\text{Here: } \frac{d\alpha}{dx} = 0$$

$$\text{and: } \tau = \frac{q}{t}$$

$$\Rightarrow \boxed{\oint \frac{q}{t} ds = 0}$$

This gives:

- a) $n - 1$ equations
 - b) 1 equation
 - c) 1 equation
- $n + 1$ equations for $n + 1$ variables

Solving these gives:

- q 's due to "Pure Shear" case
- d

--> when complete, check via:

$$\sum \text{Internal Shear loads} = \text{Applied Shear}$$

(Horizontal & vertical)

2. Consider "Pure Torsion" case

- a) Apply joint equilibrium equation at each joint
- Note: again, n joints give $n-1$ equations

Since no shear:

$$q_{out} - q_{in} = 0$$

b) Use Torque Boundary Condition

$$\sum T_{\text{internal}} = T_{\text{applied}}$$

$$T_{\text{applied}} = T + \text{or} - S_z d$$

↑
found in part 1

This gives:

- a) $n - 1$ equations
- b) 1 equation

- Need: n q 's

⇒ Solve these for q 's due to "Pure Torsion" case

3. Sum results for "Pure Shear" and "Pure Torsion" cases

(could use q_i^S, q_i^T : q_i due to pure shear = q_i^S
 q_i due to pure torsion = q_i^T)

Now have: shear flows

(get shear stresses via: $\tau_{xs} = \frac{q}{t}$)

location of shear center

(will be important to determine deflection)

(will go over sample problem of handout #6 in recitation)

for: Unsymmetric Shell Beams

- Cannot make simplifying assumptions (use equations coupling bending in y and z)
- See handout #4B

Now that the stresses (due to bending, shear and torsion) are determined, proceed to find the...

SECTION PROPERTIES

a) Bending Stiffness, EI (as before)

$$I_z = \sum A * y^2$$

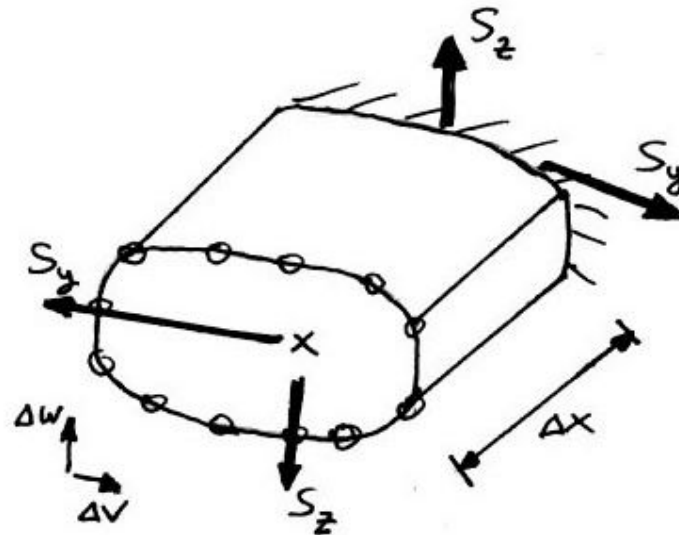
$$I_y = \sum A * z^2$$

$$I_{yz} = \sum A * y * z$$

b) Shear Stiffness, GA (have not done this before)

Consider the deflections Δv and Δw for a segment Δx with only shear forces S_y and S_z acting at the shear center

Figure 15.18 Representation of shell beam segment with only shear forces acting at the shear center



Can express the shear flows in each member as contributions due to S_y and S_z :

$$q(s) = \bar{q}_y(s) S_y + \bar{q}_z(s) S_z$$

where:

$\bar{q}_y(s)$ = shear flow due to S_y of unit magnitude

$\bar{q}_z(s)$ = shear flow due to S_z of unit magnitude

--> To determine Δv and Δw , it is best to use an Energy Method

It can be shown that:

$$A_{yy} = \frac{1}{\int \frac{(\bar{q}_y)^2}{t} ds}$$

$$A_{zz} = \frac{1}{\int \frac{(\bar{q}_z)^2}{t} ds}$$

$$A_{yz} = \frac{1}{\int \frac{\bar{q}_y \bar{q}_z}{t} ds}$$

c) Torsional Stiffness, GJ

Previously saw that:

$$\frac{d\alpha}{dx} = \frac{1}{2AG} \oint \frac{q}{t} ds \quad (\text{for closed section})$$

Let: (as just did)

$$q = \bar{\bar{q}}(s) T$$

where:

$\bar{\bar{q}}(s)$ = shear flow due to T of unit magnitude

Then, using this in the above:

$$\frac{d\alpha}{dx} = \frac{T}{G2A} \oint \frac{\bar{\bar{q}}}{t} ds$$

Recalling:

$$\frac{d\alpha}{dx} = \frac{T}{GJ}$$

$$\Rightarrow \boxed{J = \frac{2A}{\oint \frac{\bar{\bar{q}}}{t} ds}} \quad (\text{for closed section})$$

Summarizing, to find the deflections:

1. Obtain the section properties (EI , GA , GJ) and the location of the shear center
2. Decompose load into moments, shears at shear center, and torque about shear center
3. Find (independently) bending, shearing, and twisting deflections about line of shear centers (elastic axis)
4. Sum deflections to obtain total deflection

Figure 15.19