

CHAPTER II

INTERNAL BALLISTICS OF

ROCKET PROPELLANTS

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INTERNAL BALLISTICS OF SOLID ROCKET PROPELLANTS

There is a broad usage of solid rocket propellants in propulsion systems and gas generators. In such systems, there are challenging problems involving many complicated aerothermochemical processes. Solutions of such complicated systems require a good understanding of many processes like heat transfer, turbulence combustion, diffusion, chemical reactions etc. The combustion process in a solid rocket chamber helps to visualize the burning rate behaviour of solid propellant. Because the reaction rates during combustion are much faster than those of commonly observed chemical reactions and because physical changes such as velocity, temperature and pressure during combustion, a knowledge of aerothermochemistry is needed to understand combustion phenomena. It is the purpose here to present advances in combustion phenomenon of different types of propellants and to present the aerothermochemical equations applicable to solid propellants.

1. PERFORMANCE OF A SOLID ROCKET PROPELLANT¹

COMBUSTION :- It is an exothermic reaction involving rapid oxidising reactions. The combustion phenomenon of solid propellants are much more complex than those of gas because of the phase changes involved in going from a solid to a liquid to a gas.

PROPELLANT :- It is defined as the material which generates a large number of gaseous molecules at high temperature during combustion which can self-sustain without the presence of ambient oxidiser combustion. In other words propellant is a material which burns rapidly but does not explode and is self sustaining during burning. Major applications of propellants are in rockets and guns both of which use gas at high pressure

and temperature to generate propulsive forces. Rockets consists of propellant and combustion chamber within their projectiles, whereas gun projectiles usually do not. The solid propellant rockets work at operating pressures between 10 and 300 atm.

A rocket motor is a typical example of an energy transfer system which can be directly explained by thermodynamics and Newton's third law - a pressurized high temperature gas generated in the system is expanded adiabatically and the sensible energy of the gas is converted to kinetic energy. Thus the system produces a reaction force. Therefore, the thermodynamic requirement for rocket motor is to get a high pressure and high temperature gas in the motor, and to convert the sensible energy effectively into the kinetic energy through physicochemical processes. Thus, temperature in the chamber, molecular weight of combustion gases and ratio of specific heats are the performance parameters^{1,2}. However, these parameters cannot be measured. Hence these parameters are converted to the following parameters.

CHARACTERISTICS VELOCITY: It is defined as

$$C^* = \frac{\sqrt{RT_c}}{\Gamma} \quad (2.1)$$

$$\text{Experimentally, } C^* = \frac{gA_t \int P dt}{M_p} \quad (2.2)$$

It is the figure of merit of the propellant. It depends upon the combustion flame temperature and molecular weight of combustion gases. It represents the combustion efficiency of propellant.

THRUST COEFFICIENT: It is defined as,

$$C_F = \frac{F}{A_t P_c} \quad (2.3)$$

for non viscous compressible gas ,

$$C_F = \Gamma \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (2.4)$$

It depends upon the expansion ratio of the nozzle as seen from equation (2.4).

SPECIFIC IMPULSE: It is defined as the thrust per unit weight flow rate of propellant.

$$I_{sp} = \frac{\int F dt}{W_p} \quad (2.5)$$

and it is

$$I_{sp} = \frac{1}{g} \sqrt{\frac{2\gamma}{\gamma-1} \frac{RT_c}{M} \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (2.6)$$

for radially expanding gases.

BURNING RATE : It is a distance, normal to the burning surface of the propellant, travelled by a flame front per unit time. It is a key factor in determining the performance of solid propellant. In general, burning rate, called Vieille's law, is given by

$$r = aP^\eta \quad (2.7)$$

is assumed for all types of propellants.

Since the gas generation in the chamber is a function of the propellant burning and mass discharge through the nozzle, the mass balance in the rocket motor can be written as

$$\frac{d}{dt}(\rho_g V_c) = \dot{m}_g - \dot{m}_d \quad (2.8)$$

Under steady state condition, RHS of equation (2.8) vanishes. However, this term plays a dominant role in ignition transients, oscillary burning, and combustion termination. The rate of mass discharge is given by

$$\dot{m}_d = \frac{A_t P_c g}{C^*} \quad (2.9)$$

which is purely a fluid dynamical relation.

The rate of gas generation in the combustion chamber is equal to the mass burning rate of the propellant.

$$\dot{m}_g = \rho_p A_b r \quad (2.10)$$

Under the steady state,

$$\rho_p A_b r = \frac{A_t P_c g}{C^*} \quad (2.11)$$

$$P_c^{1-\eta} = \frac{a \rho_p C^* K_n}{g} \quad (2.12)$$

The combustion index n can never exceed one and it is desirable to have low or even a negative value. If n is smaller than one, after a small increase in pressure the mass flow through the nozzle becomes greater than the mass burning rate and the pressure returns towards equilibrium. The smaller the value of n , the quicker the return. On the other hand, if n is greater than unity, the mass burning rate exceeds the discharge rate and pressure increases until an explosion occurs. Inversely following a negative differential, the pressure continues to drop until combustion stops. Therefore stable burning can only be achieved if the combustion index n is smaller than one.

2. COMBUSTION PROCESS OF VARIOUS TYPES OF SOLID PROPELLANTS

It is of prime importance for solid propellants to satisfy the various combustion characteristics described in the previous section to obtain stable and high performance combustion in rocket motors. However, the physical and chemical properties of these propellants are entirely different from each other, and the combustion processes are also different. The following paragraphs describe the flame structure and burning rate characteristics of various types of solid propellants. The propellants are double base propellants, ammonium perchlorate based composite propellants, composite modified double base propellants and nitramine composite propellants.

2.1 DOUBLE BASE PROPELLANTS: Since the physical structure of a double base propellants is homogeneous, the combustion flame structure appears to be homogeneous and one dimensional along the burning direction. The decomposed gas produced at the burning surface is premixed with oxidizer and fuel on a molecular scale. The luminous flame stands some distance above the burning surface and approaches the burning surface when the pressure is raised. Between the burning surface of the propellant and luminous flame, a transparent region exists and is called the dark zone, which is equivalent to the flame stand off distance, decreases with increasing pressure, or in other words with increasing burning rates.

Following table explains this aspect.

p (atm)	r (cm/s)	thickness of flame (mm)
10	0.22	1.30
20	0.31	0.33
30	0.40	0.14

The flame structure for double base propellant is shown in fig.1

2.2 AMMONIUM PERCHLORATE COMPOSITE PROPELLANT: Since the physical structure of composite propellant is heterogeneous, the combustion wave structure appears to be heterogeneous also. The luminous flame is attached to the burning surface even at low pressures. At the burning surface, the decomposed gases from ammonium perchlorate particles and the fuel binder are interdiffused and produced diffusion flame structure. Thus, the flame structure of AP - composite propellants is complex and locally three dimensional in shape. The luminous flame stand off distance above the burning surface of AP- composite propellants is less dependent on pressure compared with double base propellants. It is evident that the diffusion process plays a significant role on the burning rate of this type of propellants. The combustion wave structure is illustrated in fig.2

2.3 COMPOSITE MODIFIED DOUBLE BASE PROPELLANTS: The specific impulse of double base propellants is less than that of AP- composite propellants. Thus, often oxidizing or high energy crystalline particles are added to double base propellants in order to increase the specific impulse. These types of propellants are called composite modified double base propellants (CMDB) propellants. In general, double base propellants contain less oxidizing fragments. Thus, the addition of AP shifts the equivalence ratio of decomposed gases to stoichiometric and T_f and I_{sp} increase.

The AP-CMDB propellants does not have the dark zone of the double base propellants, which is used as a base matrix. This is a significant difference when compared with the double base propellants. as the concentration of AP increases, the flame structure

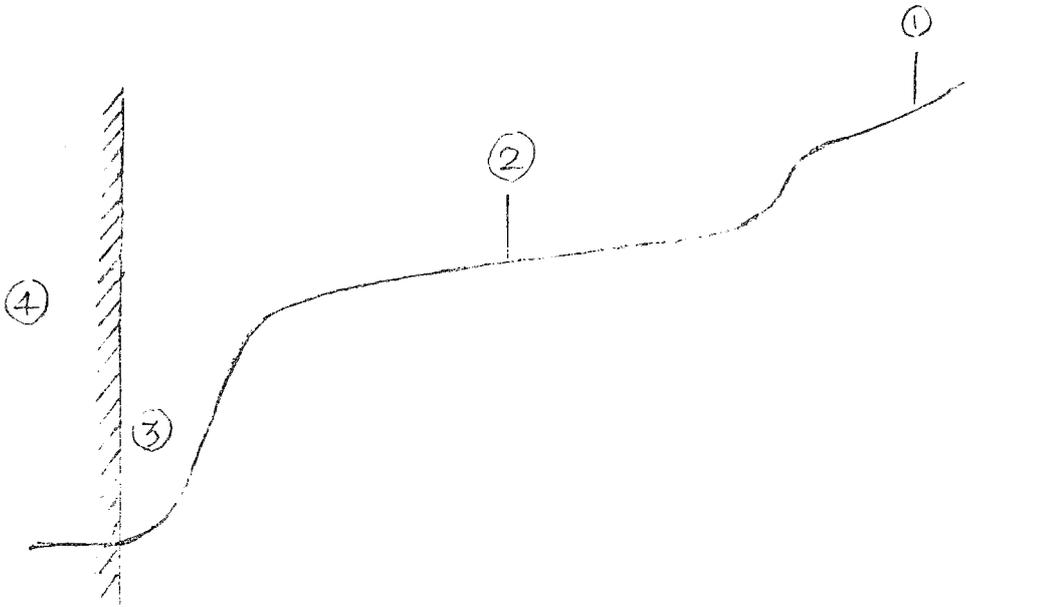


FIG 1 : FLAME STRUCTURE OF DOUBLE BASE PROPELLANT

1. FLAME ZONE T_g FLAME TEMPERATURE.
2. DARK ZONE T_f GAS TEMPERATURE
3. FIZZ ZONE T_s SURFACE TEMPERATURE
4. PROPELLANT T_0 AMBIENT TEMPERATURE

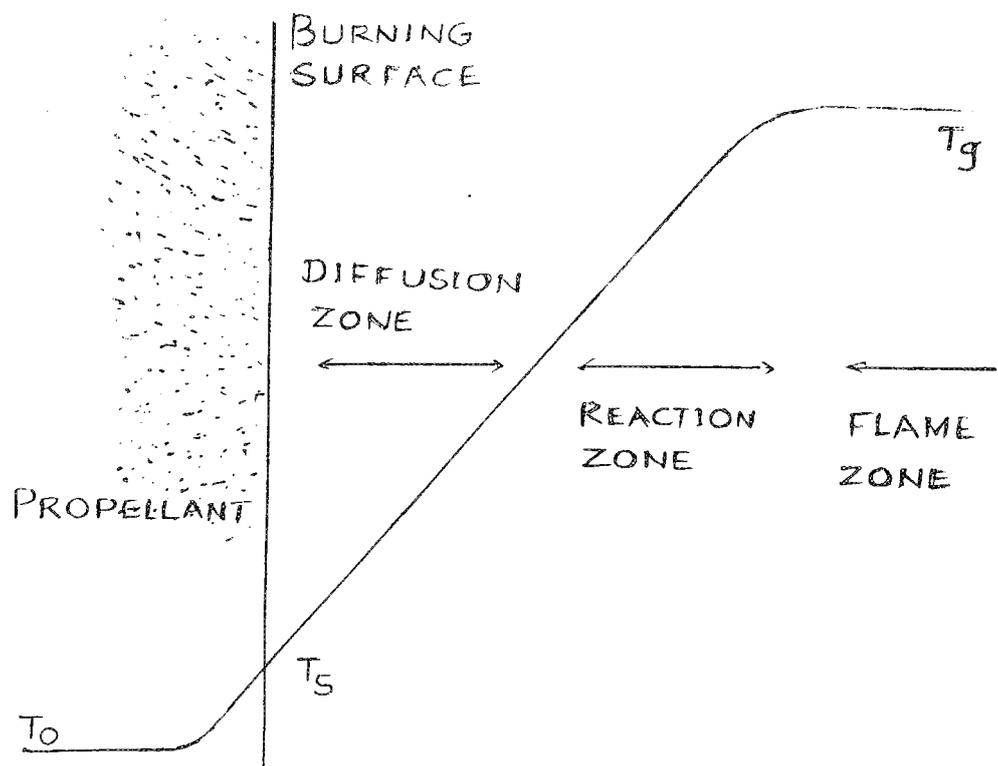


FIG 2 : COMBUSTION WAVE STRUCTURE OF AP-PROPELLANT

and the burning rate characteristics approach that of AP composite propellants, and the combustion wave becomes highly heterogeneous. The decomposed fuel-rich gas, produced from the double base propellants, and the decomposed oxidizer-rich gas, produced from AP produce diffusion flames above the particles of AP. Diffusion flames are produced around the AP monopropellant flame streams which are produced by each particle of AP. As the number of AP particles increases, the number of diffusion flamelets increases in the gas phase. An extreme case, the flame structure of double base propellants which is used as a base matrix, is eliminated completely, and the burning rate of the propellant becomes similar to that of AP-composite propellants. These characteristics are dependent on the concentration and particle size of AP. The burning rate increases as the AP concentration increases for the same particle size of AP, it also increases when the size of AP decreases. The wave structure for such type of propellant is depicted in fig 3.

When RDX is mixed with a double base propellant, the specific impulse increases because of the increased flame temperature. Since the combustion products of RDX and double base propellants are fuel-rich, exothermic chemical reactions are possible between their combustion products. Thus, the flame temperature and the specific impulse of the RDX-CMDB propellants is similar to that of double base propellants and is significantly different from that of AP-CMDB propellants. The combustion wave consists of three successive reaction zones : fizz zone and luminous flame zones. The luminous flame stands some distance above the burning surface. The flame standoff distance decreases with increasing pressure. Increasing the pressure results in the luminous flame approaching the burning surface.

The crystalline RDX particles mixed with the base matrix of double base propellants melt, decompose and gasify at the burning surface of the propellants. The decomposed RDX gas diffuses into the decomposed gas of the base matrix at and just above the burning surface. This diffusion process occurs before the RDX particles produced their monopropellant flamelets above the burning surface. Thus, a homogeneous

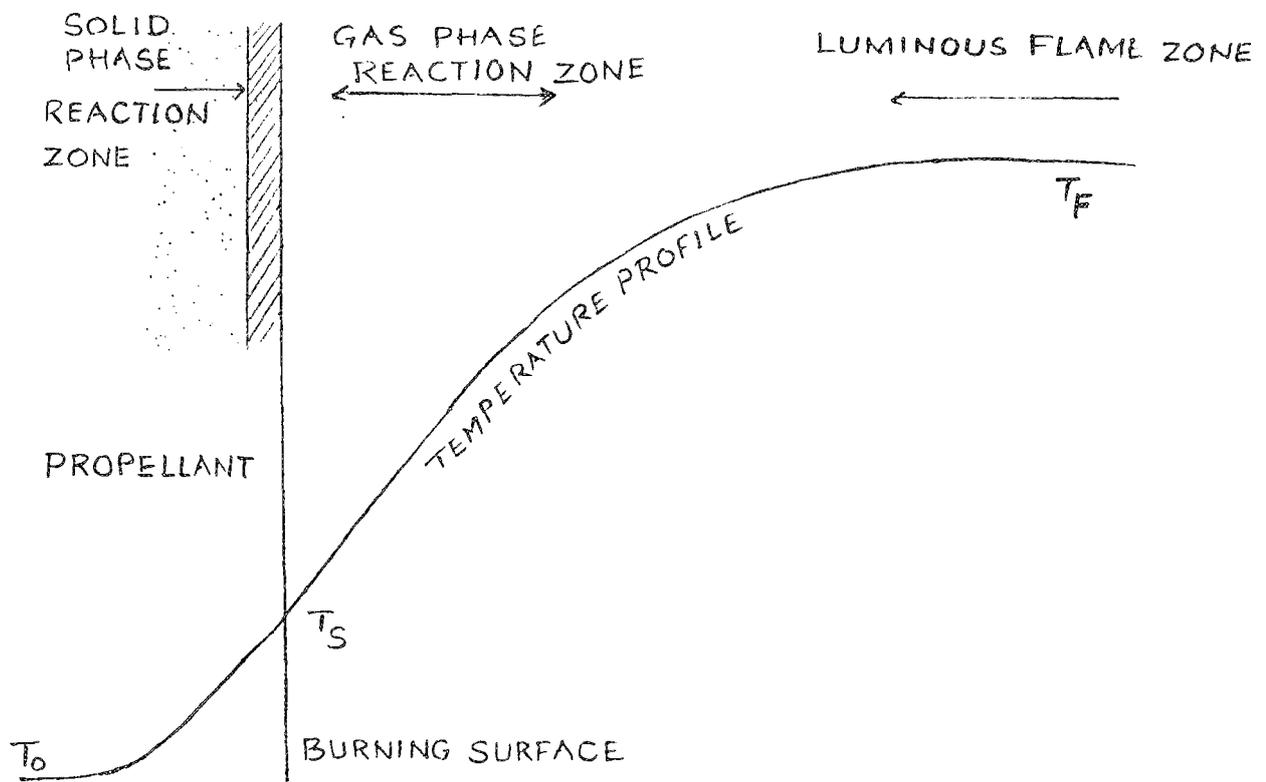


FIG 3 : COMBUSTION WAVE STRUCTURE OF CMDB PROPELLANT.

reactive gas of the mixture of the RDX and the base matrix decomposed gases is formed at the burning surface and produces the luminous flame at some distance above the burning surface. Thus the combustion wave is one-dimensional in the burning direction. The burning rate of RDX-CMDB propellant decreases when the concentration of RDX increases and is nearly independent of particle size.

2.4 NITRAMINE COMPOSITE PROPELLANTS : The physical structure of RDX composite propellants is very heterogeneous and is similar to the physical structure of AP-composite propellants. However, the combustion wave structure is homogeneous and is similar to the combustion wave structure of double base propellants. When the concentration of RDX increases, the burning rate of RDX-composite propellants increases with the burning rate relatively independent of RDX particle size.

3. GOVERNING EQUATIONS :

3.1 THE PROCESS OF PROPELLANT BURNING: As discussed earlier, combustion performance of solid propellant rocket motors can be improved by the following physico-chemical parameters :

- i) Increasing specific impulse
- ii) obtaining a spectrum of propellant burning rate
- iii) reducing the pressure exponent of burning rate and
- iv) reducing the temperature sensitivity of burning rate.

All of these items are related to the phenomena of propellant combustion mechanism. Various combustion phenomenon occurring during rocket motor operation, such as ignition, erosive burning, oscillatory burning and combustion determination must be studied using the principles of heat transfer, fluid mechanics, and chemical kinetics.

The burning process is largely dependent on the combustion wave structure. This wave structure is a function of the propellant composition, pressure and various operating conditions. Propellants produce heat and high temperature gas by the phenomenon of combustion. The heat feed back from the high temperature gas to the unburnt part of the

propellants raises this portion of the propellants to the decomposition temperature. As a result, the unburnt portion gasifies and produces the heat by an exothermic chemical reaction. This successive heat feed back process makes the propellant burning occur continuously to sustain a steady state burning.

3.2 COMBUSTION WAVE STRUCTURE : The fig.4 is a schematic drawing of a simplified combustion wave structure of solid propellant. The temperature increases rapidly from the initial temperature to the burning surface temperature T_s just beneath the burning surface. At the burning surface, either an endothermic or exothermic decomposition reaction produces reactive gases. This reaction zone is called the solid phase reaction or condensed phase reaction zone. The reactive gases emitted from the burning surface, are the gas phase above the burning surface and generate heat to form a luminous flame zone. Thus, the temperature increases in the gas phase reaction zone and reaches a maximum in the luminous flame zone. In the luminous flame zone, the exothermic reaction is completed and equilibrium combustion products are formed. The combustion products flow away continuously downstream of the combustion zone and burning surface regresses in the opposite direction of the combustion product flow. The heat balance at the burning surface is

$$\lambda_p \left(\frac{dT}{dx} \right)_s = \lambda_g \left(\frac{dT}{dx} \right)_{s'} + \rho_p r Q_s + I_g \quad (2.13)$$

3.3 CONSERVATION EQUATIONS: The conservation equations of mass, momentum, energy and the chemical species in the combustion wave for ideal gas can be written as

$$\text{MASS : } \nabla (\rho v) = 0 \quad (2.14)$$

$$\text{MOMENTUM : } \rho v \cdot \nabla v = - \nabla p \quad (2.15)$$

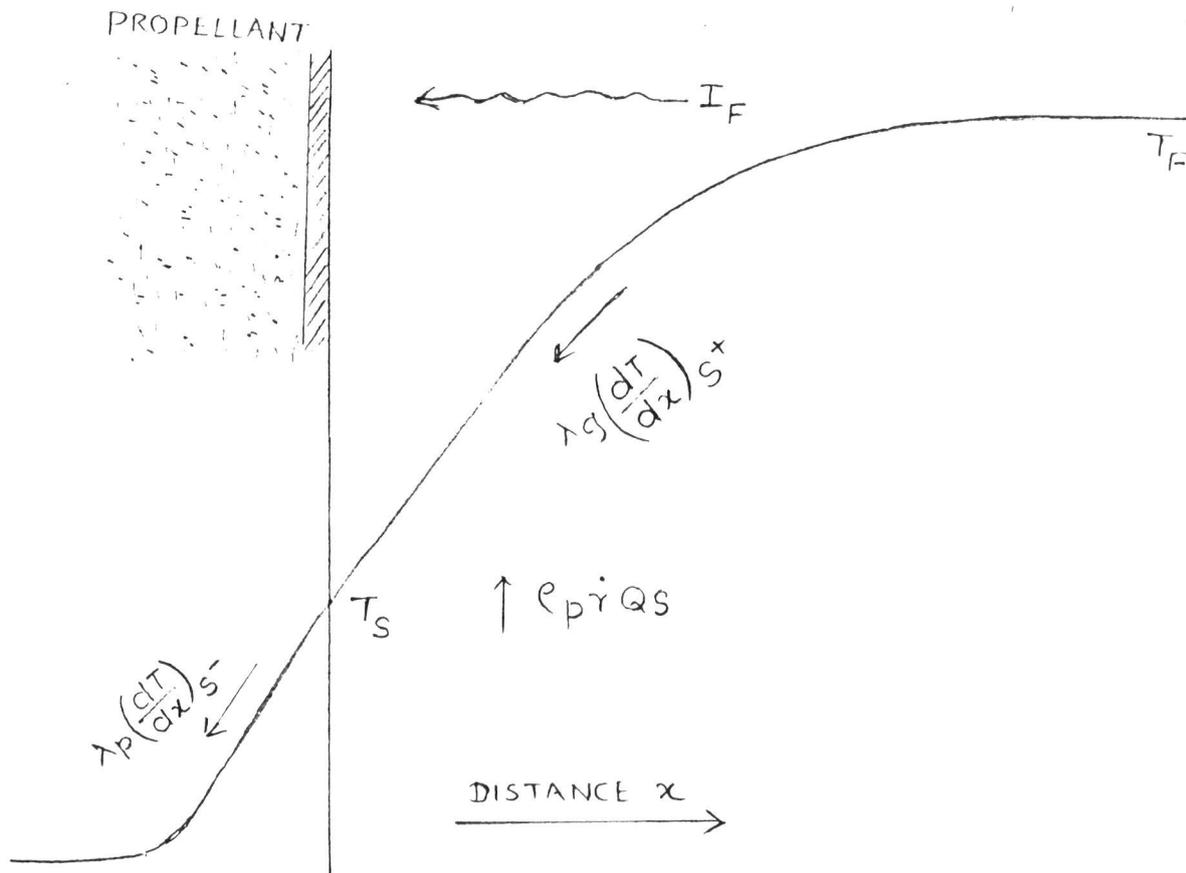


FIG 4 : HEAT BALANCE AT BURNING SURFACE

$$\text{ENERGY} : \nabla \cdot (\rho c_p T - \lambda \nabla T) = - \sum \omega_i R_i \quad (2.16)$$

$$\text{SPECIES} : \nabla \cdot (\rho v Y_i - \rho D \nabla Y_i) = \omega_i \quad (2.17)$$

3.4 HEAT TRANSFER IN A SOLID PHASE :

Assuming that the chemical reaction does not occur in the solid phase, the energy equation in the solid phase can be written as

$$\lambda \frac{d^2 T}{dx^2} - \rho_p c_p \frac{dT}{dx} = 0 \quad (2.18)$$

Using the mass balance equation in the solid and gas phase, this energy equation can be written as

$$\lambda_p \frac{d^2 T}{dx^2} - \rho_p r c_p \frac{dT}{dx} = 0 \quad (2.19)$$

where λ_p and c_p are assumed to be independent of temperature. Integrating this equation between $T = T_0$ at $x = -\infty$ and $T = T_s$ at $x = 0$. We get the heat feed back from the burning surface to the solid phase at $x = 0$ as

$$\lambda_p \left(\frac{dT}{dx} \right)_s = \rho_p r c_p (T_s - T_0) \quad (2.20)$$

Hence the temperature profile is

$$T(x) - T(0) = (T_s - T_0) \exp\left(-\frac{\rho_p c_p r x}{\lambda_p}\right) \quad (2.21)$$

3.5 HEAT TRANSFER IN A GAS PHASE : In the gas phase reaction zone, the reaction process is very complicated with a large number of chemical species and elementary reactions occurring. To understand the reaction processes occurring in the gas phase, the reaction rates for each elementary reaction must be determined and the conservation equations must be solved. However, in general, the exact solutions of the conservation equations are difficult to obtain because of the lack of the kinetic parameters in the gas phase. In order to determine the rate of heat transfer from the gas phase to the burning surface and to determine the burning rate of the propellant, simplified reaction rate models are given below.

The equation of energy can be written as

$$\lambda_g \frac{d^2 T}{dx^2} - \rho_g u_g c_{pg} \frac{dT}{dx} + \dot{\omega}_g Q_g = 0 \quad (2.22)$$

since $\dot{\omega}_g$ is a function of temperature this equation is a non-linear and analytical solution is not possible.

Assuming $\dot{\omega}_g$ is constant initially and integrating this equation we get,

$$\lambda_g \left[\frac{dT}{dx} \right]_{s^+} = Q_g \int_0^\infty \exp\left(\frac{-\rho_g u_g c_{pg} x}{\lambda_g} \right) \dot{\omega}_g dx \quad (2.23)$$

There will be three cases as shown in fig 5.

i) The chemical reactions occur in a limited region only and reaction rate is constant throughout the region between $x = x_f$ and $x = x_i$ shown in the figure above. In this case the reaction rate in the gas phase can be expressed as a step function. Hence (2.23) becomes

$$\lambda_g \left[\frac{dT}{dx} \right]_{s^+} = \frac{\lambda_g}{\rho_g u_g c_{pg}} \dot{\omega}_g Q_g \left\{ \exp\left(\frac{-\rho_g u_g c_{pg} x_i}{\lambda_g} \right) - \exp\left(\frac{\rho_g u_g c_{pg} x_f}{\lambda_g} \right) \right\} \quad (2.24)$$

For $x_i = x_f$, the thickness of the reaction zone is extremely thin and the reaction rate is assumed to be infinite. Hence in this case we have,

$$\lambda_g \left[\frac{dT}{dx} \right]_{s^+} = \rho_g u_g Q_g \exp\left[\frac{-\rho_g u_g c_{pg} x_f}{\lambda_g} \right] \quad (2.25)$$

For $x_i = 0$, the reaction starts at the burning surface and we get,

$$\lambda_g \left[\frac{dT}{dx} \right]_{s^+} = \frac{\lambda_g}{\rho_g u_g c_{pg}} \dot{\omega}_g Q_g = \frac{\lambda_g}{\rho_p r c_{pg}} \dot{\omega}_g Q_g \quad (2.26)$$

3.6 LAMINAR BOUNDARY LAYER EQUATIONS : The governing partial differential equations for chemically reacting boundary layer flow are deduced from the conservation equations for multicomponent reacting system. The order of magnitude analysis has been used to simplify the generalized momentum, energy and species equations for two dimensional laminar reacting boundary layer flow. The equations for the flow under these conditions are

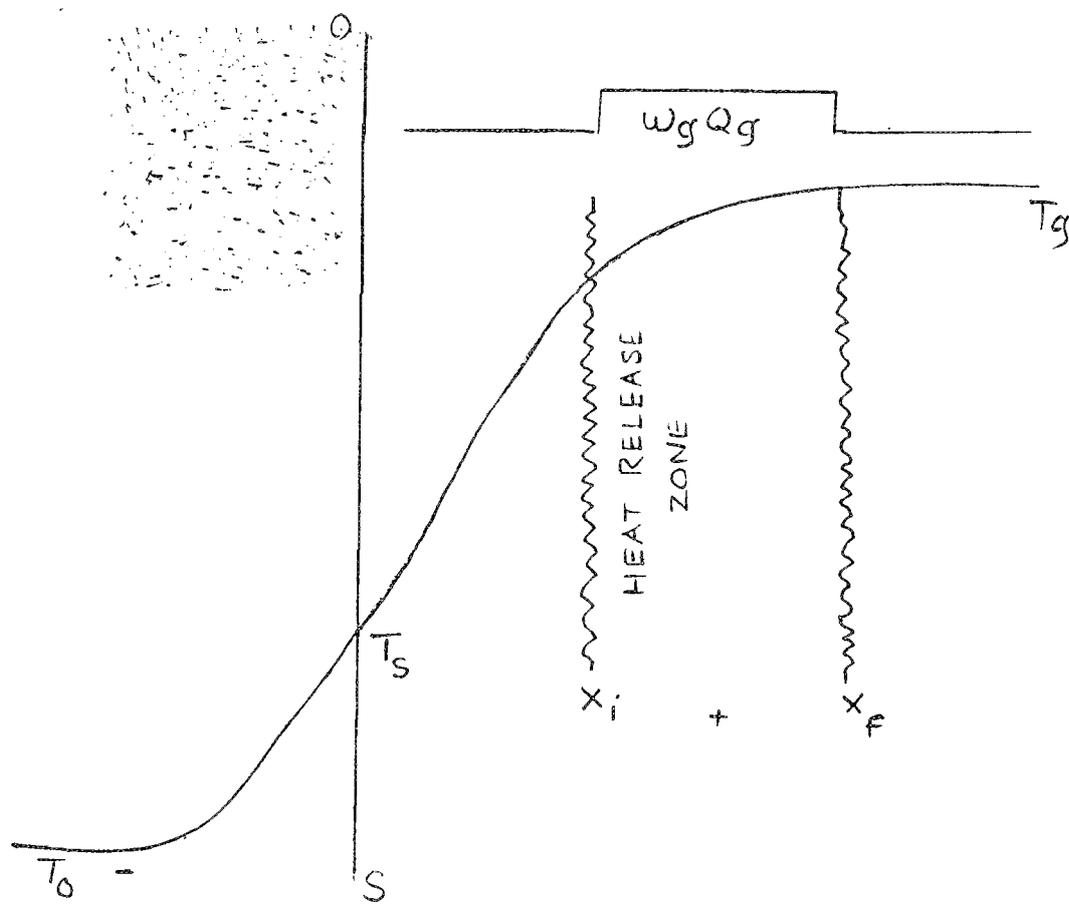


FIG 5 : GAS PHASE HEAT TRANSFER

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (2.27)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (2.28)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2.29)$$

$$\rho \left(u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} \right) = \frac{\partial}{\partial y} \left\{ \frac{\mu}{\text{Pr}} \left[\frac{\partial h}{\partial y} \right] \right\} \quad (2.30)$$

$$\rho u \frac{\partial y_i}{\partial x} + \rho v \frac{\partial y_i}{\partial y} - \frac{\partial}{\partial y} \left(\rho D_i \frac{\partial y_i}{\partial y} \right) = \dot{\omega}_i \quad (2.31)$$

$$\frac{P}{\rho} = RT \quad (2.32)$$

3.7 GOVERNING EQUATIONS OF TURBULENT BOUNDARY LAYER WITH

CHEMICAL REACTIONS : This section outlines the theoretical model for a reacting turbulent boundary layer. The theoretical model comprises of the conservation equations of mass, momentum, species, enthalpy, turbulent kinetic energy and turbulent dissipation rate and the equations of state.

The physical model considered in the theoretical analysis consists of flow of gases (a) over a flat plate or (b) inside a cylinder.

In both cases, the surface could be subjected to blowing or suction or could be a burning surface. However in the present approach, the surface is the burning surface of a solid propellant. A set of general conservation equation for a reacting turbulent compressible fluid flow are given below : The boundary layer can be assumed to be quasi-steady planar or axi-symmetric and chemically reacting. Major assumptions are :

1. The average flow properties are steady .

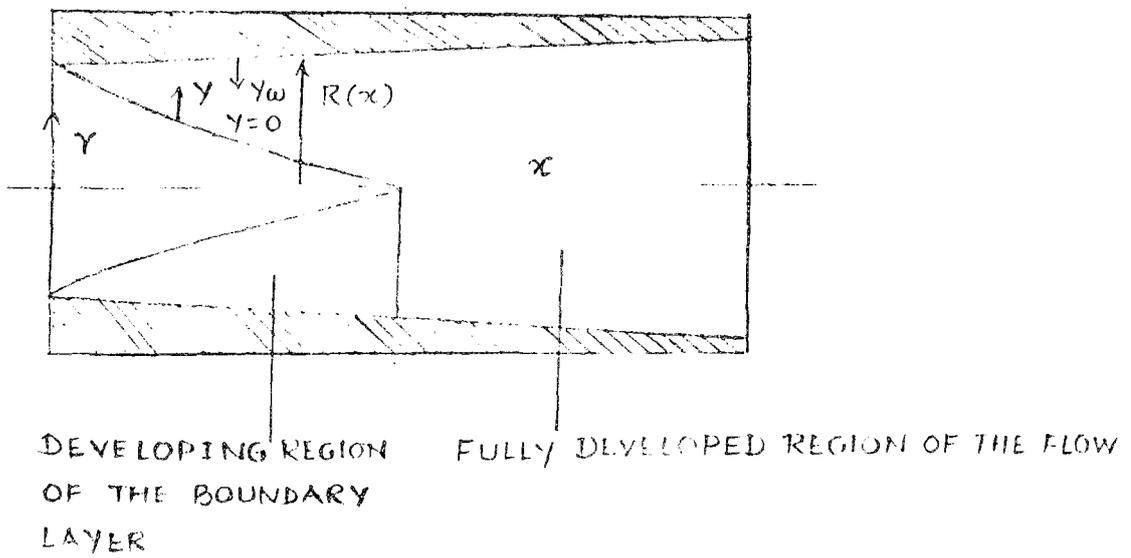


FIG 6 : PHYSICAL MODEL

2. Body forces are absent.
3. There is no radiative heat transfer.
4. The Lewis number is unity.
5. Fick's law of diffusion is valid .

A second order two equation K - ε turbulence model is used to achieve closure of turbulent flow problem. Using these assumptions and following order of magnitude analysis, conservation equations for mass, momentum, species, enthalpy, turbulent kinetic energy and turbulent diffusion and the equation of state are obtained as :

$$\frac{\partial}{\partial x}(r^m \bar{\rho} \bar{u}) + \frac{\partial}{\partial y}(r^m \bar{\rho} \bar{v}) = 0 \quad (2.33)$$

$$\bar{\rho} \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{\rho} \bar{v} \frac{\partial \bar{u}}{\partial y} = \frac{1}{r^m} \frac{\partial}{\partial y} \left[r^m \mu_{eff} \frac{\partial \bar{u}}{\partial y} \right] - \frac{\partial \bar{P}}{\partial x} \quad (2.34)$$

$$\bar{\rho} \bar{u} \frac{\partial \bar{Y}_i}{\partial x} + \bar{\rho} \bar{v} \frac{\partial \bar{Y}_i}{\partial y} = \frac{1}{r^m} \frac{\partial}{\partial y} \left[r^m \left(\frac{\mu}{Sc} \right)_{eff} \frac{\partial \bar{Y}_i}{\partial y} \right] + \bar{\omega} \quad (2.35)$$

$$\bar{\rho} \bar{u} \frac{\partial \bar{H}}{\partial x} + \bar{\rho} \bar{v} \frac{\partial \bar{H}}{\partial y} = \frac{1}{r^m} \frac{\partial}{\partial y} \left[r^m \left(\frac{\mu}{Pr} \right)_{eff} \frac{\partial \bar{H}}{\partial y} + \left\{ \mu_{eff} - \left(\frac{\mu}{Pr} \right)_{eff} \right\} \frac{\partial (\bar{u}^2 / 2)}{\partial y} \right] \quad (2.36)$$

$$\bar{\rho} \bar{u} \frac{\partial K}{\partial x} + \bar{\rho} \bar{v} \frac{\partial K}{\partial y} = \frac{1}{r^m} \frac{\partial}{\partial y} \left[r^m \left(\mu + \frac{\mu_t}{c_1} \right) \frac{\partial K}{\partial y} \right] + \mu_t \left(\frac{\partial \bar{u}}{\partial y} \right)^2 - \bar{\rho} \varepsilon \quad (2.37)$$

$$\bar{\rho} \bar{u} \frac{\partial \varepsilon}{\partial x} + \bar{\rho} \bar{v} \frac{\partial \varepsilon}{\partial y} = \frac{1}{r^m} \frac{\partial}{\partial y} \left[r^m \left(\mu + \frac{\mu_f}{c_2} \right) \frac{\partial \varepsilon}{\partial y} \right] + c_3 \mu_t \left(\frac{\partial \bar{u}}{\partial y} \right)^2 \frac{\varepsilon}{K} - c_4 \bar{\rho} \frac{\varepsilon^2}{K} \quad (2.38)$$

$$\bar{P} = \frac{\bar{\rho} R u \bar{T}}{W} \quad (2.39)$$

4. PROCESS OF DIFFUSION

A diffusion flame may be defined as any flame in which the fuel and oxidizer are initially separated (non-premixed). Diffusion flames are either laminar or turbulent.

4.1 LAMINAR DIFFUSION : In this type of flame, fuel and air flow with the same linear velocity. Molecules behave in an orderly manner and the flow is almost stagnant.

4.2 TURBULENT DIFFUSION : When the reactants enter in a separate streams, oxidizer and fuel being non-premixed, the resulting flames are called the turbulent diffusion flames. In this type of flow the molecules behave in disorderly manner, there will be eddies formation.

4.3 FICK'S LAW OF DIFFUSION⁴⁶: The mass diffusion velocity v_i in the y-direction is given by

$$v_i = -\frac{D_i}{Y_i} \frac{\partial Y_i}{\partial y} \quad (2.40)$$

These equations have been obtained under the assumption that the body force and pressure gradient diffusion are neglected.

5. DIMENSIONAL ANALYSIS⁴⁷

Dimensional Analysis is a mathematical technique which makes use of the study of dimensions as an aid to the solution of several physical problems. Each physical phenomenon can be expressed by an equation composed of variables (or physical quantities) which may be dimensional or non-dimensional quantities. Dimensional analysis helps in determining a systematic arrangement of variables in the physical relationship and combining dimensional variables to form non-dimensional parameters. In the study of fluid mechanics the dimensional analysis has been found to be useful in both analytical and experimental investigations. This technique is useful in

- (1) Testing the dimensional homogeneity of any equation of fluid motion.
- (2) Deriving the equation expressed in terms of non- dimensional parameters to show the relative importance of each parameter .

(3) Planning model terms and presenting experimental results in a systematic manner in terms of non- dimensional parameters.

5.1 METHODS OF DIMENSIONAL ANALYSIS: Following are the two methods of dimensional analysis generally used.

(a) RAYLEIGH METHOD : In this method, a functional relationship of more variables is expressed in the form of an exponential equation which must be dimensionally homogeneous. Thus if X is some function of variables X_1, X_2, \dots, X_n , the functional equation can be written as

$$X = f(X_1, X_2, \dots, X_n), \quad (2.41)$$

where X is a dependent variable, X_1, X_2, \dots, X_n are independent variables. This equation may be expressed as

$$X = CX_1^a X_2^b \dots X_n^n \quad (2.42)$$

where C is a dimensionless constant which may be determined either from the physical characterization of the problem or from the experimental measurements. The exponentials a, b, c, \dots are then evaluated on the basis that the equation is dimensionally homogeneous. The dimensionless parameters are formed by grouping together the variables with like powers.

(b) BUCKINGAM THEOREM: The theorem makes use of the following assumptions :

- (i) It is always possible to select m independent fundamental units in a physical phenomenon. (In dynamics of viscous compressible fluids $m = 4$, i.e., mass, length, time and temperature).
- (ii) There exist n quantities, say Q_1, Q_2, \dots, Q_n involved in a physical phenomenon whose dimensional formulae may be expressed in terms of m fundamental units.
- (iii) there exists a functional relationship between the n dimensional quantities Q_1, Q_2, \dots, Q_n , say

$$f(Q_1, Q_2, \dots, Q_n) = 0, \quad (2.43)$$

and this equation is independent of the types of units chosen and is dimensionally homogeneous.

The Π - theorem may now be stated as follows :

An equation in physical variables, which is dimensionally homogeneous, can be reduced to a relationship among a complete set of dimensionless products.

5.2 METHOD OF FINDING OUT Π - PRODUCTS: The following steps may be adopted in finding out the Π -products in a complete set :

- (i) Write down the dimensional matrix of n physical quantities, involving in a physical phenomenon, having m independent fundamental units.
- (ii) Determine the rank of the dimensional matrix. If the rank is r (say) then the number of Π 's will be $(n-r)$.
- (iii) Select r quantities out of the n physical quantities as base quantities, keeping in view that these r quantities should have different dimensions and the dimension of any of the fundamental unit should not be zero in all of them.
- (iv) Express $\Pi_1, \Pi_2, \dots, \Pi_{n-r}$ as power products of these r quantities raised to arbitrary integer exponents and one of the excluded, but different in different Π 's, $(n-r)$ quantities.
- (v) Equate to zero the total sum of dimensions of each fundamental unit in each Π -product to get the integer exponents.

6. SUMMARY

This chapter forms a basis for the work presented in the thesis. With a short introduction in this chapter, basic concepts in the field of internal ballistics of solid rocket propellants have been discussed. The performance parameters like characteristic velocity, thrust coefficient, and specific impulse have been defined. The combustion process of composite, double base, composite modified double base propellant has been presented in

detail. The governing equations of laminar flow and turbulent flow have been presented. The expressions for heat transfer in solid and gas phase have also been developed. These expressions along with the structure of combustion wave have been utilised to develop a generalized burn rate model for solid rocket propellant using the technique of dimensional analysis.