Time Response of First-Order Systems to Unit Step and Unit Ramp Inputs: Derivation of Error and Output

First-Order Systems:

First-order systems are characterized by a single energy storage element, such as a capacitor or inductor, and a first-order differential equation governing their behavior.

They are commonly encountered in various engineering applications, including control systems, circuits, and mechanical systems.

Unit Step Input:

A unit step input is a sudden change in the input signal from zero to a constant value at t = 0.

The step function is denoted as u(t), and a unit step input is represented by u(t - t0), where t0 is the time delay, if any.

The Laplace transform of a unit step function is U(s) = 1/s.

Derivation of Error:

The error (e(t)) in a first-order system is defined as the difference between the input and the output of the system.

For a unit step input, the input (U(s)) is 1/s, and the Laplace transform of the output (Y(s)) is given by the transfer function (G(s)) multiplied by the input (U(s)).

Thus, Y(s) = G(s) \* U(s) = G(s) / s.

To obtain the error, we subtract the output from the input, e(s) = U(s) - Y(s) = 1/s - G(s) / s.

Taking the inverse Laplace transform of e(s) gives the time-domain expression for the error, e(t).

Derivation of Output:

To obtain the time-domain expression for the output (y(t)), we take the inverse Laplace transform of Y(s) = G(s) \* U(s).

The inverse Laplace transform of G(s) / s can be found using the final value theorem or by applying the inverse Laplace transform to the transfer function directly.

The time-domain expression for the output, y(t), can be found by convolving the inverse Laplace transform of G(s) / s with the unit step function u(t - t0).

Unit Ramp Input:

A unit ramp input is a linearly increasing input signal with a slope of 1.

The ramp function is denoted as r(t), and a unit ramp input is represented by t \* u(t - t0).

The Laplace transform of a unit ramp function is  $R(s) = 1/s^2$ .

Derivation of Error and Output for Unit Ramp Input:

The derivation for the error and output of a first-order system to a unit ramp input follows a similar procedure as for the unit step input.

The Laplace transform of the output (Y(s)) for a unit ramp input is given by  $Y(s) = G(s) * R(s) = G(s) / s^2$ .

The error (e(s)) can be obtained by subtracting Y(s) from the Laplace transform of the input (R(s)),  $e(s) = R(s) - Y(s) = 1/s^2 - G(s) / s^2$ .

Taking the inverse Laplace transform of e(s) gives the time-domain expression for the error, e(t).

The time-domain expression for the output (y(t)) can be found by convolving the inverse Laplace transform of  $G(s) / s^2$  with the unit ramp function t \* u(t - t0).

Understanding the time response of first-order systems to unit step and unit ramp inputs allows for the analysis and design of various systems. The derived expressions for error and output provide insights into the system's transient and steady-state behavior.