

## Time Response of Second-Order System to Unit Step Input: Derivation of Output for Underdamped, Overdamped, and Critically Damped Cases

### Second-Order Systems:

Second-order systems are characterized by two energy storage elements and a second-order differential equation governing their behavior.

These systems commonly arise in engineering applications, including control systems, mechanical systems, and electrical circuits.

### Unit Step Input:

A unit step input is a sudden change in the input signal from zero to a constant value at  $t = 0$ .

The step function is denoted as  $u(t)$ , and a unit step input is represented by  $u(t - t_0)$ , where  $t_0$  is the time delay, if any.

The Laplace transform of a unit step function is  $U(s) = 1/s$ .

### Derivation of Output:

To derive the time response of a second-order system to a unit step input, we consider three cases: underdamped, overdamped, and critically damped.

#### Underdamped Case:

In the underdamped case, the system has complex conjugate poles, denoted as  $s_1 = -\zeta\omega_n + j\omega_d$  and  $s_2 = -\zeta\omega_n - j\omega_d$ , where  $\omega_n$  is the natural frequency and  $\zeta$  is the damping ratio.

The transfer function of the system is given by  $G(s) = K / [(s + \zeta\omega_n)^2 + \omega_d^2]$ .

The inverse Laplace transform of  $G(s) / s$  gives the time-domain expression for the output,  $y(t)$ , which can be found using partial fraction decomposition and inverse Laplace transform techniques.

#### Overdamped Case:

In the overdamped case, the system has two distinct real poles,  $s_1 = -\alpha$  and  $s_2 = -\beta$ , where  $\alpha$  and  $\beta$  are positive real numbers.

The transfer function of the system is given by  $G(s) = K / [(s + \alpha)(s + \beta)]$ .

The inverse Laplace transform of  $G(s) / s$  gives the time-domain expression for the output,  $y(t)$ , which can be found using partial fraction decomposition and inverse Laplace transform techniques.

Critically Damped Case:

In the critically damped case, the system has two identical real poles,  $s_1 = s_2 = -\alpha$ , where  $\alpha$  is a positive real number.

The transfer function of the system is given by  $G(s) = K / (s + \alpha)^2$ .

The inverse Laplace transform of  $G(s) / s$  gives the time-domain expression for the output,  $y(t)$ , which can be found using partial fraction decomposition and inverse Laplace transform techniques.

Understanding the time response of second-order systems to unit step inputs is crucial for analyzing and designing control systems, mechanical systems, and electrical circuits. The derived expressions for the output provide insights into the transient and steady-state behavior of the system in different damping scenarios.