**Resonant Peak:** 

The resonant peak is a characteristic feature of a second-order system's frequency response. It represents the maximum magnitude of the response at the system's natural frequency. Here's a derivation of the resonant peak:

Transfer Function: Consider a second-order system with the transfer function H(s) given by:

 $H(s) = K / (s^2 + 2\zeta \omega n s + \omega n^2)$ 

Where K is the system gain,  $\zeta$  is the damping ratio, and  $\omega$ n is the undamped natural frequency.

Frequency Response: The frequency response of the system can be obtained by substituting  $s = j\omega$  (where j is the imaginary unit) into the transfer function. This yields:

 $H(j\omega) = K / (-\omega^2 + j2\zeta\omega\omega n - \omega n^2)$ 

Magnitude of Frequency Response: The magnitude of the frequency response is given by:

 $|H(j\omega)| = |K / (-\omega^2 + j2\zeta\omega\omega n - \omega n^2)|$ 

Maximum Magnitude (Resonant Peak): To find the maximum magnitude, we differentiate the magnitude with respect to  $\omega$  and set the derivative equal to zero:

 $d|H(j\omega)|/d\omega = 0$ 

By solving this equation, we can find the value of  $\omega$  that corresponds to the maximum magnitude.

Simplifying the Derivative: To simplify the derivative calculation, it is convenient to work with the squared magnitude:

 $|H(j\omega)|^{2} = (K / (-\omega^{2} + j2\zeta\omega\omega n - \omega n^{2})) * (K / (-\omega^{2} - j2\zeta\omega\omega n - \omega n^{2}))$ 

 $= K^2 / [(\omega n^2 - \omega^2)^2 + (2\zeta \omega \omega n)^2]$ 

Calculating the Derivative: Differentiating  $|H(j\omega)|^2$  with respect to  $\omega$  yields:

 $d|H(j\omega)|^2/d\omega = (4\zeta\omega n^2\omega - 4\omega\zeta^2\omega n^3) / [(\omega^2 - \omega n^2)^2 + (2\zeta\omega\omega n)^2]$ 

Setting the Derivative to Zero: Set the derivative equal to zero and solve for  $\omega$  to find the value that gives the maximum magnitude. This gives the resonant frequency,  $\omega r$ , which is the frequency at which the resonant peak occurs.

Bandwidth:

The bandwidth of a second-order system represents the range of frequencies over which the system's response is significant. It is typically defined as the range between the two frequencies where the response is attenuated by a specified amount (e.g., -3 dB from the maximum). Here's a derivation of the bandwidth:

Damping Ratio: The damping ratio,  $\zeta$ , is a crucial parameter that influences the system's bandwidth. Higher values of  $\zeta$  result in a smaller bandwidth, indicating a faster decay of the system's response.

Undamped Natural Frequency: The undamped natural frequency,  $\omega n$ , determines the overall scale of the system's response. It does not directly affect the bandwidth.

-3 dB Frequency Points: The -3 dB frequency points correspond to the frequencies at which the magnitude of the response is reduced by 3 dB (approximately 70.7% of the maximum magnitude). Let's call these frequencies  $\omega 1$  and  $\omega 2$ .

Calculation of Bandwidth: The bandwidth, BW, is calculated as the difference between  $\omega 1$  and  $\omega 2$ :

 $\mathsf{BW} = \omega 2 - \omega 1$ 

Calculation of  $\omega 1$  and  $\omega 2$ : The frequencies  $\omega 1$  and  $\omega 2$  can be determined by solving the following equation:

 $|H(j\omega)|^{2} = (K / (-\omega^{2} + j2\zeta\omega\omega n - \omega n^{2}))^{2} = 0.5$ 

Solving this equation gives the values of  $\omega 1$  and  $\omega 2$ , which correspond to the -3 dB points.

It's important to note that the resonant peak and bandwidth of a system depend on the system's parameters, such as the damping ratio and undamped natural frequency. By analyzing these characteristics, we can gain insights into the system's frequency response and behavior.