

CENTURION UNIVERSITY OF TECHNOLOGY AND MANAGEMENT



Centurion
UNIVERSITY
Shaping Lives...
Empowering Communities...

School of Engineering and Technology

Paralakhemundi

Assignment -2

Subject Name-Laplace & Fourier Transform (CUTM1002)

SHORT QUESTIONS:

1. Find the value of $\Gamma(5)$.
2. What is the crucial property of the Laplace transform that makes it suitable for solving differential equation?

3. Evaluate the integral $\int_0^{\infty} e^{-x} x^{\frac{3}{2}} dx$ [CUTM- 2020]

4. Define unit step function. What is the Laplace transform of the unit step function? [CUTM- 2019]
5. What is the Laplace transform of $f(t) = t \sin 2t$? (CUTM -2022).
6. What is the Laplace transform of the function

$$f(t) = t^2 \text{ if } 0 < t < 1$$
$$= 0 \text{ otherwise.}$$

7. Find the Laplace transform of $L(\cos(\omega t))$ using the result $L(e^{i\omega t}) = \frac{1}{s - i\omega}$.
8. Show that $L(f''(t)) = s^2 L[f(t)] - sf(0) - f'(0)$.
9. Find the inverse Laplace transform of $\frac{1}{s^2(s-1)}$.
10. Find the Laplace transform of $5e^{2t} \sinh 2t$.
11. Find the inverse Laplace transform of $\frac{1}{s^2 + 4s}$ by integration.
12. Find the Laplace transform of π^t .

13. Find the inverse Laplace transformation of $F(s) = \frac{4}{s^2 + 2s + 5}$.

14. What is the subsidiary equation? How is it used?

15. What is Dirac's delta function? How did we use it?

LONG QUESTIONS:

1. Find the Laplace transform of the following function:

$$f(t) = \begin{cases} 2 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \sin t & \text{if } t > 2\pi \end{cases} .$$

2. Find the Laplace transform of the periodic function $f(t) = t, 0 < t < 2$ (with period 2).

3. Find the inverse Laplace transformation of $\frac{1-7s}{(s-3)(s-1)(s+2)}$.

4. Solve the initial value problem using Laplace transforms $y'' - 2y' + y = e^t$

5. Find the inverse Laplace transform of $F(s) = \frac{s}{(s^2 + \pi^2)^2}$ using

the convolution theorem.

6. Find the function $f(t)$ whose Laplace transform is $\frac{6}{s^2(s^2 - 9)}$.

7. If $f * g = \int_0^t f(t-\gamma)g(\gamma)d\gamma$, then show that $f * g = g * f$

8. Solve the differential equation by using Laplace transformation:

$$\frac{d^2 y}{dx^2} + 6\frac{dy}{dx} + 8y = e^{-3t} - e^{-5t}, \text{ where } y(0) = 0 \text{ and } y'(0) = 0.$$

9. Using convolution theorem, find the inverse Laplace transform of the following function:

$$F(s) = \frac{1}{s(s^2 + 4)}$$

10. Using Laplace transform solve the following integral equation:

$$y(t) = \sin t - \int_0^t \sin(t - \tau)y(\tau)d\tau .$$

SHORT QUESTIONS:

1. Find the smallest positive period of $\sin x$ & $\tan x$. (CUTM- 2012).
2. Find the smallest positive period of $\cos \frac{2\pi x}{k}$ & $\sin \frac{2\pi x}{k}$.
3. If $f(x)$ is a periodic function of period 2π then find the period of $f(2x)$ & $f(x/3)$.
4. Write the formula for Fourier Transform of a function $f(x)$. (CUTM -2012).
5. What are half-range expansions? What are they good for?
6. Do you think any function can have a Fourier expansion? Give reason for your answer.
7. Explain how Fourier transform is used to solve initial value problems.
8. Expand $\sin^2 3x + 3\cos 3x \sin 4x$ in Fourier series over 2π period.
9. Check which function is even or odd
 - i) $f(x) = \sinh x$
 - ii) $f(x) = x(1 - \cos x)$
10. What are even and odd periodic extension of a function?
11. Find the smallest positive period of the function $\sin 2\pi x$ and $\cos 2\pi x$.
12. Identify which of the functions are even in the specified period:

$$f(x) = \begin{cases} x-2, 1 \leq x \leq 2 \\ 2-x, 2 \leq x \leq 3 \end{cases}$$

13. Identify which of the functions are even in the specified period:

$$f(x) = \begin{cases} 2-x, 1 \leq x \leq 2 \\ 2-x, 2 \leq x \leq 3 \end{cases}$$

14. Check whether the following functions are odd or even or neither:

i) e^x ii) $x|x|$

iii) $x \cos x$ iv) $|x|^3$

15. Find the Fourier coefficients of the periodic function

$$f(x) = \sin \frac{x}{2}, \cos \frac{x}{2}, -\pi \leq x \leq \pi$$

LONG QUESTIONS:

1. Show that the function $f(x) = \text{const}$ is a periodic function of period p for every positive p .

2. Find the Fourier series of the function $f(x) = x^2, -\pi < x < \pi$

3. Find the Fourier series expansion of the following function:

$$f(x) = \begin{cases} k & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

And hence show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$

4. Expand the following periodic function in Fourier series $f(x) = 1 + x, x \in (0, 2\pi)$.

5. Find the cosine series of the following function:

$$f(x) = x^2, 0 < x < \pi \quad (\text{CUTM -2012}).$$

6. Find the Fourier series expansion of the following function which is assumed to be periodic with period 2

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 1 - x & \text{if } 1 < x < 2 \end{cases}$$

7. Show that $\frac{\pi}{4} = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2 - 1}$ by finding the Fourier series of the periodic

function:

$$f(x) = \begin{cases} \sin x, 0 \leq x \leq \pi \\ 0, \pi \leq x \leq 2\pi \end{cases}$$

8. Find the Fourier series of the periodic function

$$f(x) = \begin{cases} x, 0 \leq x \leq \pi \\ 2\pi - x, \pi \leq x \leq 2\pi \end{cases}$$

9. Find the half- range expansion for the even extension of the function:

$$f(x) = \begin{cases} x, 0 < x < \frac{\pi}{2} \\ \pi - x, \frac{\pi}{2} < x < \pi \end{cases}$$

10. Find the Fourier series of the following periodic function:

$$f(x) = \begin{cases} x, & -\pi \leq x \leq 0 \\ x, & 0 \leq x \leq \pi \end{cases}$$