

## DIRAC'S DELTA FUNCTION (UNIT IMPULSE FUNCTION):

Forces that produce large effects on a system when applied for a very short time interval can be represented by an impulse function which is discontinuous function. it is used in problems where a large force acts over a very small area(like hammer blow or earthquake).

The Dirac's delta function is denoted as  $\delta(t - a)$  and is defined as  $\delta(t - a) = \begin{cases} \infty & \text{if } t = a \\ 0 & \text{if } t \neq a \end{cases}$  and

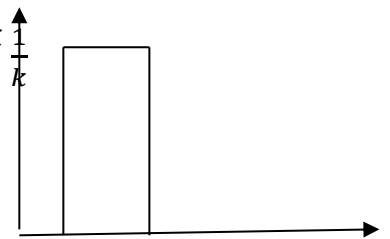
$$\int_{-\infty}^{\infty} \delta(t - a) dt = 1 \text{ .since } \delta_k(t - a) = \begin{cases} \frac{1}{k} & \text{if } a \leq t \leq a + k \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} \delta_k(t - a) dt = \int_a^{a+k} \frac{1}{k} dt = 1$$

$$\text{Also } L\{\delta(t - a)\} = e^{-as}$$

$$\text{If } a = 0 \text{ then } L\{\delta(t)\} = 1 \Rightarrow L^{-1}\{1\} = \delta(t)$$

$$\text{Example: } L\{\delta(t - 3)\} = e^{-3s} .$$



## MULTIPLICATION BY $t^n$ :

If  $L\{f(t)\} = F(s)$  then  $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [F(s)]$ , where  $n = 1, 2, 3, \dots$

$$L^{-1}\{F(s)\} = f(t), \text{ then } t f(t) = -L^{-1}\{F(s)\}$$

**Example8:** Find Laplace transform of the following functions

$$(a) t^2 \sin at \quad (b) te^{-t} \sin 3t \quad (c) te^{-3t} \cos 2t \quad (d) t^2 e^{-3t} \sin 2t$$

$$\text{Solution: (a) } L\{t^2 \sin at\} = (-1)^2 \frac{d^2}{ds^2} \left\{ \frac{a}{s^2 + a^2} \right\}$$

$$= \frac{d}{ds} \left\{ \frac{2as}{(s^2 + a^2)^2} \right\} = \frac{2a^3 - 6as^2}{(s^2 + a^2)^3}$$

$$(b) L\{te^{-t} \sin 3t\} = -\frac{d}{ds} \left\{ \frac{3}{(s+1)^2 + 9} \right\}, \text{ since } L\{e^{-t} \sin 3t\} = \frac{3}{(s+1)^2 + 9}$$

$$L\{te^{-t} \sin 3t\} = \frac{6(s+1)}{(s^2 + 2s + 10)^2}$$

$$(c) L\{te^{-3t}\cos 2t\} = -\frac{d}{ds} \left\{ \frac{s+3}{(s+3)^2+4} \right\}, \text{ since } L\{e^{-3t}\cos 2t\} = \frac{s+3}{(s+3)^2+4}$$

$$L\{te^{-3t}\cos 2t\} = \frac{s^2 + 6s + 5}{(s^2 + 6s + 13)^2}$$

$$(d) L\{t^2e^{-3t}\sin 2t\} = (-1)^2 \frac{d^2}{ds^2} \left\{ \frac{2}{(s+3)^2+4} \right\}, \text{ since } L\{e^{-3t}\sin 2t\} = \frac{2}{(s+3)^2+4}$$

$$= \frac{d}{ds} \left\{ \frac{d}{ds} \left( \frac{2}{(s+3)^2+4} \right) \right\} = -\frac{d}{ds} \left( \frac{4s+12}{(s^2+6s+13)^2} \right) = \frac{12s^2+72s+92}{(s^2+6s+13)^3}$$