

Fourier Transform

- Fourier transform is a mathematical tool that transforms a mathematical function defined in a special domain to another mathematical function in a frequency domain.
- The Fourier transform and its inverse transform are described in both continuous and discrete domains. These transforms are widely used to solve numerous problems in science and technology.

STANDARD RESULTS:

- $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$
- $\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2+b^2}$
- $\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2+b^2}$ Where $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$
- $\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}$
- $\int_0^{\infty} \frac{e^{ax} - e^{-ax}}{e^{\pi x} - e^{-\pi x}} \, dx = \frac{1}{2} \tan\left(\frac{a}{2}\right)$
- $\int_0^{\infty} \frac{e^{ax} + e^{-ax}}{e^{\pi x} - e^{-\pi x}} \, dx = \frac{1}{2} \sec\left(\frac{a}{2}\right)$
- $\int_0^{\infty} \frac{e^{-ax}}{x} \sin bx \, dx = \tan^{-1}\left(\frac{b}{a}\right), a > 0, b > 0$
- $\int_0^{\infty} \frac{\sin ax}{x} \, dx = \frac{\pi}{2} \quad \text{if } a > 0$

Fourier Complex Integral:

The Fourier complex integral form is

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iwx} \, dw \int_{-\infty}^{\infty} f(t) e^{-iwt} \, dt$$

Fourier Transform:

The Fourier complex integral form can be written in the form of Fourier transform

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iwt} f(w) \, dw$$

Where $f(w) = \int_{-\infty}^{\infty} f(t)e^{-iwt} dt$ or $\int_{-\infty}^{\infty} f(x)e^{-iwx} dx$, if $x = t$

$F(w)$ is called Fourier transform of $f(x)$ and $f(x)$ is called inverse Fourier transform of $f(w)$.

- Fourier transforms are of two types
 1. Fourier sine transform
 2. Fourier cosine transform