

## Lec-2

### EXISTENCE CONDITION FOR LAPLACE TRANSFORMS:

If  $f(t)$  is continuous and  $\lim_{t \rightarrow \infty} \{e^{-st} f(t)\}$  is finite: then Laplace transform of  $f(t)$  is  $\int_0^{\infty} e^{-st} f(t) dt$  exist for  $s > a$ . Similarly, if  $f(t)$  is piece continuous on every finite interval in the range  $t \geq 0$  and satisfies such that  $|f(t)| \leq Me^{-st}$  and some constants  $k$  and  $M$ . Then the Laplace transform of  $f(t)$  exists for all  $s > k$ .

**Example: 1.**  $L\left(\frac{1}{\sqrt{t}}\right)$  exists though  $\frac{1}{\sqrt{t}}$  is finite at  $t=0$ .

2.  $L(e^{t^2})$  is does not exist, because no matter how large we choose  $M$  and  $k$ , we have  $e^{t^2} > Me^{-st}$ , for all  $t > t_0$  where  $t_0$  is sufficiently large number.

### WORKED OUT EXAMPLES:

**Example1:** Find Laplace transform of the following functions:

- (a)  $t^2 + \sinh 2t + \cos t - 2019$       (b)  $\sin(\omega t - 3)$       (c)  $\sin^2 4t$   
(d)  $\cos^3 t$

**Solution:**

$$\begin{aligned} \text{(a)} L(t^2 + \sinh 2t + \cos t - 2019) &= L(t^2) + L(\sinh 2t) + L(\cos t) - L(2019) \\ &= \frac{2}{s^3} + \frac{2}{s^2 - 4} + \frac{s}{s^2 + 1} - \frac{2019}{s}. \end{aligned}$$

$$\begin{aligned} \text{(b)} L\{\sin(\omega t - 3)\} &= L(\sin \omega t \cos 3 - \cos \omega t \sin 3) \\ &= L(\sin \omega t \cos 3) - L(\cos \omega t \sin 3) \\ &= \cos 3 L(\sin \omega t) - \sin 3 L(\cos \omega t) = \frac{\omega \cos 3}{s^2 + \omega^2} - \frac{s \sin 3}{s^2 + \omega^2}. \end{aligned}$$

$$\begin{aligned} \text{(c)} L(\sin^2 4t) &= L\left(\frac{1 - \cos 8t}{2}\right) = \frac{1}{2} L(1 - \cos 8t) \\ &= \frac{1}{2} \{L(1) - L(\cos 8t)\} = \frac{1}{2s} - \frac{1}{2} \cdot \frac{s}{s^2 + 64}. \end{aligned}$$

$$\text{(d)} L(\cos^3 t) = L\left(\frac{1}{4} \cos 3t + \frac{3}{4} \cos t\right) = \frac{1}{4} \cdot \frac{s}{s^2 + 9} + \frac{3}{4} \frac{s}{s^2 + 1}.$$

$$\text{(since } \cos^3 t = \frac{1}{4} \cos 3t + \frac{3}{4} \cos t)$$