

Lec-3

PROPERTIES OF LAPLACE TRANSFORM:

Linearity Property: If a, b, c be any three constants and f, g, h be any function of t whose Laplace transform exist, then

$$L\{af(t) + bg(t) - ch(t)\} = aL\{f(t)\} + bL\{g(t)\} - cL\{h(t)\}.$$

Change of Scale Property: if $L\{f(t)\} = F(s)$, then $L\{f(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$ where a is positive constant.

Proof: By defⁿ of Laplace transform, we have

$$L\{f(at)\} = \int_0^{\infty} e^{-st} f(at) dt$$

$$\text{Let } at = x \Rightarrow t = \frac{x}{a} \text{ and } dt = \frac{dx}{a}$$

Also if $t \rightarrow \infty \Rightarrow x \rightarrow \infty$ and if $t \rightarrow 0 \Rightarrow x \rightarrow 0$ therefore the above integral becomes

$$L\{f(x)\} = \frac{1}{a} \int_0^{\infty} e^{-\left(\frac{s}{a}\right)x} f(x) dx = \frac{1}{a} F\left(\frac{s}{a}\right), \text{ since } \int_0^{\infty} e^{-st} f(t) dt = F(s).$$

First Shifting Property: if $L\{f(t)\} = F(s)$, then $L\{e^{at} f(t)\} = F(s - a)$, where a be any real constant.

Proof: By definition of LT, we get $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$.

$$L\{e^{at} f(t)\} = \int_0^{\infty} e^{-st} e^{at} f(t) dt$$

$$= \int_0^{\infty} e^{-(s-a)t} f(t) dt = F(s - a).$$