

Lec-4

WORKED OUT EXAMPLES:

Example2: Find Laplace transform of the following functions

- (a) $3\sinh 3t - \cos a \cos 3t$ (b) $\cos t \cos 2t \cos 3t$ (c) $\cosh^2 2t$
 (d) $e^{-2t} \sin 4t$

Solution: $L(3\sinh 3t - \cos a \cos 3t) = 3L(\sinh 3t) - \cos a L(\cos 3t)$

$$= \frac{9}{s^2 - 9} - \frac{3 \cos a}{s^2 + 9}$$

(b) $L(\cos t \cos 2t \cos 3t)$

Hear $\cos 2t \cos 3t = \frac{1}{2}(\cos 5t + \cos t)$

$$\cos t \cos 2t \cos 3t = \frac{1}{2}(\cos 5t \cos t + \cos^2 t)$$

$$= \frac{1}{4}(\cos 6t + \cos 4t + 1 + \cos 2t)$$

$$= \frac{1}{4} \left(\frac{s}{s^2 + 36} + \frac{s}{s^2 + 16} + \frac{1}{s} + \frac{s}{s^2 + 4} \right)$$

(c) $L(\cosh^2 2t) = L\left(\frac{1 + \cosh 4t}{2}\right)$, since $\cosh^2 2t = \frac{1 + \cosh 4t}{2}$

$$= \frac{1}{2s} + \frac{1}{2} \cdot \frac{s}{s^2 - 16}$$

(d) $L(e^{-2t} \sin 4t)$

Comparing with First shifting theorem $e^{-2t} = e^{at} \Rightarrow a = -2$

Hear $f(t) = \sin 4t \Rightarrow F(s) = \frac{4}{s^2 + 16}$

Therefore $L(e^{-2t} \sin 4t) = F(s + 2)$

$$= \frac{4}{(s + 2)^2 + 16} = \frac{4}{s^2 + 4s + 20}$$

APPLICATION OF FIRST SHIFTING PROPERTY:

Sl.No.	Laplace transform of $f(t)$	Inverse transform of F(s) for each case $s > a$
1	$L(e^{at} \sin bt)$ $= \frac{b}{(s - a)^2 + b^2}$	$L^{-1} \left(\frac{b}{(s - a)^2 + b^2} \right) = e^{at} \sin bt$
2	$L(e^{at} \cos bt)$ $= \frac{s - a}{(s - a)^2 + b^2}$	$L^{-1} \left(\frac{s - a}{(s - a)^2 + b^2} \right) = e^{at} \cos bt$

3	$L(e^{at} \sinh bt)$ $= \frac{b}{(s-a)^2 - b^2}$	$L^{-1} \left(\frac{b}{(s-a)^2 - b^2} \right) = e^{at} \sinh bt$
4	$L(e^{at} \cosh bt)$ $= \frac{s-a}{(s-a)^2 - b^2}$	$L^{-1} \left(\frac{s-a}{(s-a)^2 - b^2} \right) = e^{at} \cosh bt$
5	$L(e^{at} t^n) = \frac{\Gamma(n+1)}{(s-a)^{n+1}} \text{ or } \frac{n!}{(s-a)^{n+1}}$	$L^{-1} \left(\frac{\Gamma(n+1)}{(s-a)^{n+1}} \text{ or } \frac{n!}{(s-a)^{n+1}} \right)$ $= e^{at} t^n$