

TRANSFORM OF INTEGRALS:

If $L\{f(t)\} = F(s)$, then $L\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$ ($s > 0, s > k$)

If we take the inverse transform on both sides of above equation, we get

$$L^{-1}\left\{\frac{1}{s}F(s)\right\} = \int_0^t f(t)dt$$

WORKED OUT EXAMPLES:

Example 4: Find Inverse Laplace transform of the following functions

$$(a) \frac{2(s+1)}{s^2(s^2+4)} \quad (b) \frac{2(s+1)}{s^2(s-1)} \quad (c) \frac{9(s+1)}{s^2(s^2+9)} \quad (d) \frac{9}{s^2(s^2+9)}$$

SOLUTION: (a) $L^{-1}\left\{\frac{2(s+1)}{s^2(s^2+4)}\right\} = \int_0^t \int_0^t f(t)dt dt$,

where $f(t) = L^{-1}\left\{\frac{2s}{s^2+4} + \frac{2}{s^2+4}\right\} = 2\cos t + \sin t$

$$= \int_0^t \int_0^t (2\cos t + \sin t) dt dt = \int_0^t (2\sin t - \cos t + 1) dt$$

$$= t - 2\cos t - \sin t + 2.$$

(b) $L^{-1}\left\{\frac{2(s+1)}{s^2(s-1)}\right\} = \int_0^t \int_0^t f(t)dt dt$, where $f(t) = L^{-1}\left\{\frac{2(s-1)}{s-1} + \frac{4}{s-1}\right\}$

$$= L^{-1}\left\{2 + \frac{4}{s-1}\right\} = 2u(t) + 4e^t$$

$$= \int_0^t \int_0^t (2u(t) + 4e^t) dt dt = \int_0^t (4e^t - 2) dt = 4e^t - 2t - 4.$$

(c) $L^{-1}\left\{\frac{9(s+1)}{s^2(s^2+9)}\right\} = \int_0^t \int_0^t f(t)dt dt$,

where $f(t) = L^{-1}\left\{\frac{9s}{s^2+9} + \frac{9}{s^2+9}\right\} = 9\cos 3t + 3\sin 3t$

$$= \int_0^t \int_0^t (9\cos 3t + 3\sin 3t) dt dt = \int_0^t (3\sin 3t - \cos 3t + 1) dt$$

$$= t - \cos 3t - \frac{1}{3} \sin 3t + 1.$$

$$\begin{aligned} \text{(d)} L^{-1} \left\{ \frac{9}{s^2(s^2+9)} \right\} &= \int_0^t \int_0^t f(t) dt dt, \text{ where } f(t) = L^{-1} \left\{ \frac{9}{s^2+9} \right\} = 3 \sin 3t \\ &= \int_0^t \int_0^t (3 \sin 3t) dt dt = \int_0^t (-\cos 3t + 1) dt \\ &= t - \frac{1}{3} \sin 3t. \end{aligned}$$