

Module-1 LAPLACE TRANSFORM

Introduction:

Laplace Transformations were introduced by Pierre Simon Marquis De Laplace (1749-1827), a French Mathematician known as a Newton of French. Laplace Transformations is a powerful Technique; it replaces operations of calculus by operations of Algebra.

This subject is originated from the operational methods applied by English engineer Oliver Heaviside (1850-1925), to problems in electrical engineering. The method of Laplace transform has the advantage of directly giving the solution of differential equation with boundary /initial value without the necessity of first finding the general solution and other evaluating from it the arbitrary constants.

Suppose an Ordinary (or) Partial Differential Equation together with Initial conditions is reduced to a problem of solving an Algebraic Equation.

Definition of Laplace Transform: Let $f(t)$ be a given function defined for all $t \geq 0$, then the laplace transform of $f(t)$ is denoted by $\mathcal{L}\{f(t)\}$ or $F(s)$ and is defined by

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$
 Provided that improper integral exists. Here 's' is a frequency parameter which may be real or complex number. Here $f(t)$ is called inverse Laplace transform of $F(s)$.

The symbol \mathcal{L} , which transforms $f(t)$ into $F(s)$, is called Laplace Transformation Operator.

Transform of Elementary Functions: Table-1.1

Sl.No.	$f(t)$	$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$ $= F(s)$	Condition
1.	k	$\frac{k}{s}$	$s > 0$
2	t^n	$\frac{n!}{s^{n+1}}$ or $\frac{\Gamma(n+1)}{s^{n+1}}$	$n= 0,1,2,3,\dots$
3	e^{at}, e^{-at}	$\frac{1}{s-a}, \frac{1}{s+a}$	$s > a$
4.	$\sin at$	$\frac{a}{s^2 + a^2}$	$s > 0$
5.	$\sinh at$	$\frac{a}{s^2 - a^2}$	$s > a $
6.	$\cos at$	$\frac{s}{s^2 + a^2}$	$s > 0$
7.	$\cosh at$	$\frac{s}{s^2 - a^2}$	$s > a $

Note: Laplace Transform does not exist for $s \leq 0$.