

Median

1. Measure of Central Tendency.

2. The median is determined by sorting the data set from lowest to highest values and taking the data point in the middle of the sequence.

3. Middle Value In Ordered Sequence

- If Odd n , Middle Value of Sequence
- If Even n , Average of 2 Middle Value

4. Not Affected by Extreme Values

Merits:

- It is rigidly defined
- It is easy to understand and easy to calculate.
- It is not at all affected by extreme values.
- It can be calculated for distributions with open-end classes.
- Median is the only average to be used while dealing with qualitative data.
- Can be determined graphically.

Demerits:

- In case of even number of observations median cannot be determined exactly.
- It is not based on all the observations.
- It is not capable of further mathematical treatment

For ungrouped data:-

Step-1

Arranged data in ascending or descending order.

Step:-2

If total no. of observations 'n' is **odd** then used the following formula for median

$$= \frac{n+1}{2} \text{th observation.}$$

Step:-3

If total no. of observations 'n' is **even** then used the following formula for

median = arithmetic mean of two middle observations.

Median

If $X_1, X_2, X_3, \dots, \dots, X_n$ are n observations arranged in ascending order of magnitude.

$$X_{(n+1)/2} \quad \text{if } n \text{ is odd}$$

$$\text{Median} = \left\{ \begin{array}{l} X_{n/2} + X_{(n/2)+1} \\ \hline 2 \end{array} \right. \quad \text{if } n \text{ is even}$$

Calculate the median for the following series :-

2,3,5,1,4,5,8

1,2,3,4,5,5,8.

$$\text{Median} = \frac{n+1}{2} \text{th observation.}$$

$$= 7+1/2$$

$$= 4^{\text{th}} \text{ number}$$

The data on pulse rate per minute of 10 healthy individuals are 82, 79, 60, 76, 63, 81, 68, 74, 60, 75.
 $n = 10$

60, 60, 63, 68, 74, 75, 76, 79, 81, 82

$$X_{n/2} + X_{(n/2)+1} / 2$$

$$\text{Median} = \frac{74+75}{2} = 74.5$$

Find out the median for number of sports injuries happened in cricket in all teams

37, 57, 65, 46, 12, 14, 19, 23, 56, 78, 5, 33

For Grouped data:-

Median:-

$$l + \frac{\left(\frac{n}{2} - c \right) h}{f}$$

l = lower limit of class interval where the median occurs

f = Frequency of the class where median occurs

h = Width of the median class

C = Cumulative frequency of the class preceding the median class (PCF)

Find the median weight of 590 infants born in a hospital in one year from the following table.

Weight of infant in kg	No of infants
2.0-2.5	37
2.5-3.0	117
3.0-3.5	207
3.5-4.0	155
4.0-4.5	48
4.5 and above	26

Weight of infants in kg	No of infants	Cumulative frequency
2.0-2.5	37	37
2.5-3.0	117	37+117=154
3.0-3.5	207	154+207=361
3.5-4.0	155	361+155=516
4.0-4.5	48	516+48=564
4.5 and above	26	564+26=590

$$N/2 = 590/2 = 295$$

Median Class = 3.0-3.5, so L=3.0, f= 207
Cf = 154 h=0.5

$$l + \frac{\left(\frac{n}{2} - c \right) h}{f}$$

$$= 3.0 + \frac{(295-154) * 0.5}{207}$$

$$= 3.0 + 0.34$$

$$= 3.34 \text{ Median Weight}$$

For grouped Data:-

Calculate the median for the following data series:-

Class interval	Frequency
5-9	2
10-14	11
15-19	26
20-24	17
25-29	8
30-34	6
35-39	3
40-44	2
45-49	1

Solution:-

Class interval	Frequency	cumulative frequency
5-9	2	2
10-14	11	13
15-19	26	39
20-24	17	56
25-29	8	64
30-34	6	70
35-39	3	73
40-44	2	75
45-49	1	76

$$n=76$$

l = lower limit of class interval where the median occurs

$$= 15$$

h = Width of the median class

$$= 4$$

f = Frequency of the class where median occurs

$$= 26$$

C = Cumulative frequency of the class preceding the median class

$$= 13$$

$$l + \frac{\left(\frac{n}{2} - c \right) h}{f}$$

Mode

1. Measure of Central Tendency
2. The mode is the most frequently occurring value in the data set.
3. May Be No Mode or Several Modes

Merits:

- Mode is readily comprehensible and easy to calculate.
- Mode is not at all affected by extreme values.
- Mode can be conveniently located even if the frequency distribution has class intervals of unequal magnitude
- Open-end classes also do not pose any problem in the location of mode.
- Mode is the average to be used to find the ideal size.

Demerits:

- Mode is ill defined.
- It is not based upon all the observations.
- It is not capable of further mathematical treatment.
- As compared with mean, mode is affected to a great extent by fluctuations of sampling.

Mode Example

No Mode

Raw Data: 10.3 4.9 8.9 11.7 6.3 7.7

One Mode

Raw Data: 6.3 **4.9** 8.9 6.3 **4.9** **4.9**

More Than 1 Mode

Raw Data: 21 **28** **28** 41 **43** **43**

Mode for ungrouped data:-

2,2,3,4,6,7,4,4,4,4,8,9,0

mode is 4

10,10,3,3,4,2,1,6,7

mode is 10 and 3

10,34,23,12,11,3,4

no mode

Mode for Group Data

$$\text{Mode} = L_1 + \frac{F_m - F_1}{2F_m - F_1 - F_2} * c$$

Where

- F_m is mode freq.
- F_1 is freq. just lower mode class.
- F_2 is freq. after mode class.
- L_1 is lower limit of mode class
- C is class difference.

Q. Find the Mode for group data

Age group	20-30	30-40	40-50	50-60	60-70
No. of persons	3	20	27	15	9

CI	FREQU. (F)
20 - 30	3
30 - 40	20
40 - 50	27
50 - 60	15
60 - 70	9

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	CI	FREQU. (F)	
	20 - 30	3	
	30 - 40	20	F_1
L_1	40 - 50	27	F_m
	50 - 60	15	F_2
	60 - 70	9	

$$\text{Mode} = L_1 + \frac{F_m - F_1}{2F_m - F_1 - F_2} * c$$

$$\text{Mode} = 40 + \frac{27 - 20}{2 * 27 - 20 - 15} * 10 = 40 + \frac{70}{19}$$

$$\text{Mode} = 43.68$$

Calculate the mode for the following frequency distribution:-

IQ Range	Frequency
90-100	11
100-110	27
110-120	36
120-130	38
130-140	43
140-150	28
150-160	16
160-170	1

**Modal class by inspection is
130-140**

$$f_m = 43$$

$$f_1 = 38$$

$$f_2 = 28$$

$$C = 10$$

$$l = 130$$

$$l + \left(\frac{(f_m - f_1)}{2f_m - (f_1 + f_2)} \right) C$$

$$= 130.6579$$

Relationship between Mean, Median and Mode

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

Summary of Central Tendency Measures

Measure	Description
Mean	Balance Point
Median	Middle Value When Ordered
Mode	Most Frequent

Ex. Calculate Mean, Median, Mode.

Age Group	No. of Patients
25-30	4
30-35	3
35-40	2
40-45	3
45-50	4
50-55	8
55-60	6

Age Group	No. of Patients (F)	X	F*X	C.F
25-30	4	27.5	110	4
30-35	3	32.5	97.5	7
35-40	2	37.5	75	9
40-45	3	42.5	127.5	12
45-50	4	47.5	190	16
50-55	8	52.5	420	24
55-60	6	57.5	345	30
	30		1365	

$$\text{MEAN} = 1365/30 = \mathbf{45.5}$$

$$\text{MEDIAN} = 45 + (15-12)*5/4 = \mathbf{48.75}$$

$$\text{MODE} = 50 + [(8-4)*5/(2*8-4-6)] = \mathbf{53.34}$$

Ex. The following table gives the frequency distribution of marks obtained by 2300 medical students of Gujarat in MCQ of PSM exam. Find Mean, Median and Mode.

Marks	No. of students
11-20	141
21-30	221
31-40	439
41-50	529
51-60	495
61-70	322
71-80	153