

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Learning Objectives

- History.
- Introduction To RSM.
- Real Life Example.
- Why And When Use RSM.
- Experimental Strategy.

History

- In the Mead and Pike paper, they move back the origin of RSM to include use of "response curves" dating back into the 1930's.
- Then in 1935 Yates work on it.
- In the Hill And Hunter Review, they State that
- In November 1966, a paper "A Review of Response Surface Methodology" ; A literature was published by "Hill and Hunter. Its purpose was to review the practical applications of RSM in chemical and related fields.

History

- In December 1976, another paper “A Review of Response Surface Methodology From A Biometric View Point” was published by Mead and Pike appeared.
- With the passage of time many Statisticians work on RSM for Improvement.

Introduction

- Response surface methodology (RSM) uses various statistical, graphical, and mathematical techniques to develop, improve, or optimize a process, also use for modeling and analysis of problems if our response variables in influenced by several independent variables.
- Main objectives are as follow.
 - Optimize.(main objective)
 - Develop.
 - Improve. (if necessary).

Real Life Examples

- RSM is used in different fields of real life. Like Industries, Agriculture, Electronics, Medical field and many other like this. It is use where we want to get optimum response.

An Example of Medical field.

- A single tablet is introduced in market after large number of experiments. Suppose a company want to introduce a new pain killer tablet in market. The pharmacist will make the table that will be more effective and has rapid action to kill the pain, will have low price at market for patient

Why And When We Use RSM.

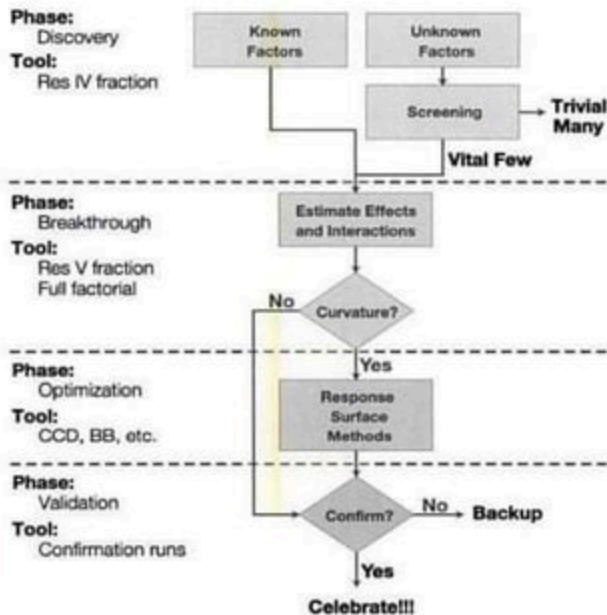


Figure 1-1. Strategy of experimentation

Experimental Strategy

1. RSM resolve around the assumption that the response is a function of a set of independent(design) variables $x_1, x_2, x_3, \dots, x_k$ and function can be approximated in some region of polynomial model.

$$y = f(x_i)$$

$$y = f(x_1, x_2 \dots \dots x_k)$$

Here response variable is “y” that depend on the “k” independent variables.

Experimental Strategy

2. If the factors are given then directly estimate the effects and interaction of model as describe in figure.
3. And if the factors are unknown then first calculate them by using the Screening method.
4. Estimate The Interaction effect using 1st order model.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

Experimental Strategy

5. If curvature is found then use the RSM. And 2nd order model will be used to approximate the response variable.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon$$

6. Make the graph and find the stationary point.
Maximum response, Minimum response or saddle point
by using the obtained values of $x_1, x_2, x_3 \dots x_4$.

Types OF Models

We use two types of model in RSM.

1. 1st Order Model.
2. 2nd Order Model.

When Use Which Model

- 1st Order Model.

Oftenly in RSM the relationship between response variable and Independent variables is not given. After screening we use 1st order model to find current situation and to find either there is curvature or not.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

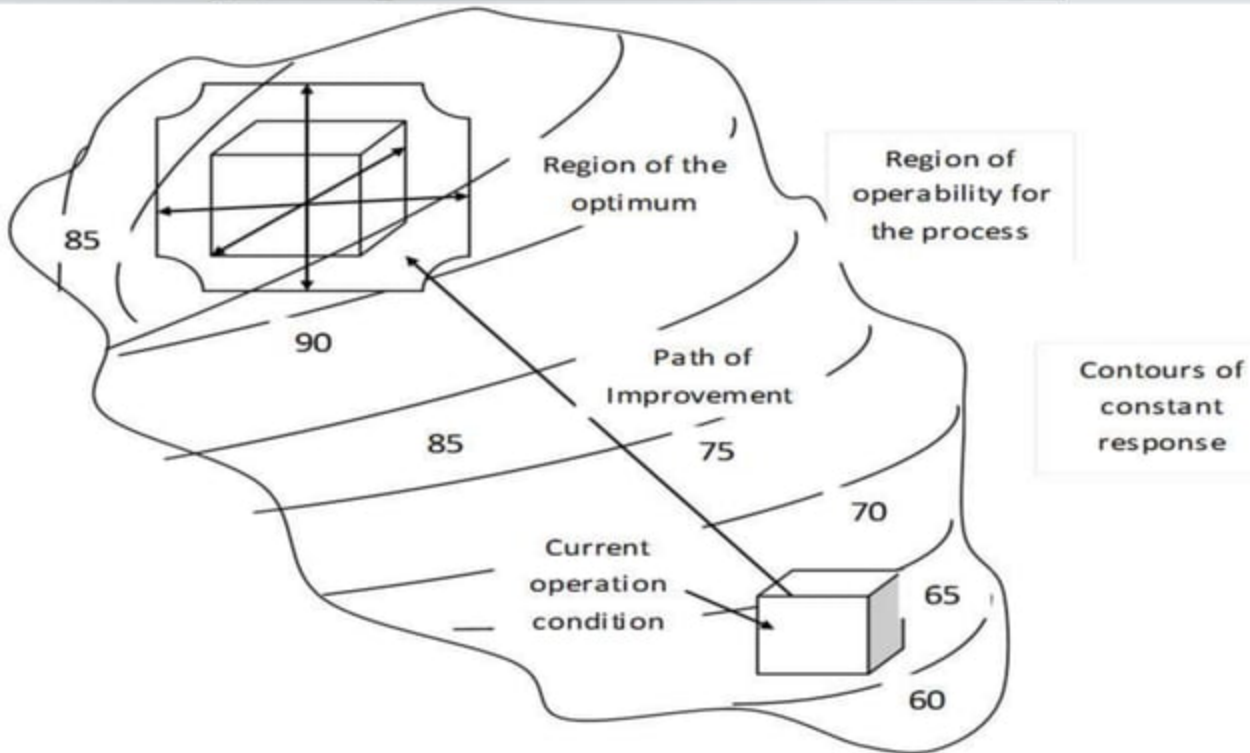
2nd Order Model

If we have find curvature after making fig from the result of 1st order model.

Then we use 2nd order model to find our optimum point.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon$$

(I) Sequential Nature Of RSM.



Sequential Nature Of RSM

RSM is **sequential procedure**. Often, when we are at a point on the response surface that is remote from the optimum, and we want to move rapidly from current point to the optimum point with sequence.

If we want the optimum point where the sources are minimum but output is maximum then that is called our optimum point. And we move rapidly toward it.

(II) Methods of RSM

- There are two methods of RSM to obtain optimum response. And we move toward our optimum point with these two method..

» **Method Of Steepest Ascent.**

» **Method Of Steepest Descent.**

Steepest Ascent Method:

This is a procedure for moving sequentially in the direction of the maximum increase in the response getting optimum response.

Steepest Descent Method :

If minimization is desired then we call this technique the “method of steepest descent”.

Steepest Ascent Method

- The initial estimate of the optimum operating condition for this will be far from the actual optimum.
- In such circumstances, the objective of the experimenter is to move rapidly to the general vicinity(nearest point) of the optimum. We wish to use a simple and economically efficient experimental procedure. When we remote from the optimum, we usually assume that a 1st order model is an adequate approximation to the true surface in a small region of the x 's.

Steepest Ascent Method

This is a procedure for moving sequentially in the direction of the maximum increase in the response getting optimum response.

$$\bar{y} = \beta_0 + \sum_{i=1}^k \bar{\beta}_i x_i$$

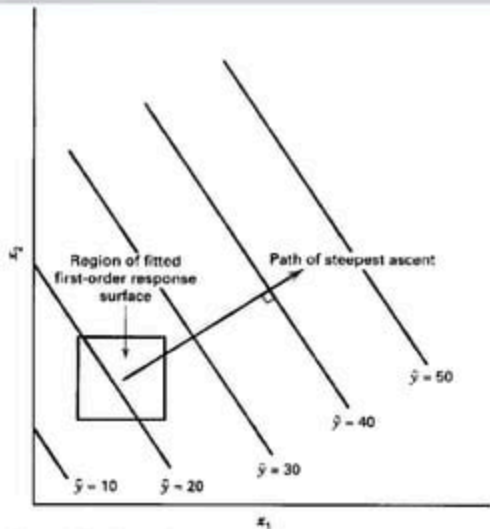


Figure 11-4 First-order response surface and path of steepest ascent.

Steepest Descent Method

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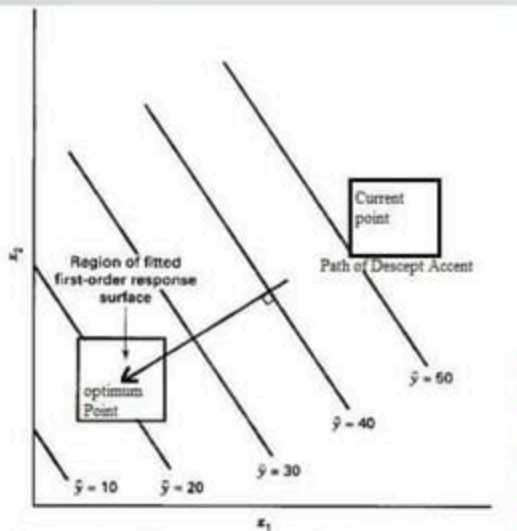


Figure 11-4 First-order response surface and path of steepest ascent.

Numerical example:

Numerical variable		Coded variable		response
Time	temp	x1	x2	y
30	150	-1	-1	39.3
30	160	-1	1	40.0
40	150	1	-1	40.9
40	160	1	1	41.5
35	155	0	0	40.3
35	155	0	0	40.5
35	155	0	0	40.7
35	155	0	0	40.2
5	155	0	0	40.6

the coded variable are

$$x1 = \frac{time - 35}{5}$$

$$x2 = \frac{temp - 155}{5}$$

A first order model may be fit to these data by least, employing the methods for two level designs, we obtain the following model in the coded variable

$$y = 40.44 + 0.775x_1 + 0.775x_2$$

Before exploring along the path of steepest ascent, the adequacy of the first order model should be investigated. The 2^2 design with center points allows the experiment to

1. Obtain an estimate of error
2. Check for interactions (cross product terms) in the model

- the replicates at the center can be used to calculate an

$$\hat{\sigma}^2 = \frac{(40.3)^2 + (40.5)^2 + (40.7)^2 + (40.2)^2 + (40.6)^2 - \frac{(202.3)^2}{5}}{4} = 0.0430$$

The first order model assume that the variable x_1 & x_2 have an additive effect on the response. Interaction b/w the variables would be represent by the coefficient β_{12} of a cross product term $x_1 x_2$ added to the model. the least square estimate of this coefficient is just one half the interaction effect calculated as in an ordinary 2^2 factorial design. Or

- $\hat{\beta}_{12} = \frac{1}{4}[(1 * 39.3) + (1 * 41.5) + (-1 * 40.0) + (-1 * 40.9)]$
 $= -0.025$

The single degree of freedom sum of square for interaction is

$$\text{SS interaction} = \frac{(-0.1)^2}{4} = 0.0025$$

Comparing SS interactions to $\hat{\sigma}^2$ gives a lack of fit statistics

$$F = \frac{\text{ss interaction}}{\hat{\sigma}^2} = 0.0025/0.0430 = 0.058$$

Which is a small ,indicating that interaction is negligible

Application

- ❖ The most frequent applications of RSM are in the industrial area.
- ❖ RSM is important in designing formulating and developing and analyzing new specific scientific studying and product.
- ❖ It is also efficient in improvements of existing studies and products
- ❖ Most common application of RSM are in industrial ,biological and clinical sciences, social sciences ,food sciences and physical and engineering sciences