



Hypothesis Testing

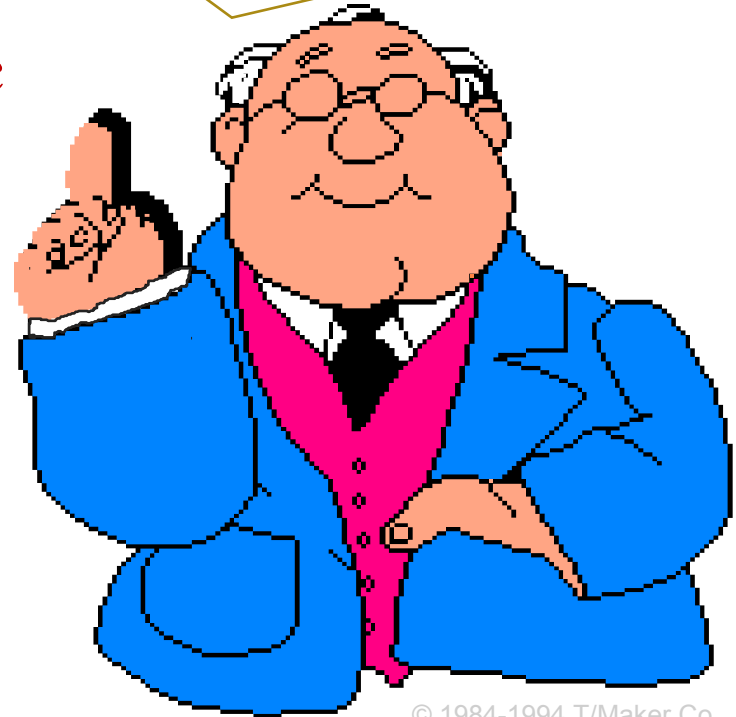
What is a Hypothesis?

A hypothesis is an assumption about the population parameter.

A parameter is characteristic of the population, like its mean or variance.

The parameter must be identified before analysis.


I assume the average weight of this class is 58 kg!



Testing of Hypothesis

A hypothesis is an assumption about the population parameter (say population mean) which is to be tested.

For that we collect sample data , then we calculate sample statistics (say sample mean) and then use this information to judge/decide whether hypothesized value of population parameter is correct or not.

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- To test the *validity of assumed or hypothetical value of population* , we gather sample data and determine the difference between hypothesized value and actual value of the sample mean.
 - Then we judge whether the difference is significant or not.
 - **The smaller the difference, the greater the likelihood that our hypothesized value for the mean is correct. The larger the difference, the smaller the likelihood.**

- In hypothesis testing the first step is to state the assumed or hypothesized(numerical) value of the population parameter.
- The assumption we wish/ want to test is called the null hypothesis. The symbol for null hypothesis is H_0 .

The Null Hypothesis, H_0

- **State the Assumption (numerical) to be tested**
- **e.g. The average weight of the semester 2 student is 58kgs ($H_0: \mu = 58$)**
- **Begin with the assumption that the null hypothesis is TRUE.**

(Similar to the notion of innocent until proven guilty)



The Alternative Hypothesis, H_1

- **Is the opposite of the null hypothesis**

eg. The average weight of the students is not equal to 58kgs. ($H_1: \mu \neq 58$)

Procedure of Hypothesis Testing

The Hypothesis Testing comprises the following steps:

Step 1

Set up a hypothesis.

Step 2

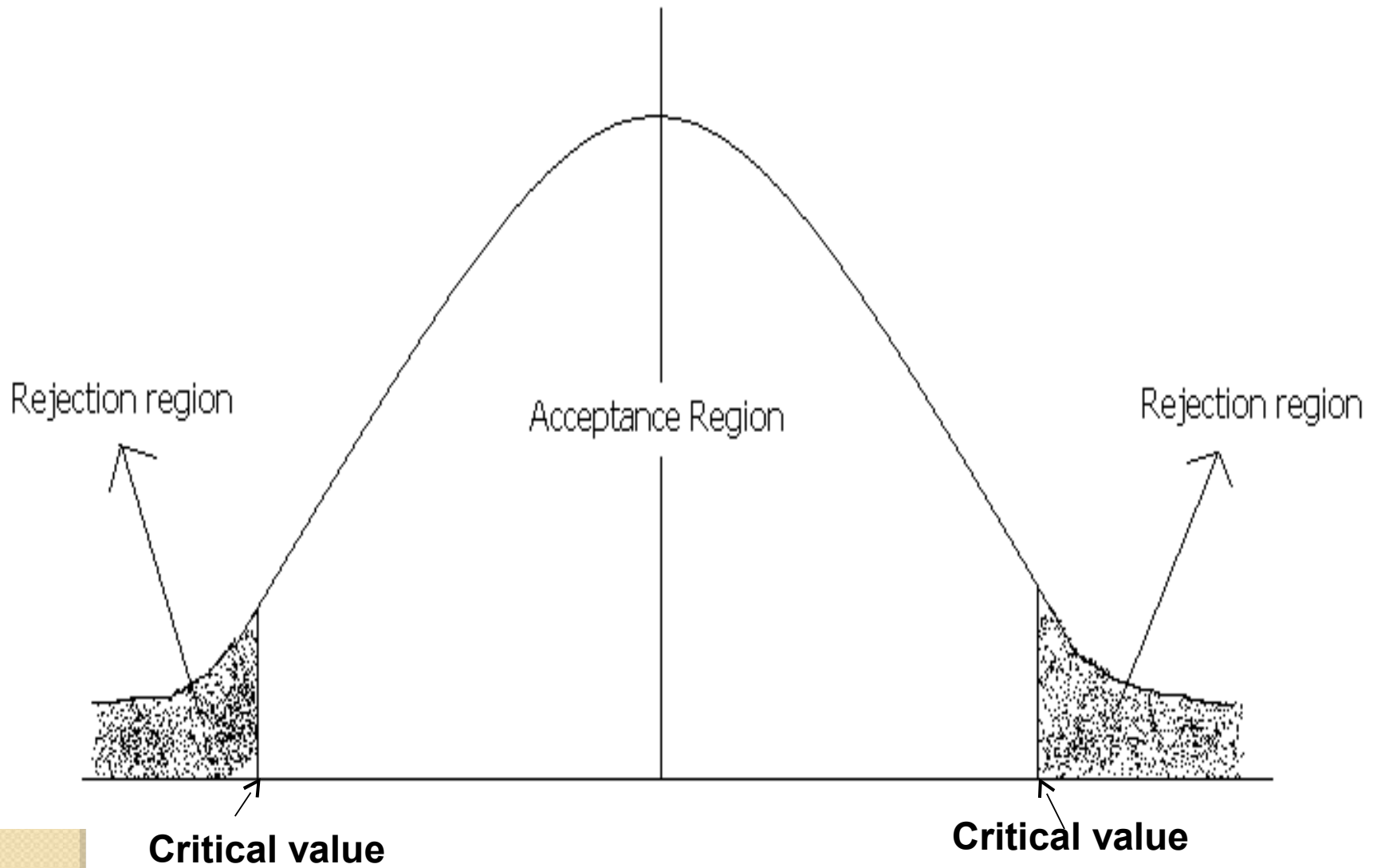
Set up a suitable significance level.

The confidence with which an experimenter rejects or accepts Null Hypothesis depends on the significance level adopted. Level of significance is the **rejection region** (which is outside the confidence or acceptance region).The level of significance, usually denoted by the **α** .

Selecting a significance level

Though any level of significance can be adopted, **in practice we either take 5% or 1% level of significance .**

When we take 5% level of significance($\alpha = .05$), then there are about 5 chances out of 100 that we would reject the null hypothesis. In other words out of 100, 95% chances are there that the null hypothesis will be accepted i.e. we are about 95% confident that we have made the right decision.



If our sample statistic(calculated value) fall in the non-shaded region(acceptance region), then it simply means that there is no evidence to reject the *null hypothesis.*

It proves that null hypothesis (H_0) is true. Otherwise, it will be rejected.

Step 3

Determination of suitable test statistic: For example Z, t Chi-Square or F-statistic.

Step 4

Determine the critical value from the table.

Step 5

After doing computation, check the sample result.

Compare the calculated value(sample result) with the value obtained from the table.(tabulated or critical value)

Step 6

Making Decisions

Making decisions means either accepting or rejecting the null hypothesis.

If computed value(absolute value) is more than the tabulated or critical value, then it falls in the critical region. In that case, reject null hypothesis, otherwise accept.

Type I and Type II Errors

When a statistical hypothesis is tested, there are 4 possible results:

- (1) The hypothesis is true but our test accepts it.
- (2) The hypothesis is false but our test rejects it.
- (3) The hypothesis is true but our test rejects it.
- (4) The hypothesis is false but our test accepts it.


Obviously, the last 2 possibilities lead to errors.

Rejecting a null hypothesis when it is true is called a *Type I error*.

Accepting a null hypothesis when it is false is called *Type II error*.

Example 1 - Court Room Trial

In court room, a defendant is considered not guilty as long as his guilt is not proven. The prosecutor tries to prove the guilt of the defendant. Only when there is enough charging evidence the defendant is condemned. In the start of the procedure, there are two hypotheses H_0 : "the defendant is not guilty", and H_1 : "the defendant is guilty". The first one is called *null hypothesis*, and the second one is called *alternative hypothesis*.



	Null Hypothesis (H_0) is true He is not guilty	Alternative Hypothesis (H_1) is true He is guilty
Accept Null Hypothesis	Right decision	Wrong decision Type II Error
Reject Null Hypothesis	Wrong decision Type I Error	Right decision

One-Tailed and Two-Tailed Tests

Two-Tailed Test is that where the hypothesis about the population parameter is rejected for the value of sample statistic falling into **either tail** of the distribution.(fig3)

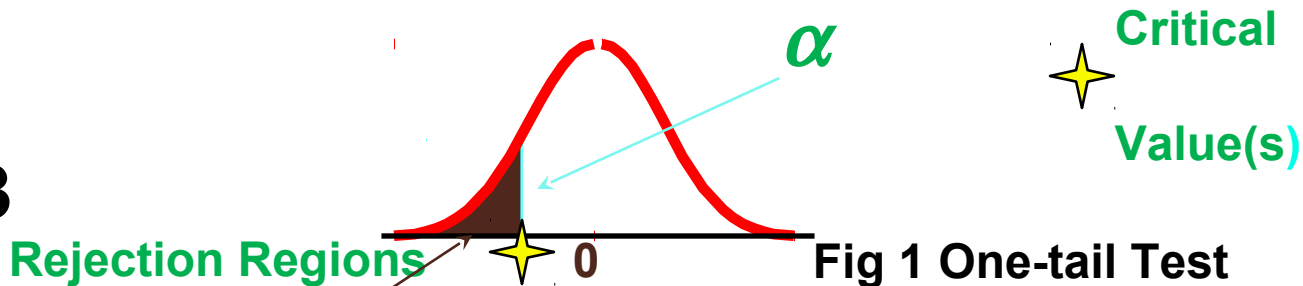
When the hypothesis about the population parameter is rejected for the value of sample statistic falling into **one side tail** of the distribution, then it is known as **one-tailed test**.

If the rejection area falls on the right side, then it is called right-tailed test.(fig 2) On the other hand If the rejection area falls on the left side, then it is called left-tailed test.(fig 1)

Two-tail and One-tail Test

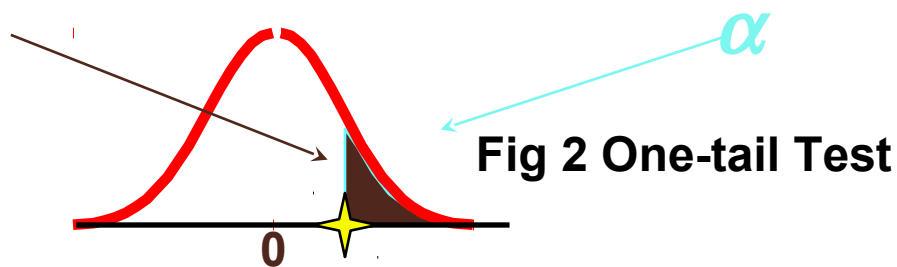
$$H_0: \mu \geq 3$$

$$H_1: \mu < 3$$



$$H_0: \mu \leq 3$$

$$H_1: \mu > 3$$



$$H_0: \mu = 3$$

$$H_1: \mu \neq$$

3

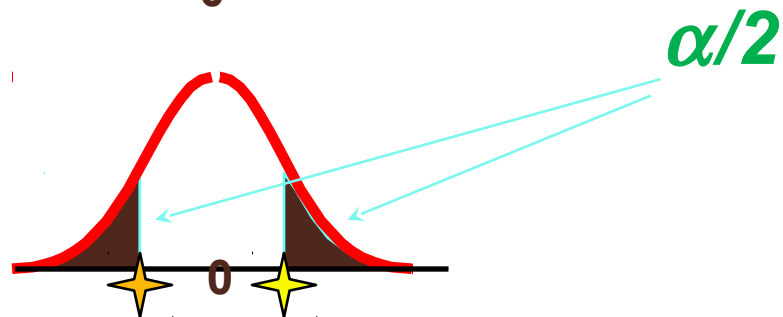


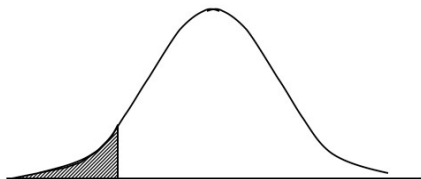
Fig-3 two-tail test

Summary of One- and Two-Tail Tests...

One-Tail Test
(left tail)

$$H_0 : \mu = \mu_0$$

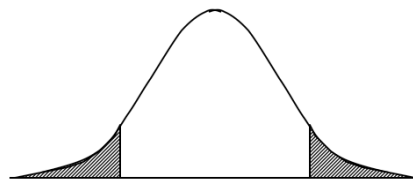
$$H_1 : \mu < \mu_0$$



Two-Tail Test

$$H_0 : \mu = \mu_0$$

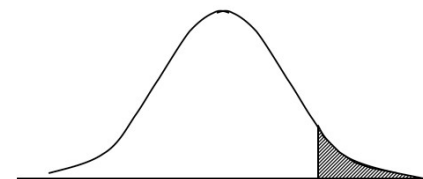
$$H_1 : \mu \neq \mu_0$$



One-Tail Test
(right tail)

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu > \mu_0$$



The following table gives critical values of Z for both one-tailed and two-tailed tests at various levels of significance.

level of significance	0.10	.05	.01
critical value of z for one-tailed test	1.28 or -1.28	1.645 or -1.645	2.33 Or -2.33
critical value of z for two-tailed test	1.645 or -1.645	1.96 or -1.96	2.58 or -2.58