

Binary

Addition & Subtraction



## Elementary school concepts

* + Add columns of numbers and keep track of the carry

over to the next column

* + We normally use the decimal number system
		- Digits: 0-9
		- Carry over is in sets of 10x

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 1 | 1 | 1 |
| 245 | 245 | 245 | 245 | 245 | 245 |
| + 189 | + 189 | + 189 | + 189 | + 189 | + 189 |
|  | 14 | 4 | 134 | 34 | 434 |

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## Extend elementary school concepts

* + Add columns of numbers and keep track of the carry

over to the next column

* + Use the binary number system
		- Digits: 0-1
		- Carry over is in sets of 2x

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 1 | 1 | 1 |  |
| 101 | 101 | 101 | 101 | 101 | 101 | 101 |
| + 011 | + 011 | + 011 | + 011 | + 011 | + 011 | + 011 |
|  | 2 | 0 | 20 | 00 | 200 | 1000 |
|  | (10) |  | (10) |  | (10) |  |

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## Example

2 sets of 2

2

zero 1’s

1 set of 2

2

1

one 1

2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 101101 | 101101 | 101101 |  | 101101 | 101101 | 101101 |  | 101101 |
| 100101 | 100101 | 100101 |  | 100101 | 100101 | 100101 |  | 100101 |
| 010101 | 010101 | 010101 |  | 010101 | 010101 | 010101 |  | 010101 |
| + 101011 | + 101011 | + 101011 |  | + 101011 | + 101011 | + 101011 |  | + 101011 |
|  | 4 | 4 |  | 0 | 3 0 | 30 |  | 10 |

(100) (11)

1

101101

100101

010101

+ 101011

410

2

101101

100101

010101

+ 101011

010

2

101101

100101

010101

+ 101011

4010

2

101101

100101

010101

+ 101011

0010

2

101101

100101

010101

+ 101011

30010

1

101101

100101

010101

+ 101011

10010

1

101101

100101

010101

+ 101011

410010

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 2 | 2 | 1 | 1 |  |
| 101101 | 101101 | 101101 | 101101 | 101101 |
| 100101 | 100101 | 100101 | 100101 | 100101 |
| 010101 | 010101 | 010101 | 010101 | 010101 |
| + 101011 | + 101011 | + 101011 | + 101011 | + 101011 |

Common

010010

2010010

4 0010010

10010010

10010010

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## Overflow

* + In elementary school we did not care how many digits

the answer required

745

+ 589

1334

* + In binary addition – we are generally representing something that ultimately is to be executed in hardware
		- Our hardware cannot change the number of bits (wires) it can hold
		- We must establish a maximum number size and create an error

when the result of the addition exceeds this size

* + - The error is called an overflow 101101

100101

00101101

00100101

3 bit addition

101

1. bit addition

00101

1. bit addition

010101

8 bit addition

00010101

+ 011

1000

overflow

+ 00011

01000

OK

+ 101011

10010010

overflow

+ 00101011

10010010

OK

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## Signed binary addition is done the same way as unsigned addition

* + Signed – means 2’s complement representation
	+ Overflow is determined differently
		- carry-in of the msb  carry-out of the msb  overflow

8 bit signed addition

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 |  | 1 | 1 | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  | 1 | 1 | 1 |  |
| **0** | **1** | **0** | **0** | **0** | **1** | **1** | **1** | **71** |  | **1** | **1** | **0** | **0** | **0** | **1** | **1** | **1** | **-57** |

 **+ 0 0 0 1 1 1 0 1** + **29**

**0 1 1 0 0 1 0 0 100**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |
| **0** | **1** | **1** | **0** | **0** | **1** | **1** | **1** | **103** |

 **+ 1 1 1 1 0 1 0 1** + **-11**

**1 0 1 1 1 1 0 0 -68**

1 1 1

1

**1 1 0 0 0 1 1 1 -57**

 **+ 1 0 0 0 0 1 0 1** + **-123**

**0 1 0 0 0 1 0 0 -180**

|  |  |
| --- | --- |
|  **+ 0 0 1 1 1 1 0 1**  |  + **61** |
| **1 0 1 0 0 1 0 0**overflow -28Common | **164** |

overflow 68

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## Elementary school concepts

* + Subtract columns of numbers and keep track of how

much is borrowed from the next column

* + This is very difficult to implement in hardware

245 minuend

- 189 subtrahend

56 difference

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 315 |  | 113 |  |
| 245 | 245 | 235 | 235 | 135 | 245 |
| - 189 | - 189 | - 189 | - 189 | - 189 | - 189 |
|  | 6 | 6 | 56 | 056 | 56 |

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## Binary Subtraction - signed

* + Signed – means 2’s complement representation
	+ Negate the subtrahend and add
	+ Overflow – same rule during addition

|  |  |  |  |
| --- | --- | --- | --- |
| 71 | 71 | 7 | 7 |
| - 29 | + (-29) | - (-99) | + 99 |

**0 1 0 0 0 1 1 1 71**

 **- 0 0 0 1 1 1 0 1** - **29**

**0 0 0 0 0 1 1 1 7**

 **- 1 0 0 1 1 1 0 1** - **(-99)**

1 1 1 1

1

**0 1 0 0 0 1 1 1 71**

 **+ 1 1 1 0 0 0 1 1** + **(-29)**

**0 0 1 0 1 0 1 0 42**

1 1 1

**0 0 0 0 0 1 1 1 7**

0

 **+ 0 1 1 0 0 0 1 1** + **99**

**0 1 1 0 1 0 1 0 116**



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