

Deformation of solids

Unit 2

Deformation of Solids

- It is possible to change the shape or size (or both) of an object through the application of external forces
- It is described as strain.
- As deformation occurs internal inter molecular forces arise that oppose the applied force.
- If the applied force is not too great these internal forces are sufficient to completely resist the applied force and allow the object to assume a new equilibrium state and to return to its original state when the load is removed.
- A larger applied force may lead to a permanent deformation of the object or even to its structure failure.
- Depending upon the type of material, size and geometry of the object and the forces applied various deformation may result.

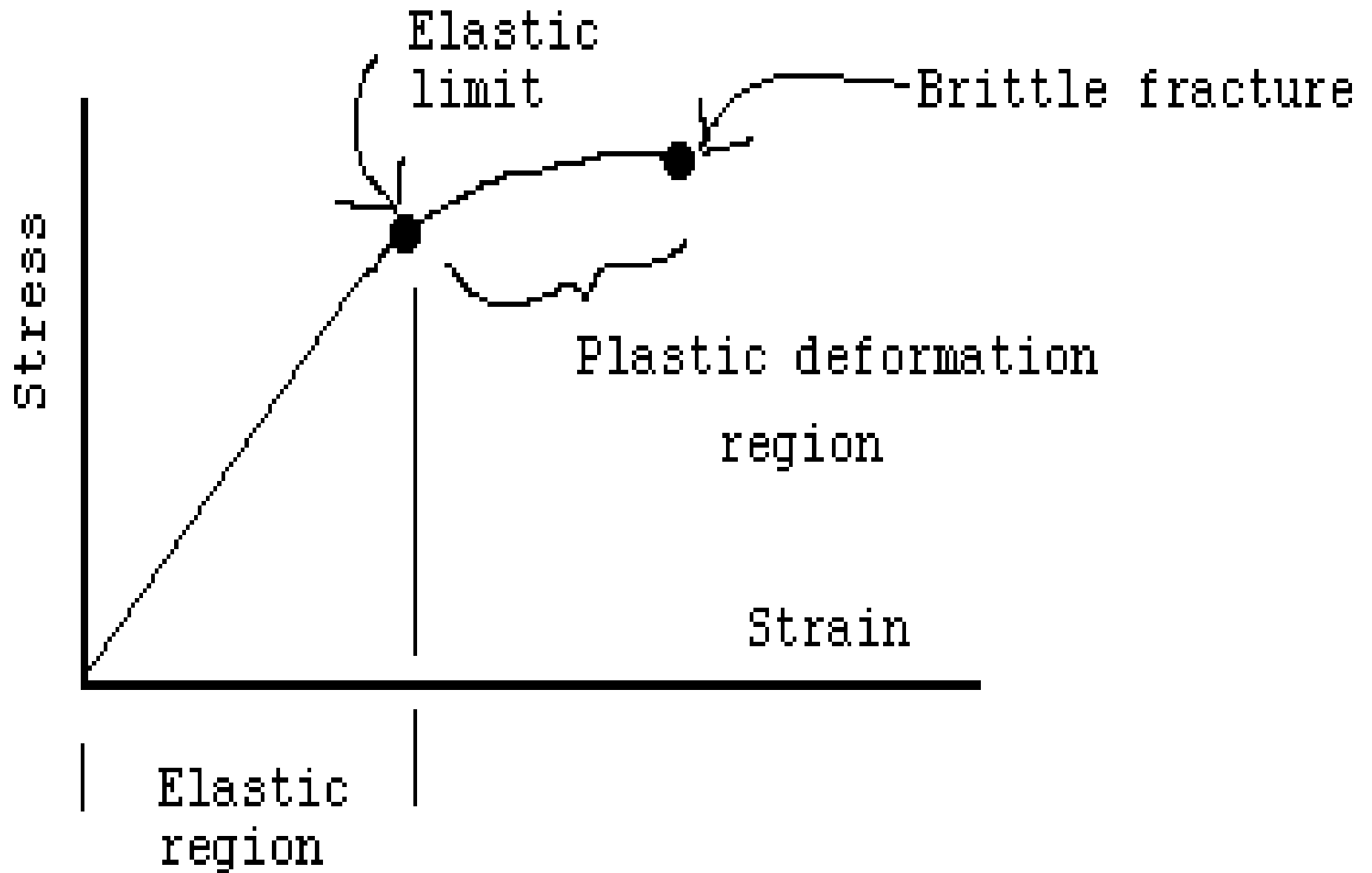
Elastic Deformation

- It is reversible.
- when the forces are removed, the object tends to its original shape.
- Elastomers and shape memory metals such as nitinol exhibit large elastic deformation ranges as does rubber. Elasticity is non linear. Metals and ceramics show linear elasticity.
- Linear elastic deformation is governed by **Hooke's Law**

$$\sigma = E \epsilon$$

Where σ is applied stress, E is material constant called young's modulus or elastic modulus and ϵ is the resulting strain.

- The relationship indicates that the slope of stress vs. strain curve can be used to find the elastic or young modulus E .
- The elastic range ends when the material reaches its yield strength.
- At this point plastic deformation begins.



Elastic Modulus

- The *elastic modulus* is the constant of proportionality between stress and strain
 - For sufficiently small stresses, the stress is directly proportional to the strain
 - The constant of proportionality depends on the material being deformed and the nature of the deformation
- The elastic modulus can be thought of as the stiffness of the material
 - A material with a large elastic modulus is very stiff and difficult to deform

Elastic Modulus

stress = Elastic modulus \times strain

Young's Modulus

- Tensile stress is the ratio of the external force to the cross-sectional area
- SI units of stress are Pascals, Pa
 - $1 \text{ Pa} = 1 \text{ N/m}^2$
- The tensile strain is the ratio of the change in length to the original length
 - Strain is dimensionless
 - The elastic modulus is called *Young's modulus*

$$\frac{F}{A} = Y \frac{\Delta L}{L_o}$$

Plastic Deformation

- It is irreversible.
- Object in plastic deformation range will first have undergone elastic deformation which is reversible so the object will partly return to its original shape.
- Soft thermoplastic materials have rather large plastic deformation range as do ductile metals such as copper, silver and gold. An example of a material with a large plastic deformation range is a wet chewing gum which can be stretched dozens of its times its original length.
- Hard thermosetting plastics, rubber and ceramics have minimal plastic deformation ranges.

- Under the tensile stress plastic deformation is characterized as
 1. **Strain hardening region** – material becomes stronger through the movement of atomic dislocations
 2. **Necking region** – reduction in cross sectional area of specimen. It begins after the ultimate strength is reached. Material can no longer withstand the maximum stress and strain in the specimen rapidly increases
 3. **Fracture**- indicates the end of the plastic deformation

Breaking

- If stress continues, it surpasses its *ultimate strength*
 - The ultimate strength is the greatest stress the object can withstand without breaking
- The **breaking point**
 - For a brittle material, the breaking point is just beyond its ultimate strength
 - For a ductile material, after passing the ultimate strength the material thins and stretches at a lower stress level before breaking

Shear Modulus

- It defines the change in shape of an object caused by pushing its top and bottom surfaces in opposite directions.
- Shear stress is the force per unit area exerted on the top and bottom of solid.
- Shear strain is the distance the top surface has moved relative to the bottom surface divided by the perpendicular distance between them.

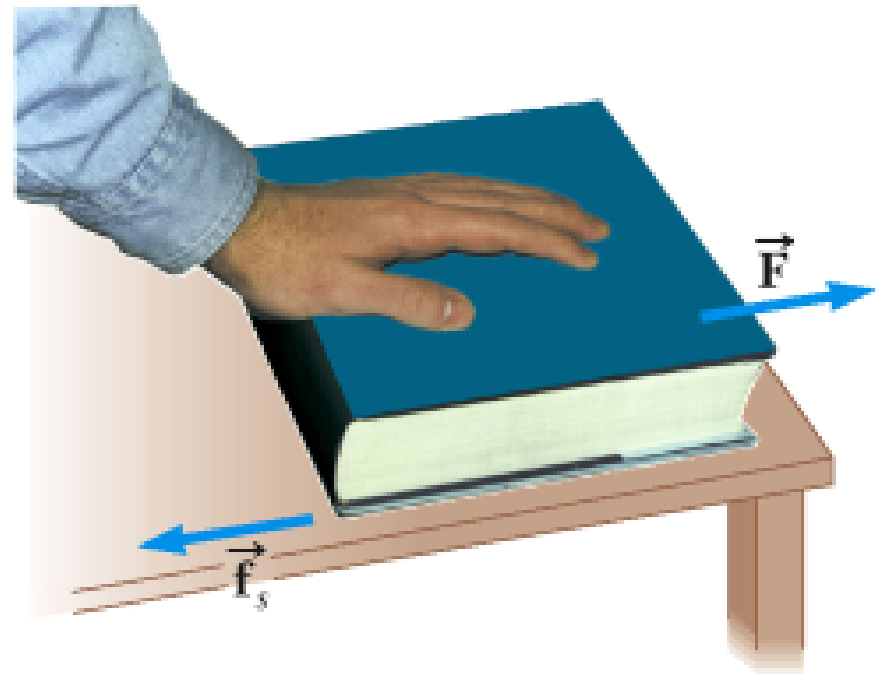
- Shear Modulus is the ratio of shear stress to shear strain and has a unit of N/m^2 .
- $S = \text{Shear stress} / \text{Shear Strain}$
 $= \frac{F/A}{\Delta x/h}$

Where

F is applied force: **A** is area, **Δx** is the distance the top surface has moved relative to bottom surface, and **h** is perpendicular distance between the top and bottom surfaces.

Shear Modulus, final

- $shear\ stress = \frac{F}{A}$
- $shear\ strain = \frac{\Delta x}{h}$
- $\frac{F}{A} = S \frac{\Delta x}{h}$
- S is the shear modulus
- A material having a large shear modulus is difficult to bend

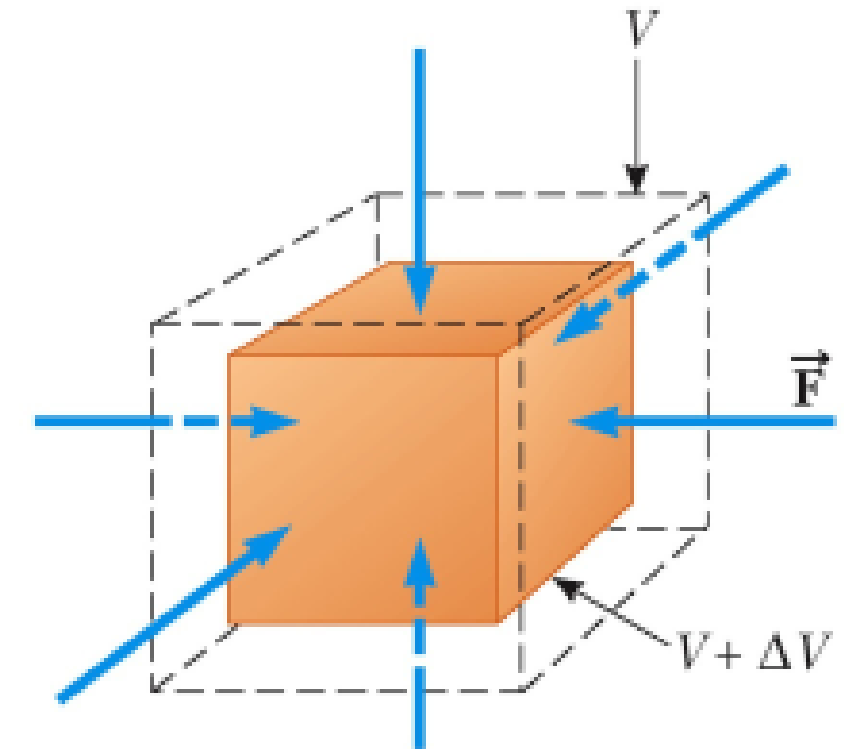


Bulk Modulus: Volume Elasticity

- Bulk modulus characterizes the response of an object to uniform squeezing
 - Suppose the forces are perpendicular to, and act on, all the surfaces
 - Example: when an object is immersed in a fluid
- The object undergoes a change in volume without a change in shape

Bulk Modulus, cont.

- Volume stress, ΔP , is the ratio of the force to the surface area
 - This is also the Pressure
- The volume strain is equal to the ratio of the change in volume to the original volume



Heckel Equation

- The variations in volume with applied pressure are defined by way of numerous equations among which Heckel theory is most vital. Heckel measured that decline in the voids follows the first order kinetics connection with applied pressure. For compression method heckel has suggested following equation

- $$\ln(V/V - V_d) = KP + (V_0/V_0 - V_d)$$

- Where V = volume at the applied pressure P
- V_0 = original volume of the powder including the voids
- V_a = volume of the powder excluding the voids
- K = constant related to the yield pressure of the powder
- P = Applied pressure

as we know **porosity E** is the ratio of the total volume of the void space to the bulk volume of the powdered material

$$E = \frac{V - V_a}{V}$$

Replacing the value in previous equation it becomes

$$\ln(1/E) = KP + \left(\frac{V_0}{V_0 - V_a} \right)$$

Bulk Modulus, final

$$\Delta P = -B \frac{\Delta V}{V}$$

- A material with a large bulk modulus is difficult to compress
- The negative sign is included since an increase in pressure will produce a decrease in volume
 - B is always positive
- The *compressibility* is the reciprocal of the bulk modulus