**Congruences: Definition - Basic properties of congruences**

Congruence is a relation between two integers that represents their equality modulo a given integer. In other words, two integers "a" and "b" are congruent modulo "n" if their difference "a - b" is divisible by "n". This is denoted as "a ≡ b (mod n)".

Here are some key details about congruences:

1. Definition: Given three integers "a," "b," and "n" (where "n" is a positive integer), we say that "a" is congruent to "b" modulo "n" if "a - b" is divisible by "n". It can be expressed as "a ≡ b (mod n)".
2. Modular Arithmetic: Congruences are closely related to modular arithmetic. In modular arithmetic, we perform arithmetic operations (addition, subtraction, multiplication, and exponentiation) on remainders obtained when dividing by a modulus. Congruences help us understand the properties of modular arithmetic and provide a concise way to express them.
3. Basic Properties:
   * Reflexivity: Every integer "a" is congruent to itself modulo any modulus "n". For example, "a ≡ a (mod n)".
   * Transitivity: If "a ≡ b (mod n)" and "b ≡ c (mod n)", then "a ≡ c (mod n)". In other words, if two integers are congruent modulo "n", and one of them is congruent modulo "n" to a third integer, then the first and third integers are congruent modulo "n".
   * Addition Property: If "a ≡ b (mod n)" and "c ≡ d (mod n)", then "a + c ≡ b + d (mod n)". This property states that if two pairs of integers are congruent modulo "n", then their sum is also congruent modulo "n".
   * Multiplication Property: If "a ≡ b (mod n)" and "c ≡ d (mod n)", then "a \* c ≡ b \* d (mod n)". This property states that if two pairs of integers are congruent modulo "n", then their product is also congruent modulo "n".
4. Congruence Classes: Congruence classes, also known as residue classes, are sets of integers that are congruent to each other modulo a given modulus "n". For any integer "a", the congruence class of "a" modulo "n" is denoted as "[a]". It represents all integers that are congruent to "a" modulo "n". For example, the congruence class of 2 modulo 5 is represented as "[2]" and includes integers like 2, 7, 12, -3, etc., that leave the same remainder when divided by 5.

Congruences and modular arithmetic have extensive applications in various areas of mathematics, cryptography, and computer science. They provide a powerful framework for solving equations, simplifying computations, and understanding the properties of numbers. The basic properties of congruences help us establish relationships and perform operations within congruence classes.