**Euler totient function –**

The Euler totient function, also known as Euler's phi function or simply the totient function, is a mathematical function that counts the number of positive integers less than or equal to a given number "n" that are coprime (relatively prime) to "n". It is denoted by the Greek letter phi (φ) and the function is often written as φ(n).

Here are some key details about the Euler totient function:

1. Definition: The Euler totient function φ(n) is defined as the number of positive integers less than or equal to "n" that are coprime to "n". Two numbers are considered coprime if their greatest common divisor (GCD) is 1. For example, φ(9) = 6 because the numbers 1, 2, 4, 5, 7, and 8 are coprime to 9.
2. Prime Number Property: For any prime number "p", φ(p) = p - 1. This property arises from the fact that every positive integer less than "p" is coprime to "p" since no smaller number can divide a prime number other than 1.
3. Multiplicative Property: The Euler totient function has a multiplicative property, which means that if "a" and "b" are coprime positive integers (i.e., GCD(a, b) = 1), then φ(ab) = φ(a) × φ(b). This property allows for efficient computation of φ(n) for large numbers by breaking them down into their prime factors.

**Euler's Theorem**:

Euler's theorem is a significant result related to the Euler totient function. It states that if "a" and "n" are coprime positive integers, then a^φ(n) ≡ 1 (mod n), where "≡" denotes congruence modulo "n". In simpler terms, if "a" and "n" have no common factors, raising "a" to the power of φ(n) and taking the remainder when divided by "n" gives a result of 1.

Applications: The Euler totient function has applications in various areas of mathematics, particularly in number theory and cryptography. It is used in the RSA encryption algorithm for key generation, modular exponentiation, and primality testing. The knowledge of φ(n) is crucial for implementing secure cryptographic systems.

The Euler totient function and Euler's theorem are fundamental concepts in number theory that play a crucial role in various mathematical computations and cryptographic algorithms. They provide insights into the properties of coprime numbers and modular arithmetic, enabling efficient calculations and facilitating the development of secure systems.