**Prime numbers-**

Prime numbers are positive integers greater than 1 that have no divisors other than 1 and themselves. In other words, a prime number cannot be divided evenly by any other number except 1 and itself.

Here are some key points about prime numbers:

1. Definition: A prime number is a natural number greater than 1 that has exactly two distinct positive divisors: 1 and itself. For example, 2, 3, 5, 7, 11, and 13 are prime numbers.
2. Prime Factorization: Every positive integer greater than 1 can be expressed as a unique product of prime numbers, known as its prime factorization. For example, the prime factorization of 24 is 2 × 2 × 2 × 3, where 2 and 3 are prime factors.
3. Infinite Set: There are infinitely many prime numbers. This was proved by the ancient Greek mathematician Euclid using a proof by contradiction known as Euclid's theorem. The proof shows that if we assume there is a finite set of prime numbers, we can always find a larger prime number not in that set.
4. Distribution: Prime numbers do not follow a predictable pattern and become less frequent as numbers increase. However, they are found in abundance throughout the entire range of positive integers.
5. Sieve of Eratosthenes: The Sieve of Eratosthenes is an ancient algorithm used to find all prime numbers up to a given limit. It works by iteratively sieving out multiples of primes, leaving only the prime numbers.
6. Importance in Cryptography: Prime numbers are extensively used in cryptography, particularly in public-key cryptography algorithms like RSA. The security of these algorithms relies on the difficulty of factoring large composite numbers into their prime factors.
7. Open Question: The distribution of prime numbers is a topic of ongoing research in number theory. The Riemann Hypothesis, one of the most famous unsolved problems in mathematics, provides a conjectured relationship between the distribution of prime numbers and the behavior of the Riemann zeta function.

Prime numbers have captivated mathematicians for centuries due to their intriguing properties and role in number theory. They are the building blocks of integers and play a vital role in various mathematical concepts and applications.