**Rings – Sub rings-**

In abstract algebra, a ring is an algebraic structure that consists of a set equipped with two binary operations: addition and multiplication. A subring is a subset of a ring that forms a ring itself, inheriting the same operations and satisfying the ring axioms.

Here are some key details about rings and subrings:

1. Ring Definition: A ring is a set R equipped with two binary operations, usually denoted as "+" for addition and "·" (or simply concatenation) for multiplication. The set R must satisfy the following properties:

a. R is an abelian group under addition, meaning that it is closed under addition, has an identity element (usually denoted as 0), and every element has an additive inverse. b. R is closed under multiplication, meaning that the product of any two elements in R is also in R. c. Multiplication is associative, meaning that for any three elements a, b, and c in R, the equation (a · b) · c = a · (b · c) holds. d. Distributive properties hold, meaning that for any three elements a, b, and c in R, the equations a · (b + c) = a · b + a · c and (a + b) · c = a · c + b · c hold.

1. Subring Definition: A subring of a ring R is a subset S of R that forms a ring under the same operations of addition and multiplication defined in R. To be a subring, S must satisfy the following properties:

a. S is closed under addition, meaning that the sum of any two elements in S is also in S. b. S is closed under multiplication, meaning that the product of any two elements in S is also in S. c. S contains the additive identity of R. d. S contains the additive inverses of its elements. e. S is closed under subtraction, meaning that the difference of any two elements in S is also in S.

1. Examples: Here are a few examples of rings and subrings:

a. The set of integers (Z) forms a ring under ordinary addition and multiplication. The set of even integers (2Z) is a subring of Z. b. The set of rational numbers (Q) forms a ring under ordinary addition and multiplication. The set of integers (Z) is a subring of Q. c. The set of real numbers (R) and the set of complex numbers (C) both form rings under ordinary addition and multiplication. Any subfield of R or C is also a subring.

1. Subring Properties: a. A subring inherits the same ring properties as the original ring, such as associativity, distributivity, and closure under addition and multiplication. b. The subring may or may not have a multiplicative identity. If the original ring has a multiplicative identity, it is inherited by the subring.
2. Subring Test: One way to determine if a subset S is a subring of a ring R is to verify that S is itself a ring using the same operations of addition and multiplication defined in R. This involves checking closure under the operations and satisfying the ring axioms.

Subrings provide a way to study and analyze certain subsets of a given ring that possess the algebraic structure of a ring themselves. They allow for the exploration of common properties and behaviors within the context of a larger ring.