

# Large-Scale Path Loss

## I. Problems Unique to Wireless (not wired) systems:

- Paths can vary from simple line-of-sight to ones that are severely obstructed by buildings, mountains, and foliage.
- Radio channels are extremely random and difficult to analyze.
- Interference from other service providers
  - out-of-band non-linear Tx emissions

- Interference from other users (same network)
  - CCI due to frequency reuse
  - ACI due to Tx/Rx design limitations & large # users sharing finite BW
- Shadowing
  - Obstructions to line-of-sight paths cause areas of weak received signal strength

- Fading

- When no clear line-of-sight path exists, signals are received that are reflections off obstructions and diffractions around obstructions
- Multipath signals can be received that interfere with each other
- Fixed Wireless Channel → random & unpredictable
  - must be characterized in a statistical fashion
  - field measurements often needed to characterize radio channel performance

- \*\* The Mobile Radio Channel (MRC) has unique problems that limit performance \*\*
  - A mobile Rx in motion influences rates of fading
    - the faster a mobile moves, the more quickly characteristics change

## II. Radio Signal Propagation

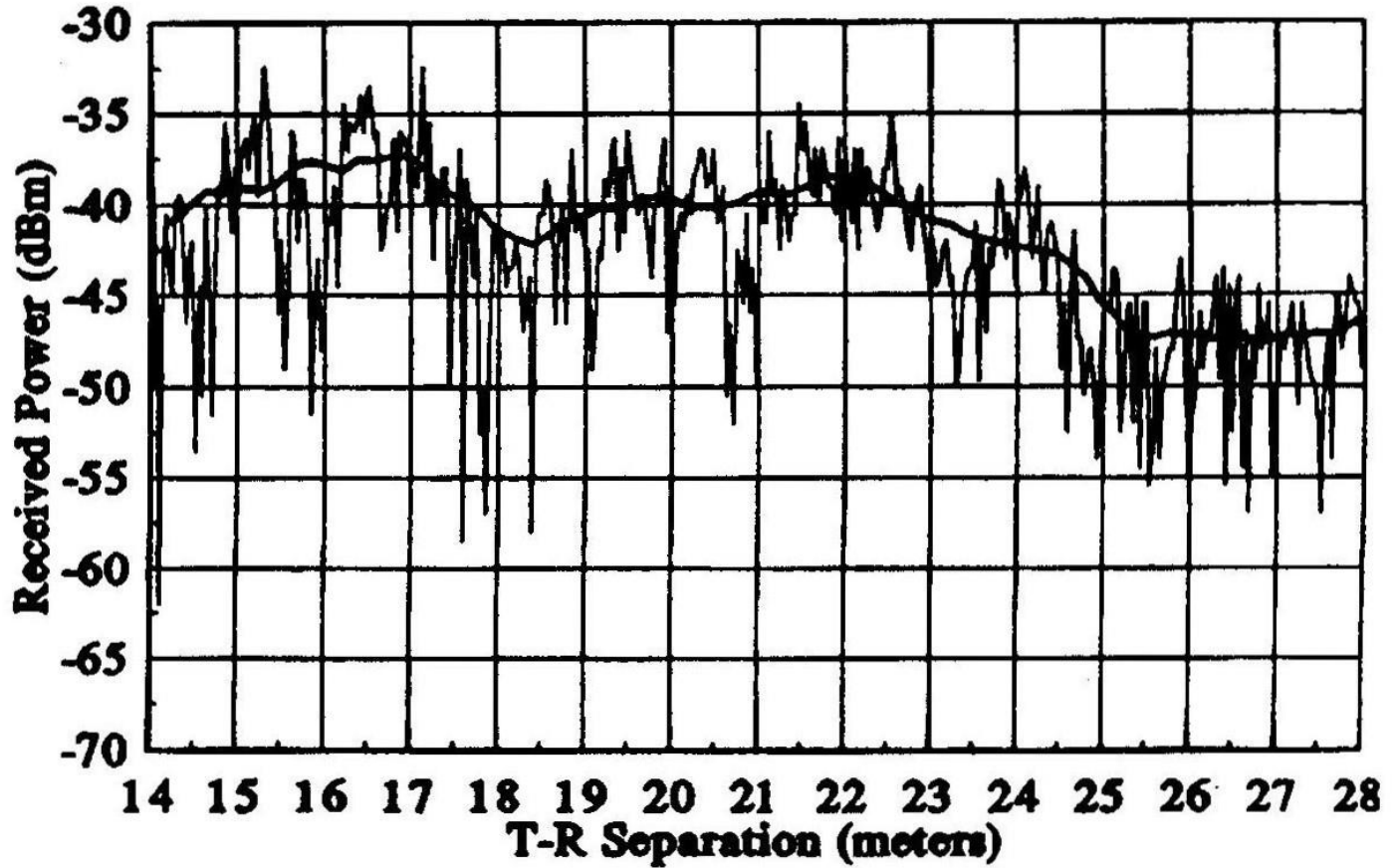


Figure 4.1 Small-scale and large-scale fading.

- The smoothed line is the average signal strength. The actual is the more jagged line.
- Actual received signal strength can vary by more than 20 dB over a few centimeters.
- The average signal strength decays with distance from the transmitter, and depends on terrain and obstructions.

- Two basic goals of propagation modeling:
  - 1) Predict magnitude and rate (speed) of received signal strength fluctuations over short distances/time durations
    - “short” → typically a few wavelengths ( $\lambda$ ) or seconds
    - at 1 GHz,  $\lambda = c/f = 3 \times 10^8 / 1 \times 10^9 = 0.3$  meters
    - received signal strength can vary drastically by 30 to 40 dB



- **small-scale** fluctuations → called \_\_\_\_\_ (Chapter 5)
- caused by received signal coming from a sum of many signals coming together at a receiver
- multiple signals come from reflections and scattering
- these signals can destructively add together by being out-of-phase

## 2) Predict average received signal strength for given Tx/Rx separation

- characterize received signal strength over distances from 20 m to 20 km
- *Large-scale radio wave propagation model* models
- needed to estimate coverage area of base station
- in general, large scale path loss decays *gradually* with distance from the transmitter
- will also be affected by geographical features like hills and buildings

- Free-Space Signal Propagation

- clear, unobstructed line-of-sight path → satellite and fixed microwave
- Friis transmission formula → Rx power ( $P_r$ ) vs. T-R separation ( $d$ )

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

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where

- $P_t$  = Tx power (W)
- $G$  = Tx or Rx antenna gain (unitless)
  - relative to isotropic source (ideal antenna which radiates power uniformly in all directions)
  - in the \_\_\_\_\_ of an antenna (beyond a few meters)
  - *Effective Isotropic Radiated Power (EIRP)*

$$EIRP = P_t G_t$$

- Represents the max. radiated power available from a Tx in the direction of max. antenna gain, as compare to an isotropic radiator

$$\lambda^2/4\pi$$

- $\lambda = \text{wavelength} = c / f$  (m). A  $\lambda^2/4\pi$  term is related to antenna gain.
- So, as frequency increases, what happens to the propagation characteristics?
- $L = \text{system losses}$  (antennas, transmission lines between equipment and antennas, atmosphere, etc.)
  - unitless
  - $L = 1$  for zero loss
  - $L > 1$  in general

- $d$  = T-R separation distance (m)
  - Signal fades in proportion to  $d^2$
  - We can view signal strength as related to the density of the signal across a large sphere.
  - This is the surface area of a sphere with radius  $d$ .
  - So, a term in the denominator is related to distance and density of surface area across a sphere.

- $\Rightarrow$  Path Loss ( $PL$ ) in dB:

$$\begin{aligned} PL_{dB} &= 10 \log\left(\frac{P_t}{P_r}\right) = -10 \log\left(\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2 L}\right) \\ &= -10 \log\left(\frac{G_t G_r \lambda^2}{(4\pi)^2 L}\right) + 10 \log(d^2) \\ &= -10 \log\left(\frac{G_t G_r \lambda^2}{(4\pi)^2 L}\right) + 20 \log(d) \end{aligned}$$

- $d^2 \rightarrow$  power law relationship
  - $P_r$  decreases at rate of proportional to  $d^2$
  - $P_r$  decreases at rate of 20 dB/decade (for line-of-sight, even worse for other cases)
  - For example, path loses 20 dB from 100 m to 1 km
  - Comes from the  $d^2$  relationship for surface area.
- Note: Negative “loss” = “gain”



- Example:
  - Path loss can be computed in terms of a link budget calculation.
  - Compute path loss as a sum of dB terms for the following:
    - Unity gain transmission antenna.
    - Unity gain receiving antenna.
    - No system losses
    - Carrier frequency of 3 GHz
    - Distance = 2000 meters
    -

$$\log\left((4\pi)^2\right) = 2.20$$

- Close in reference point ( $d_o$ ) is used in large-scale models

$$P_r(d) = P_r(d_o) \left( \frac{d_o}{d} \right)^2 \quad \text{for } d > d_o > d_f$$

- $d_o$ : known received power reference point - typically 100 m or 1 km for outdoor systems and 1 m for indoor systems
- $d_f$ : far-field distance of antenna, we will always work problems in the far-field

$$d_f = \frac{2D^2}{\lambda} \quad d_f \gg D \quad d_f \gg \lambda$$

- D: the largest physical linear dimension of antenna

- Reference Point Example:
  - Given the following system characteristics for large-scale propagation, find the reference distance  $d_0$ .
    - Received power at  $d_0 = 20 \text{ W}$
    - Received power at 5 km = 13 dBm
  - Using Watts:
  - Using dBm:

# III. Reflections

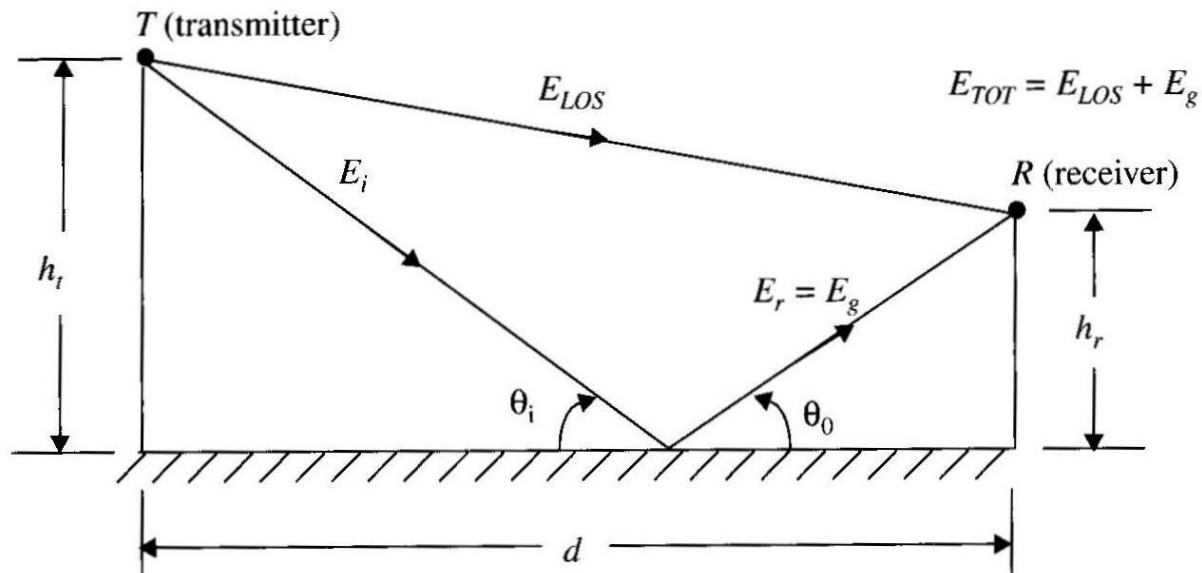
- There are three basic propagation mechanisms in addition to line-of-sight paths
  - Reflection - Waves bouncing off of objects of large dimensions
  - Diffraction - Waves bending around sharp edges of objects
  - Scattering - Waves traveling through a medium with small objects in it (foliage, street signs, lamp posts, etc.) or reflecting off rough surfaces

- Reflection occurs when RF energy is incident upon a boundary between two materials (e.g air/ground) with different electrical characteristics
  - Permittivity  $\mu$
  - Permeability  $\epsilon$
  - Conductance  $\sigma$
- Reflecting surface must be large relative to  $\lambda$  of RF energy
- Reflecting surface must be smooth relative to  $\lambda$  of RF energy
  - “specular” reflection

- What are important reflecting surfaces for mobile radio?
- Fresnel reflection coefficient  $\rightarrow \Gamma$ 
  - describes the magnitude of reflected RF energy
  - depends upon material properties, polarization, & angle of incidence

## IV. Ground Reflection (2-Ray) Model

- Good for systems that use tall towers (over 50 m tall)
- Good for line-of-sight microcell systems in urban environments



**Figure 4.7** Two-ray ground reflection model.

- ETOT is the electric field that results from a **combination** of a direct line-of-sight  $\vec{E}_{TOT} = \vec{E}_{LOS} + \vec{E}_g$

let  $E_0$  be  $|\vec{E}|$  at reference point  $d_0$  then

$$\vec{E}(d, t) = \left( \frac{E_0 d_0}{d} \right) \cos \left( \omega_c \left( t - \frac{d}{c} \right) \right) \quad d > d_0$$

- $\frac{E_0 d_0}{d}$  the amplitude of the electric field at distance  $d$
- $\omega_c = 2\pi f_c$  where  $f_c$  is the carrier frequency of the signal
- Notice at different distances  $d$  the wave is at a different phase because of the form similar to  $\vec{E}(d, t) = A \cos(\omega_c t + \theta)$



- For the direct path let  $d = d'$  ; for the reflected path  $d = d''$  then

$$\vec{E}_{TOT}(d, t) = \left( \frac{E_0 d_0}{d'} \right) \cos \left( \omega_c \left( t - \frac{d'}{c} \right) \right) + \Gamma \left( \frac{E_0 d_0}{d''} \right) \cos \left( \omega_c \left( t - \frac{d''}{c} \right) \right)$$

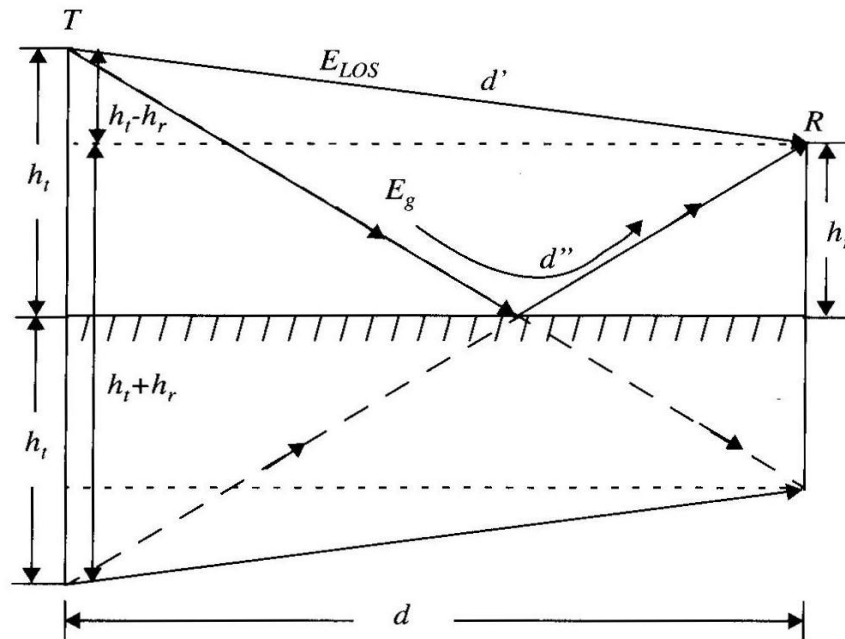
- for large T-R separation :  $\theta_i$  goes to 0 (angle of incidence to the ground of the reflected wave) and

$$\Gamma = -1$$

- **Phase difference** can occur depending on the phase difference between direct and reflected E fields
- The **phase difference** is  $\theta_\Delta$  due to **Path difference** ,  $\Delta = d'' - d'$ , between

$$\vec{E}_{LOS} \text{ and } \vec{E}_g$$

$$\text{Equation (4.40): } \Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$



**Figure 4.8** The method of images is used to find the path difference between the line-of-sight and the ground reflected paths.

- $\Delta$  can be expanded using a Taylor series expansion

$$\begin{aligned} f(x) = \sqrt{1+x} &= f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \dots \\ &= 1 + x \frac{1}{2} (1+0)^{-1/2} + \frac{x^2}{2!} \left( -\frac{1}{4} \right) (1+0)^{-3/2} + \dots \\ &= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots \end{aligned}$$

if  $x$  is small  $f(x) \approx 1 + \frac{1}{2}x$

$$\begin{aligned}
\Delta = d'' - d' &= \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2} \\
&= d \sqrt{\left(\left(\frac{h_t + h_r}{d}\right)^2 + 1\right)} - d \sqrt{\left(\left(\frac{h_t - h_r}{d}\right)^2 + 1\right)} \\
&\approx d \left(1 + \frac{1}{2} \left(\frac{h_t + h_r}{d}\right)^2\right) - d \left(1 + \frac{1}{2} \left(\frac{h_t - h_r}{d}\right)^2\right) \\
&\approx \frac{1}{2d} \left( (h_t + h_r)^2 - (h_t - h_r)^2 \right) \\
&\approx \frac{1}{2d} \left( (h_t^2 + 2h_t h_r + h_r^2) - (h_t^2 - 2h_t h_r + h_r^2) \right) \\
&\approx \frac{2h_t h_r}{d}
\end{aligned}$$

• WHICH WORKS WELL FOR  $d \gg (h_t + h_r)$ , WHICH MEANS

and  $\frac{h_t - h_r}{d}$  are small

$$\left(\frac{h_t + h_r}{d}\right)^2, \quad \left(\frac{h_t - h_r}{d}\right)^2$$

- the phase difference between the two arriving signals is

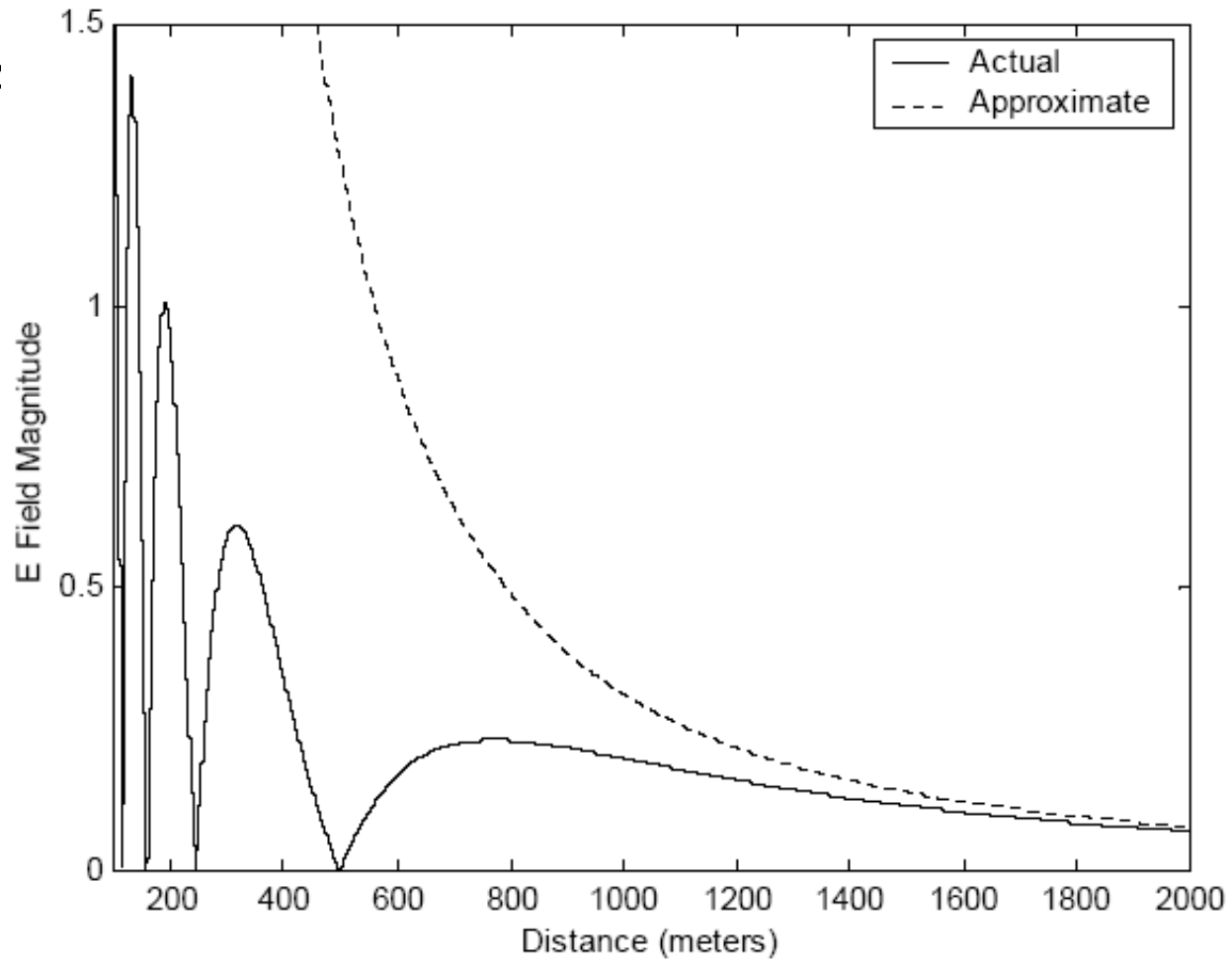
$$\theta_{\Delta} \text{ radians} = \left( \frac{\Delta}{\lambda} \text{ wavelengths} \right) \left( \frac{2\pi \text{ radians}}{\text{wavelength}} \right) = \frac{2\pi\Delta}{\lambda} = \frac{2\pi\Delta}{\left( \frac{c}{f_c} \right)} = \frac{\omega_c \Delta}{c}$$

$$|E_{TOT}(t)| = 2 \frac{E_0 d_0}{d} \sin \left( \frac{\theta_{\Delta}}{2} \right)$$

$$\frac{\theta_{\Delta}}{2} \approx \frac{2\pi h_r h_t}{\lambda d} < 0.3 \text{ rad}$$

$$E_{TOT}(t) \approx 2 \frac{E_0 d_0}{d} \frac{2\pi h_r h_t}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

• For  $\epsilon$



at  $t=0$

- note that the magnitude is with respect to a reference of  $E_0=1$  at  $d_0=100$  meters, so near 100 meters the signal can be stronger than  $E_0=1$ 
  - the second ray adds in energy that would have been lost otherwise
- for large distances it can be shown that

$$d \gg \sqrt{h_t h_r}$$

$$\left| \bar{E}_{TOT}(d) \right| \approx \frac{4\pi E_0 d_0 h_t h_r}{\lambda d^2}$$

$$P_r \approx \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}$$

Two-ray path loss model:

$$PL \text{ (dB)} = 40 \log d - [10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r]$$

Now  $d^4$  instead of  $d^2$  for free space

- $P_r \propto \frac{1}{d^4} P_t$

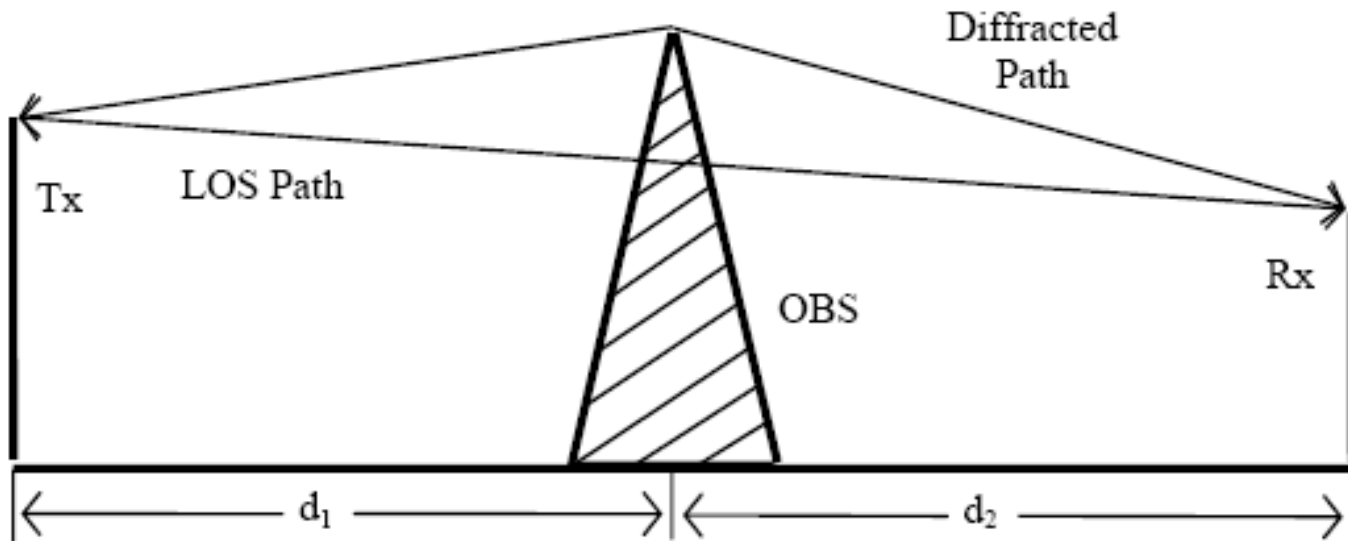
- $P_r(d) = P_r(d_0) \left( \frac{d_0}{d} \right)^4$



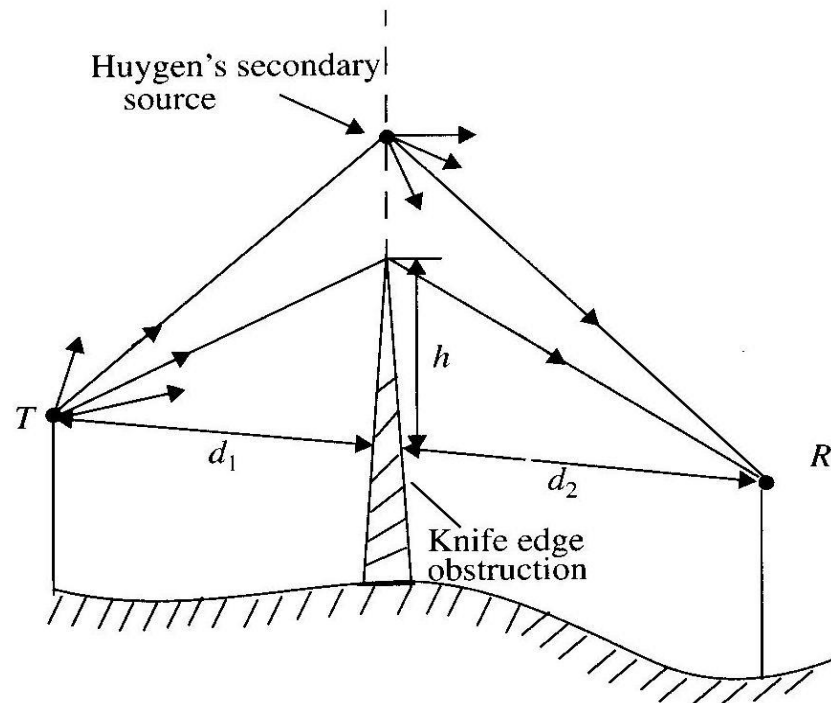
# V. Diffraction

- RF energy can propagate:
  - around the curved surface of the Earth
  - beyond the line-of-sight horizon
  - Behind obstructions
- Although EM field strength decays rapidly as Rx moves deeper into “shadowed” or obstructed (OBS) region
- The diffraction field often has sufficient strength to produce a useful signal

- Huygen's principle says points on a wavefront can be considered sources for additional wavelets.



- The wavefront on top of an obstruction generates secondary (weaker) waves.



**Figure 4.13** Illustration of knife-edge diffraction geometry. The receiver  $R$  is located in the shadow region.

- The difference between the direct path and diffracted path, call *excess path length*

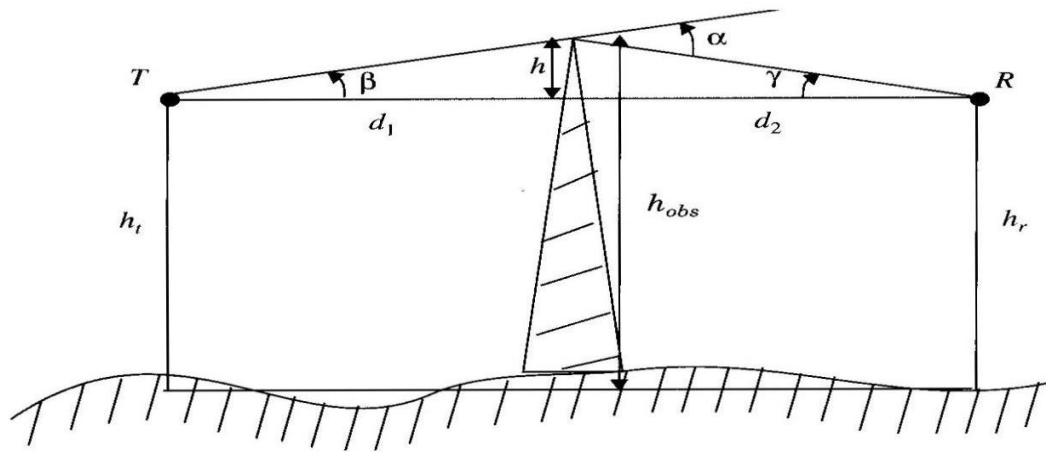
$$\Delta \approx \frac{h^2 (d_1 + d_2)}{2 d_1 d_2}$$

- *Fresnel-Kirchoff* diffraction parameter

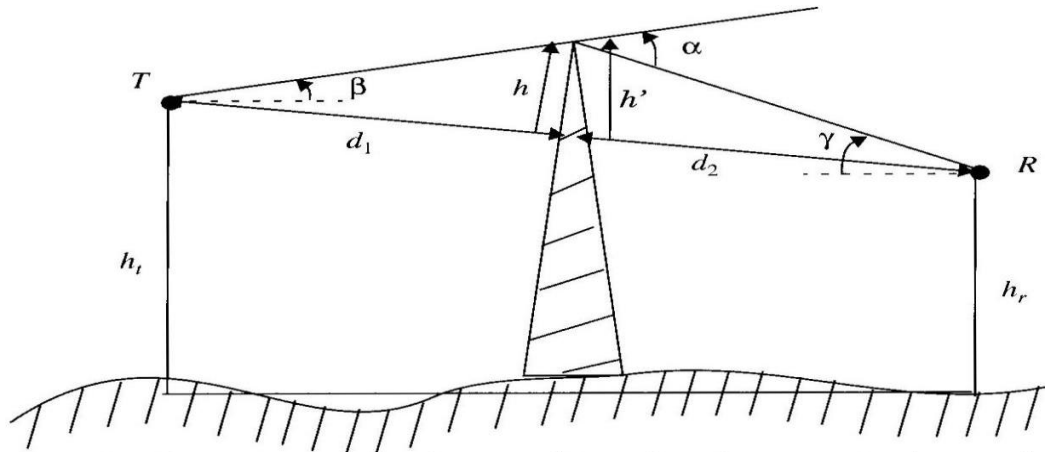
$$v = h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \alpha \sqrt{\frac{2d_1 d_2}{\lambda (d_1 + d_2)}}$$

- The corres,

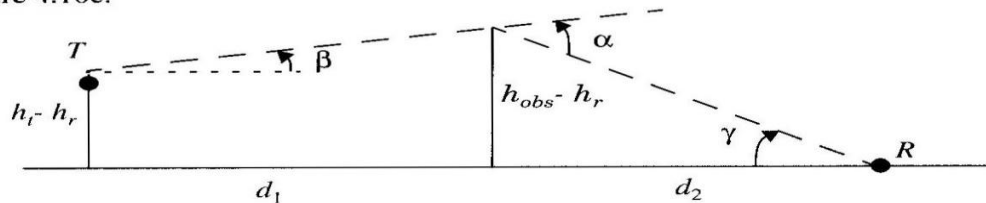
$$\phi = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{\lambda} \frac{h^2}{2} \frac{(d_1 + d_2)}{d_1 d_2} \qquad \phi = \frac{\pi}{2} v^2$$



(a) Knife-edge diffraction geometry. The point  $T$  denotes the transmitter and  $R$  denotes the receiver, with an infinite knife-edge obstruction blocking the line-of-sight path.



(b) Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if  $\alpha$  and  $\beta$  are small and  $h \ll d_1$  and  $d_2$ , then  $h$  and  $h'$  are virtually identical and the geometry may be redrawn as shown in Figure 4.10c.

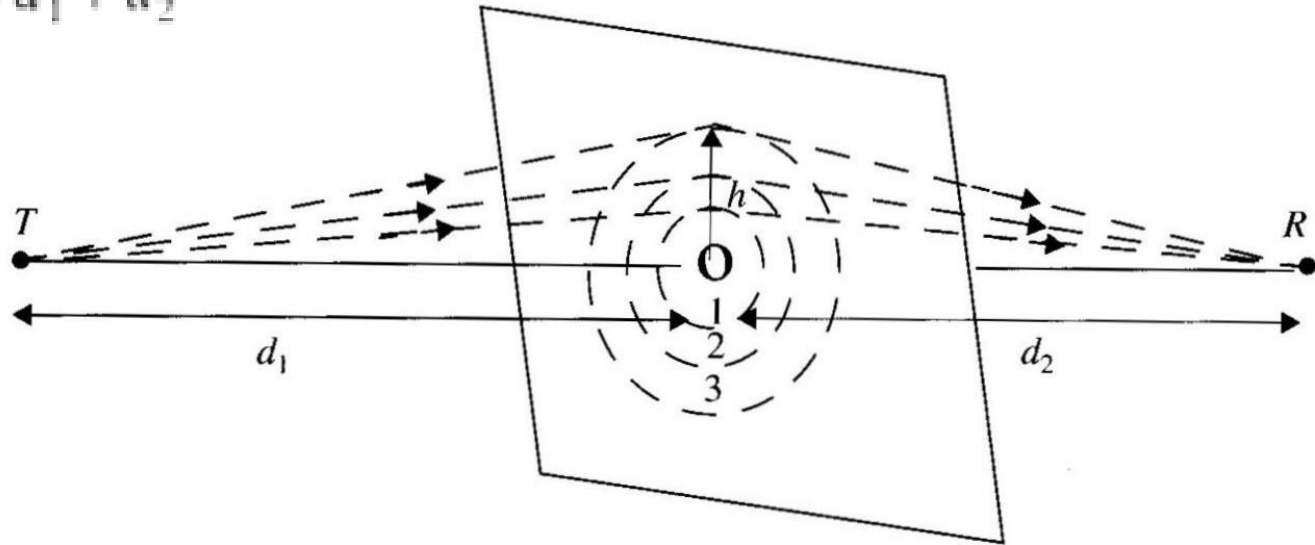


(c) Equivalent knife-edge geometry where the smallest height (in this case  $h_r$ ) is subtracted from all other heights.

**Figure 4.10** Diagrams of knife-edge geometry.

- The excess total path length traversed by a ray passing through each circle is  $n\lambda/2$

$$r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$

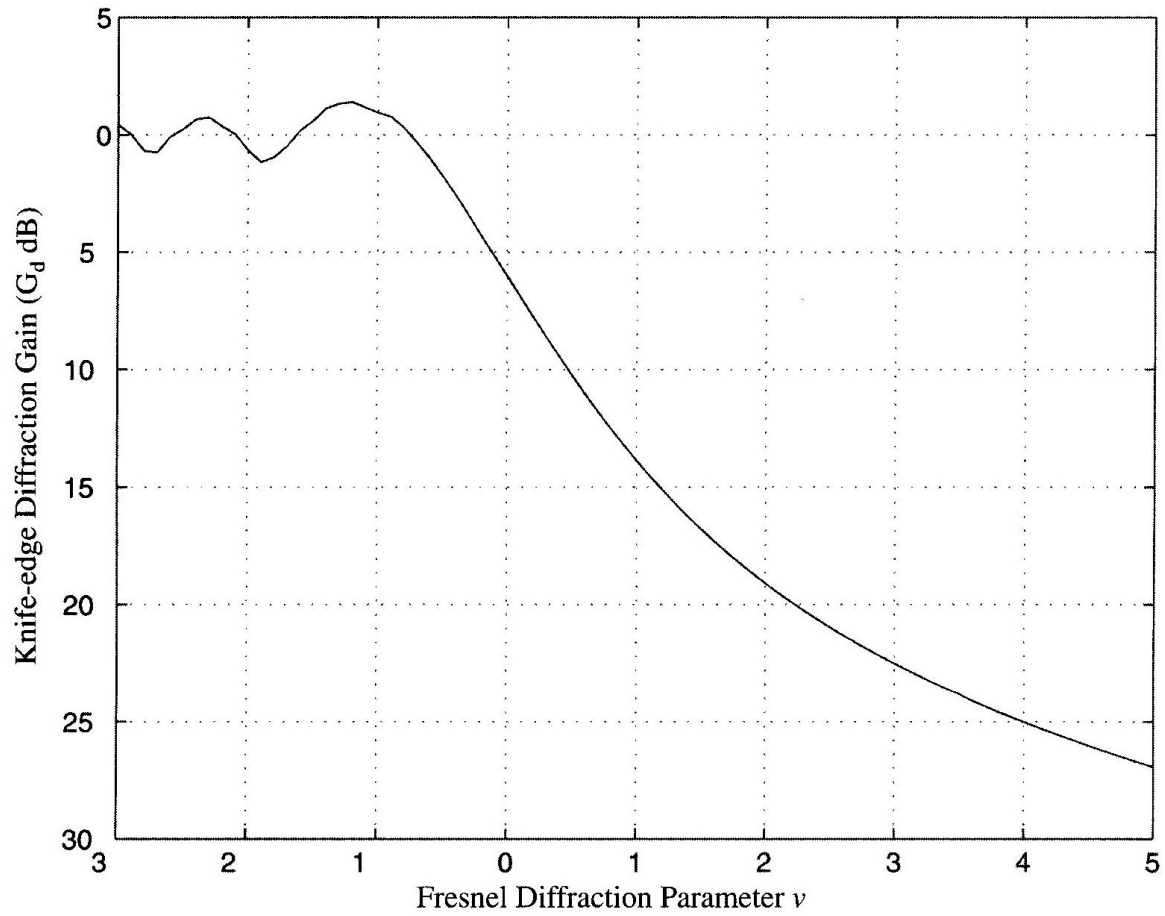


**Figure 4.11** Concentric circles which define the boundaries of successive Fresnel zones.

- The diffraction gain due to the presence of a knife edge, as compared to the free space E-field

$$G_d(\text{dB}) = 20\log|F(v)|$$

$$\frac{E_d}{E_0} = F(v) = \frac{(1+j)}{2} \int_v^{\infty} \exp((-j\pi t^2)/2) dt$$



**Figure 4.14** Knife-edge diffraction gain as a function of Fresnel diffraction parameter  $\nu$ .



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**Example 4.7**

Compute the diffraction loss for the three cases shown in Figure 4.12. Assume  $\lambda = 1/3$  m,  $d_1 = 1$  km,  $d_2 = 1$  km, and (a)  $h = 25$  m, (b)  $h = 0$ , (c)  $h = -25$  m. Compare your answers using values from Figure 4.14, as well as the approximate solution given by Equation (4.61.a)–(4.61.e). For each of these cases, identify the Fresnel zone within which the tip of the obstruction lies.

Given:

$$\lambda = 1/3 \text{ m}$$

$$d_1 = 1 \text{ km}$$

$$d_2 = 1 \text{ km}$$

(a)  $h = 25 \text{ m}$

Using Equation (4.56), the Fresnel diffraction parameter is obtained as

$$v = h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = 25 \sqrt{\frac{2(1000 + 1000)}{(1/3) \times 1000 \times 1000}} = 2.74.$$

From Figure 4.14, the diffraction loss is obtained as 22 dB.

Using the numerical approximation in Equation (4.61.e), the diffraction loss is equal to 21.7 dB.

The path length difference between the direct and diffracted rays is given by Equation (4.54) as

$$\Delta = \frac{h^2(d_1 + d_2)}{2 d_1 d_2} = \frac{25^2(1000 + 1000)}{2 \times 1000 \times 1000} = 0.625 \text{ m.}$$

To find the Fresnel zone in which the tip of the obstruction lies, we need to compute  $n$  which satisfies the relation  $\Delta = n\lambda/2$ . For  $\lambda = 1/3 \text{ m}$ , and  $\Delta = 0.625 \text{ m}$ , we obtain

$$n = \frac{2\Delta}{\lambda} = \frac{2 \times 0.625}{0.3333} = 3.75.$$

Therefore, the tip of the obstruction completely blocks the first three Fresnel zones.

(b)  $h = 0$  m

Therefore, the Fresnel diffraction parameter  $v = 0$ .

From Figure 4.14, the diffraction loss is obtained as 6 dB.

Using the numerical approximation in Equation (4.61.b), the diffraction loss is equal to 6 dB.

For this case, since  $h = 0$ , we have  $\Delta = 0$ , and the tip of the obstruction lies in the middle of the first Fresnel zone.

(c)  $h = -25$  m

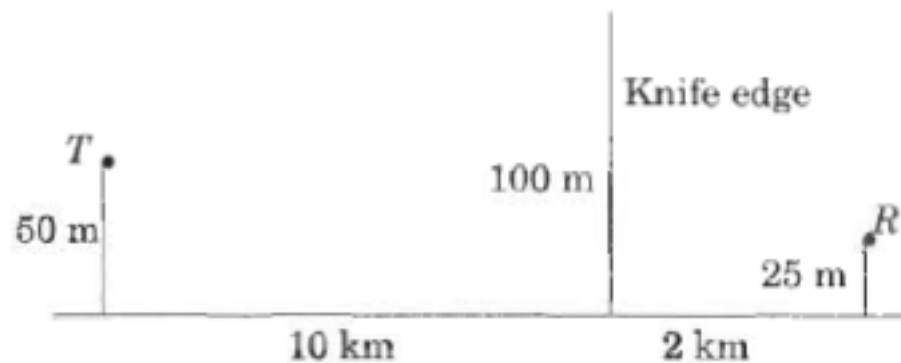
Using Equation (4.56), the Fresnel diffraction parameter is obtained as  $-2.74$ .

From Figure 4.14, the diffraction loss is approximately equal to 1 dB.

Using the numerical approximation in Equation (4.61.a), the diffraction loss is equal to 0 dB.

### Example 4.8

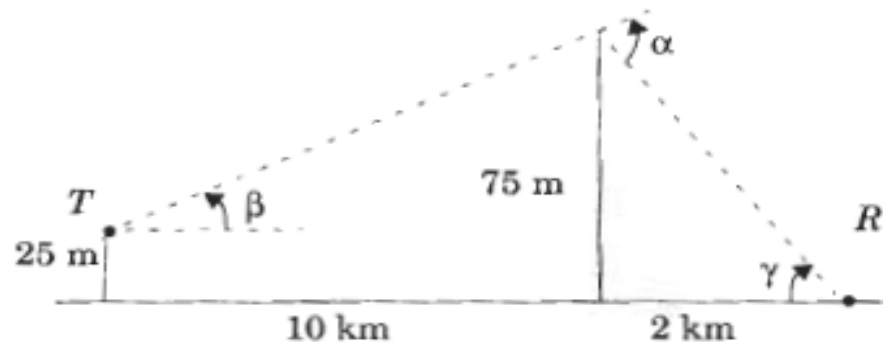
Given the following geometry, determine (a) the loss due to knife-edge diffraction, and (b) the height of the obstacle required to induce 6 dB diffraction loss. Assume  $f = 900$  MHz.



### Solution

(a) The wavelength  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3}$  m.

Redraw the geometry by subtracting the height of the smallest structure.



$$\beta = \tan^{-1}\left(\frac{75 - 25}{10000}\right) = 0.2865^\circ$$

$$\gamma = \tan^{-1}\left(\frac{75}{2000}\right) = 2.15^\circ$$

and

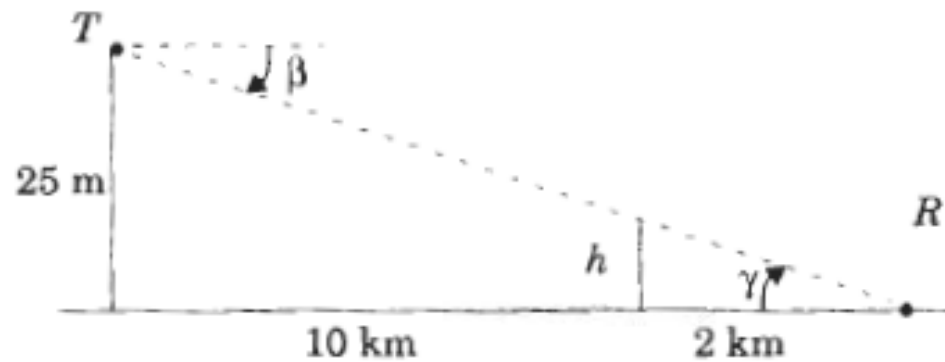
$$\alpha = \beta + \gamma = 2.434^\circ = 0.0424 \text{ rad}$$

Then using Equation (4.56)

$$v = 0.0424 \sqrt{\frac{2 \times 10000 \times 2000}{(1/3) \times (10000 + 2000)}} = 4.24.$$

From Figure 4.14 or (4.61.e), the diffraction loss is 25.5 dB.

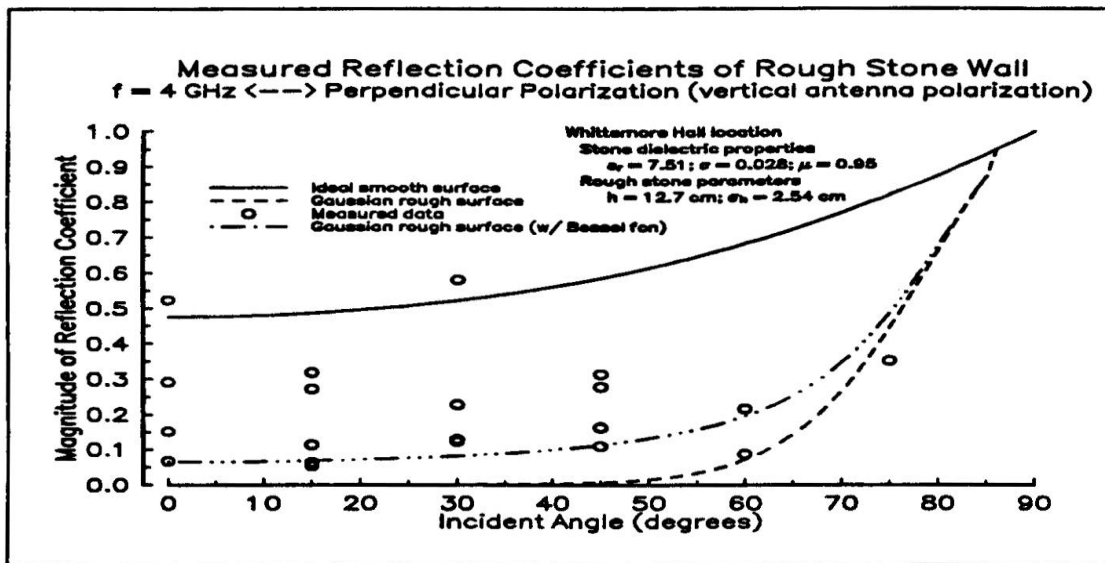
- (b) For 6 dB diffraction loss,  $v = 0$ . The obstruction height  $h$  may be found using similar triangles ( $\beta = \gamma$ ), as shown below.



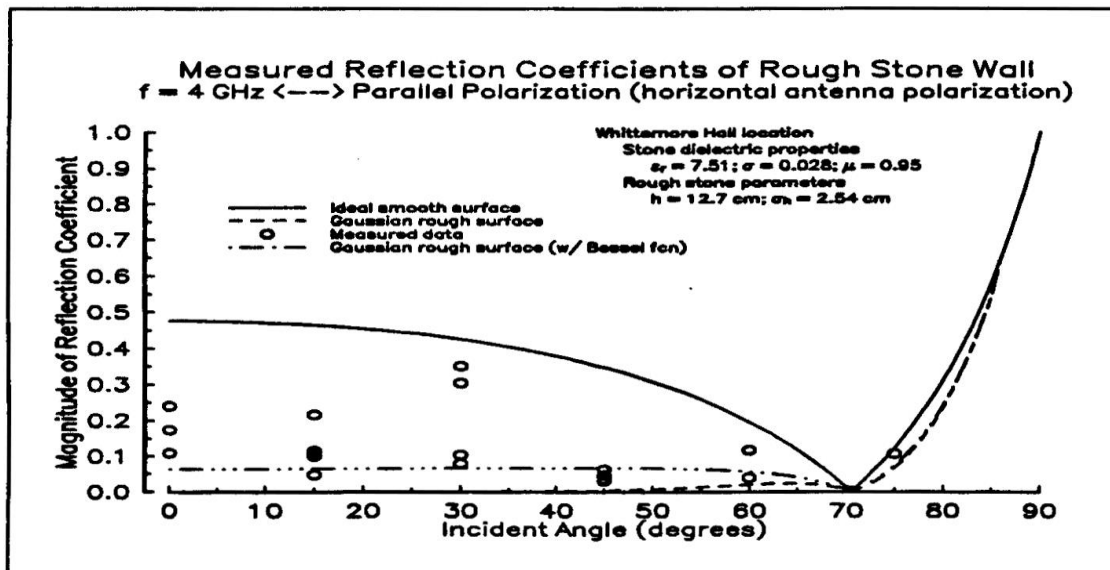
It follows that  $\frac{h}{2000} = \frac{25}{12000}$ , thus  $h = 4.16$  m.

## VI. Scattering

- Received signal strength is often stronger than that predicted by reflection/diffraction models alone
- The EM wave incident upon a rough or complex surface is **scattered** in **many** directions and provides more energy at a receiver
  - energy that would have been absorbed is instead reflected to the Rx.
- Scattering is caused by trees, lamp posts, towers, etc.
- flat surface → EM reflection (one direction)
- rough surface → EM scattering (many directions)



(a) E-field in the plane of incidence (parallel polarization).



(b) E-field normal to plane of incidence (perpendicular polarization).

**Figure 4.16** Measured reflection coefficients versus incident angle at a rough stone wall site. In these graphs, incident angle is measured with respect to the normal, instead of with respect to the surface boundary as defined in Figure 4.4. These graphs agree with Figure 4.6 [Lan96].



## VII. Path Loss Models

- We wish to predict large scale coverage using analytical and empirical (field data) methods
- It has been repeatedly measured and found that  $P_r @ Rx$  decreases logarithmically with distance

$\therefore PL(d) = (d/d_o)^n$  where  $n$  : path loss exponent or

$$PL(\text{dB}) = PL(d_o) + 10 n \log(d/d_o)$$

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- “bar” means the **average** of many  $PL$  values at a given value of  $d$  (T-R sep.)

- $n$  depends on the propagation environment

- “† **Table 4.2** Path Loss Exponents for Different Environments

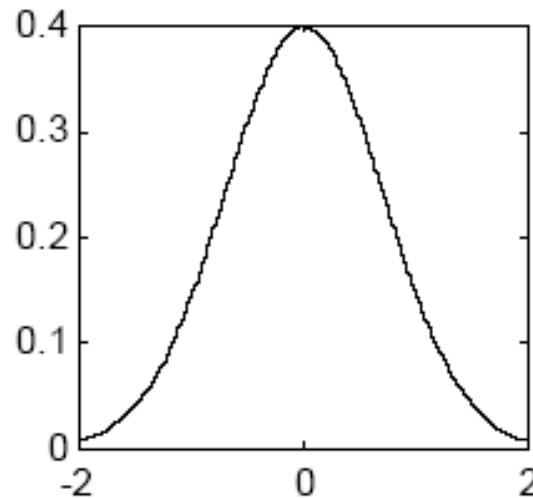
Environment	Path Loss Exponent, $n$
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

- At any specific  $d$  the measured values vary drastically because of variations in the surrounding environment (obstructed vs. line-of-sight, scattering, reflections, etc.)
- Some models can be used to describe a situation generally, but specific circumstances may need to be considered with detailed analysis and measurements.

- Log-Normal Shadowing

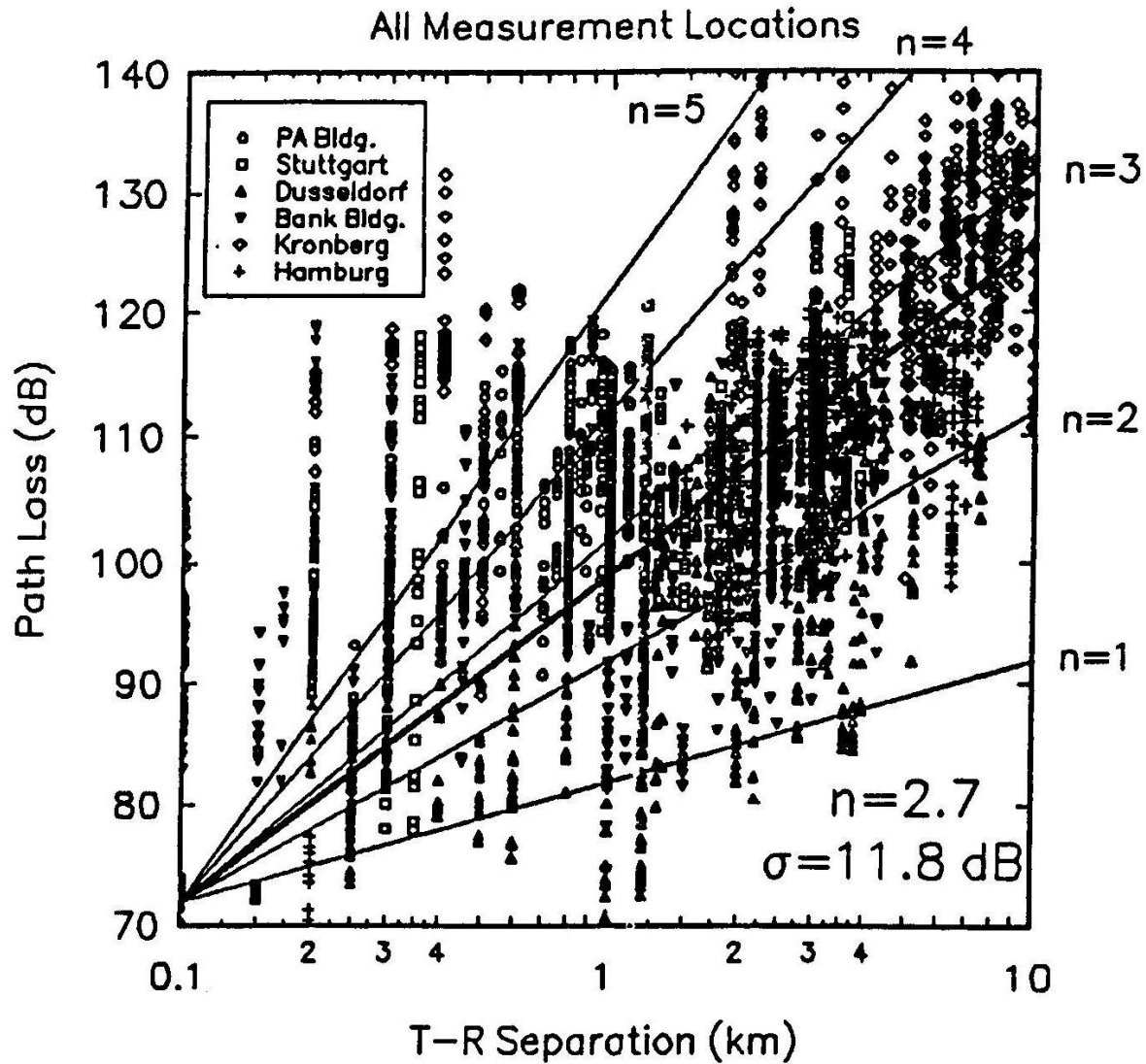
$$PL(d) = PL(d_o) + 10n \log(d/d_o) + X_\sigma$$

- describes how the path loss at any specific location may vary from the average value



- has a the large-scale path loss component we have already seen plus a random amount  $X_\sigma$ .

- $X_\sigma$  : zero mean Gaussian random variable, a “bell curve”
- $\sigma$  is the standard deviation that provides the second parameter for the distribution
- takes into account received signal strength variations due to shadowing
  - measurements verify this distribution
- $n$  &  $\sigma$  are computed from measured data for different area types
- any other path loss models are given in your book.
  - That correlate field measurements with models for different types of environments.



**Figure 4.17** Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germany. For this data,  $n = 2.7$  and  $\sigma = 11.8 \text{ dB}$  [from [Sei91] © IEEE].