

Introduction

Example of combined motion

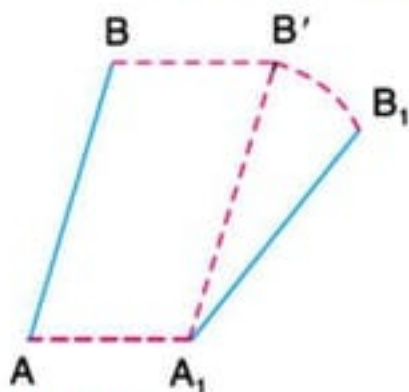


Fig. (a)

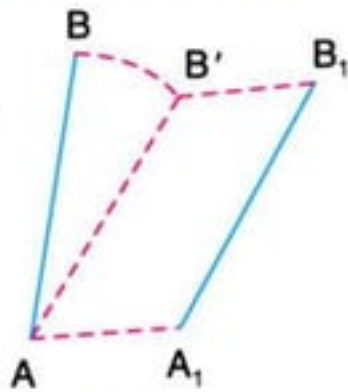


Fig. (b)

- Consider a rigid link AB, which moves from its initial position AB to A₁B₁ as shown in Fig. (a).
- A little consideration will show that the link neither has complete motion of translation nor complete rotational, but a combination of the two motions.

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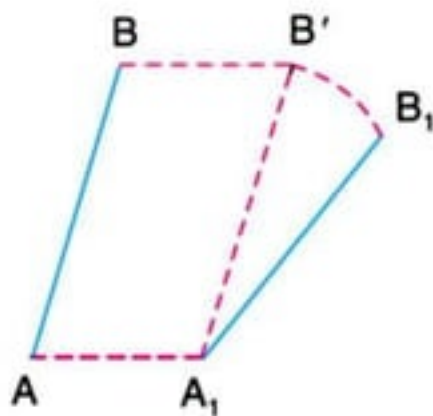
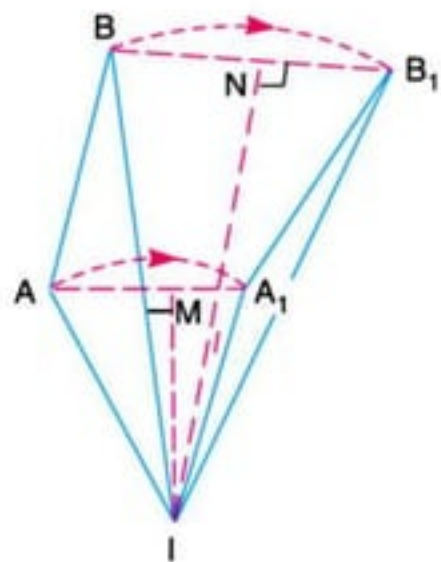


Fig. (a)

Combined motion of **rotation** and **translation** of the **link AB** may be assumed to be a motion of pure rotation about some centre **I**, known as the **instantaneous centre of rotation (centro or virtual centre)**.

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Location of position of instantaneous centre

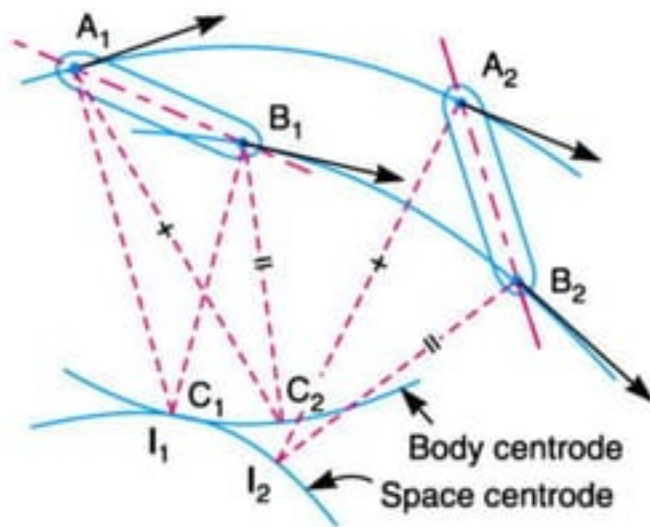


- The points A and B of the link has moved to A_1 and B_1 respectively under the motion of rotation (as assumed).
 - Therefore the position of the centre of rotation must lie on the intersection of the right bisectors of chords $A A_1$ and $B B_1$.
 - Let these bisectors intersect at I as shown in Fig. which is the instantaneous centre of rotation of the link AB .
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- As position of the link AB goes on changing, therefore the centre about which the motion is assumed to take place (i.e. the instantaneous centre of rotation) also goes on changing.

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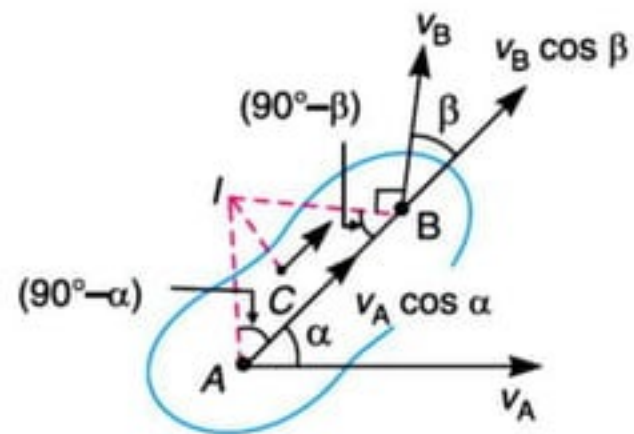
- Instantaneous centre of a moving body may be defined as **that centre which goes on changing from one instant to another.**
- The locus of all such instantaneous centres is known as **centrode.**
- A line drawn through an **instantaneous centre** and perpendicular to the plane of motion is called **instantaneous axis.**

Space and Body Centroides



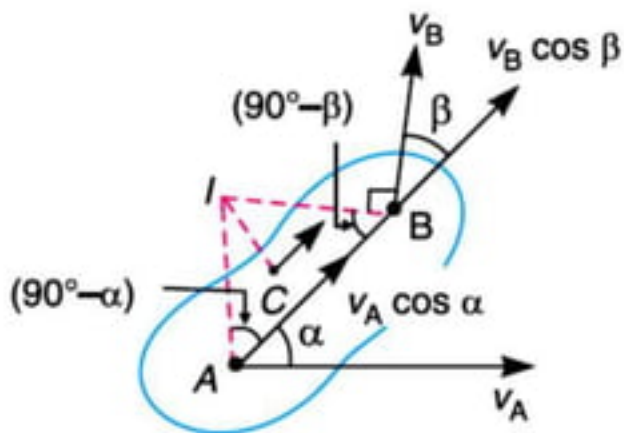
- ✓ The locus of the instantaneous centre in space during a definite motion of the body is called the **space centroide**.
- ✓ The locus of the instantaneous centre relative to the body itself is called the **body centroide**.

Methods for determining the Velocity of a Point on a Link



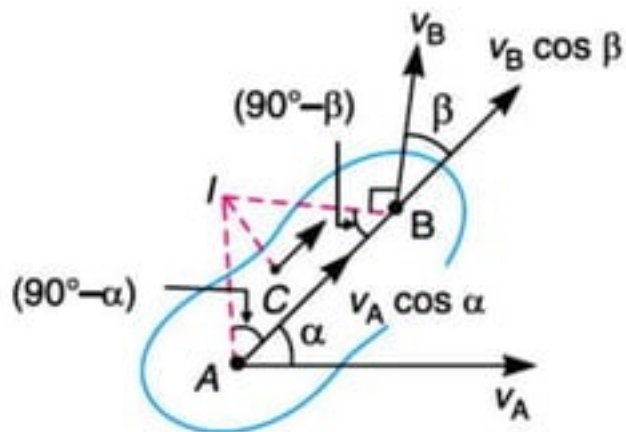
- Consider two points A and B on a rigid link.
- Let v_A and v_B be the velocities of points A and B , whose directions are given by angles α and β as shown in Fig.
- If v_A is known in magnitude and direction and v_B in direction only, then the magnitude of v_B may be determined by the instantaneous centre method

Methods for determining the Velocity of a Point on a Link



1. Draw **AI** and **BI** perpendiculars to the directions **v_A** and **v_B** respectively.
2. Let these lines intersect at **I**, which is known as instantaneous centre of the link.
3. The link is to **rotate or turn** about the centre **I**.
4. Since **A** and **B** are the points on a rigid link, therefore there cannot be any relative motion between them along the line **AB**.

Methods for determining the Velocity of a Point on a Link



Now resolving the velocities along AB ,

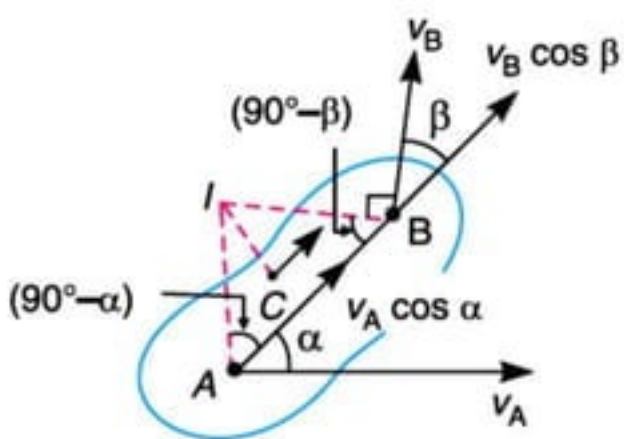
$$v_A \cos \alpha = v_B \cos \beta$$

$$\frac{v_A}{v_B} = \frac{\cos \beta}{\cos \alpha} = \frac{\sin(90^\circ - \beta)}{\sin(90^\circ - \alpha)} \quad (i)$$

Applying Lami's theorem to triangle ABI ,

$$\frac{AI}{\sin(90^\circ - \beta)} = \frac{BI}{\sin(90^\circ - \alpha)}$$

$$\frac{AI}{BI} = \frac{\sin(90^\circ - \beta)}{\sin(90^\circ - \alpha)} \quad (ii)$$



From equation (i) and (ii),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega$$

where

ω = Angular velocity of the rigid link.

If C is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI}$$

From the above equation,

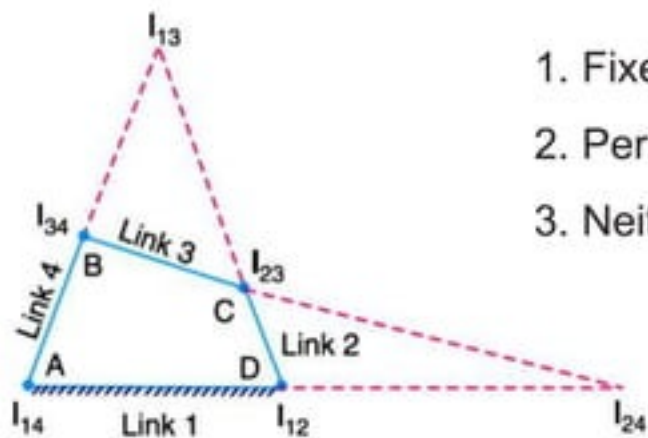
1. v_A is known in magnitude and direction and v_B in direction only, then **velocity of point B** or any other **point C** lying on the same link may be determined in magnitude and direction.
2. The magnitude of velocities of the points on a link is inversely proportional to the distances from the points to the instantaneous centre and is **perpendicular** to the line joining the point to the instantaneous centre.

Number of Instantaneous Centres in a Mechanism

- The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links.

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

Types of Instantaneous Centres



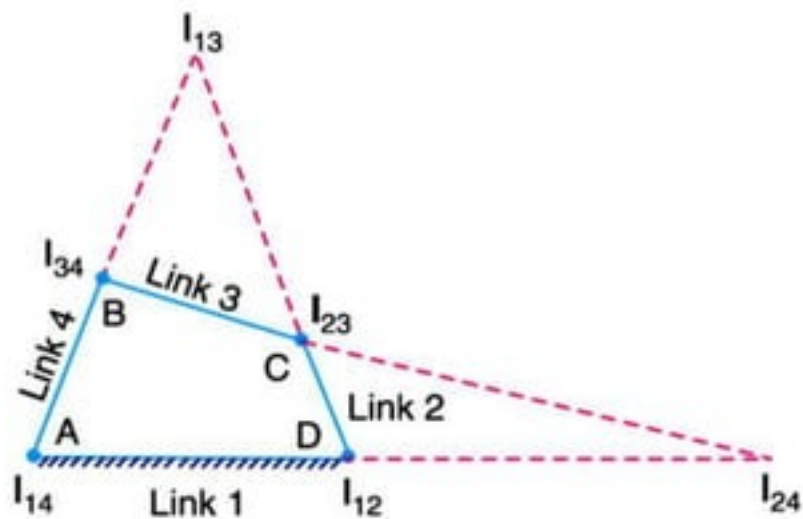
1. Fixed instantaneous centres.
2. Permanent instantaneous centres.
3. Neither fixed nor permanent instantaneous centres.

- The first two types i.e. fixed and permanent instantaneous centres are together known as **primary instantaneous centres**.
- The third type is known as **secondary instantaneous centres**.

Types of Instantaneous Centres

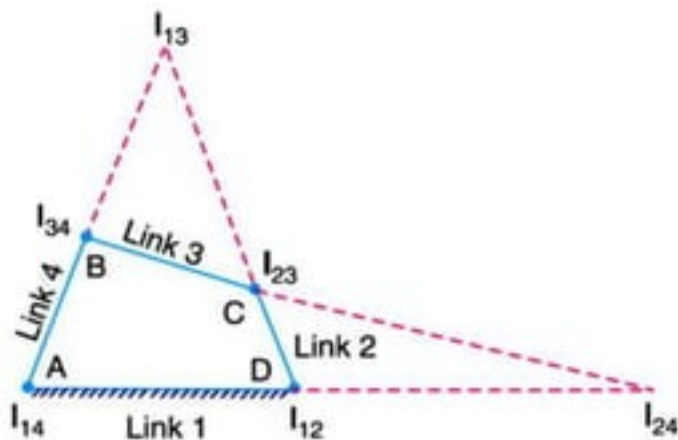
Consider a four bar mechanism as shown in fig.

- The number of instantaneous centres (N) in a four bar mechanism is given by,



$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

Types of Instantaneous Centres

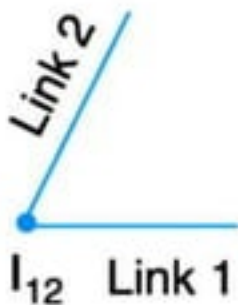


- The instantaneous centres I_{12} and I_{14} are called the **fixed instantaneous centres** as they remain in the same place for all configurations of the mechanism.
- The instantaneous centres I_{23} and I_{34} are the **permanent instantaneous centres** as they move when the mechanism moves, but the joints are of permanent nature.
- The instantaneous centres I_{13} and I_{34} are **neither fixed nor permanent instantaneous centres** as they vary with the configuration of the mechanism.

Location of Instantaneous Centres

The following rules may be used in locating the instantaneous centres in a mechanism :

1. When the two links are connected by a pin joint, the instantaneous centre lies on the centre of the pin as shown in Fig. Such a instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.

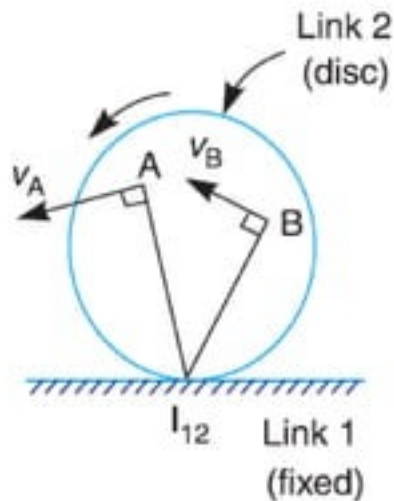


Location of Instantaneous Centres

2. When the two links have a pure rolling contact, the instantaneous centre lies on their point of contact, as shown in Fig.

- The velocity of any point A on the link 2 relative to fixed link 1 will be perpendicular to $I_{12}A$ and is proportional to $I_{12}A$.

$$\frac{v_A}{v_B} = \frac{I_{12}A}{I_{12}B}$$

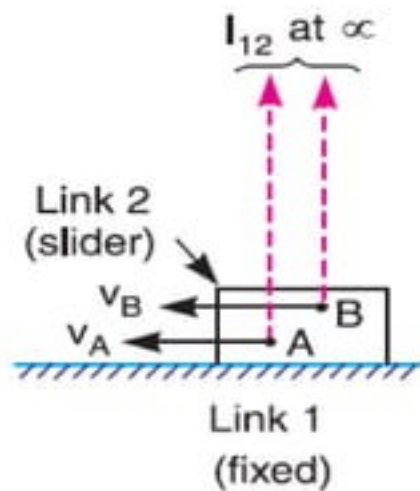


Location of Instantaneous Centres

3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.

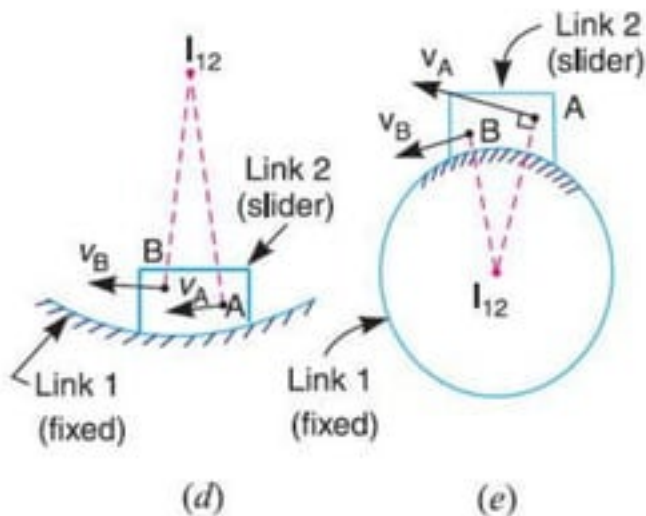
- Consider the following three cases :

(a) When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig.(c), the instantaneous centre lies at infinity and each point on the slider have the same velocity.



(b) When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig. (d), the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

(c) When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. (e), the instantaneous centre lies at the centre of curvature.

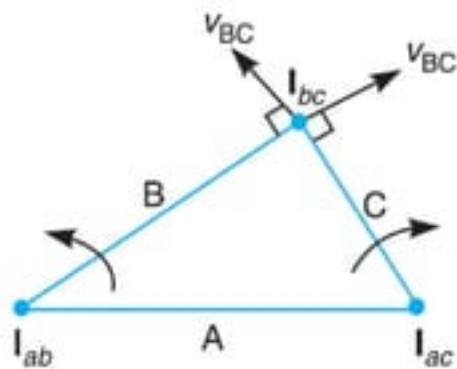


Aronhold Kennedy (or Three Centres in Line) Theorem

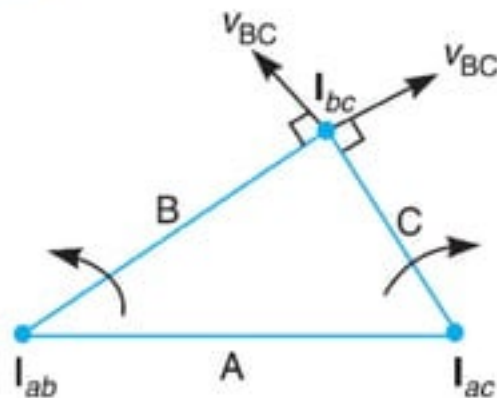
The Aronhold Kennedy's theorem states that "if three bodies move relative to each other, they have three instantaneous centres and lie on a straight line".

Consider three kinematic links A , B and C having relative plane motion. The number of instantaneous centres (N) is given by,

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

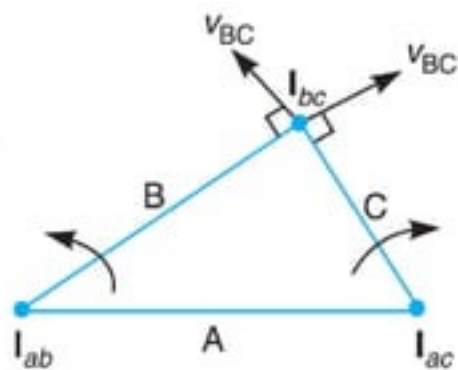


- The two instantaneous centres at the pin joints of B with A, and C with A (i.e. I_{ab} and I_{ac}) are the permanent instantaneous centres.
- According to **Aronhold Kennedy's theorem**, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac} .

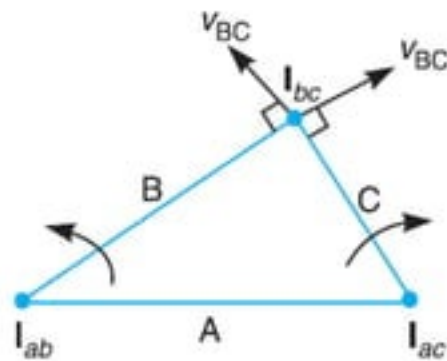


In order to prove this;

- Consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig.
- The point I_{bc} belongs to both the links B and C.
- Now consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc} .
- Now consider the point I_{bc} on the link C.
- Its velocity v_{bc} must be perpendicular to the line joining I_{ac} and I_{bc} .



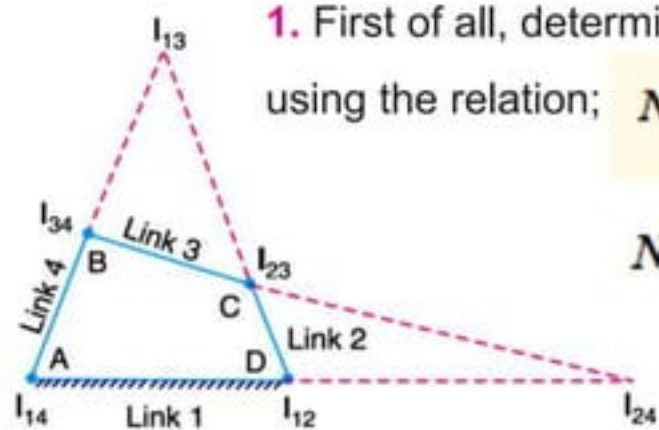
- WKT, the velocity of the instantaneous centre is same whether it is a point on the first link or a point on the second link.
- Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab}I_{bc}$ and $I_{ac}I_{bc}$, unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac} .
- Thus, the three instantaneous centres (I_{ab} , I_{ac} and I_{bc}) must lie on the same straight line.
- The exact location of I_{bc} on line $I_{ab}I_{bc}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A .



Method of Locating Instantaneous Centres in a Mechanism

1. First of all, determine the number of instantaneous centres (N) by using the relation; $N = \frac{n(n-1)}{2}$, where n = Number of links.

$$N = \frac{4(4-1)}{2} = 6$$



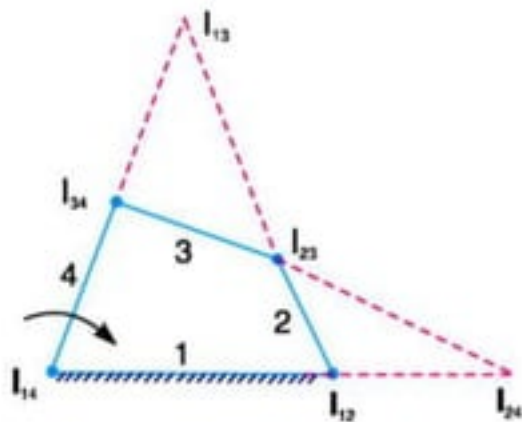
2. Make a list of all the instantaneous centres in a mechanism.

Book-keeping table.

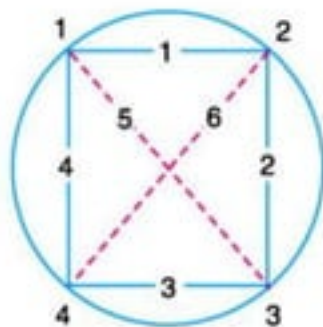
Links	1	2	3	4
Instantaneous centres	12	23	34	-
(6 in number)	13	24		
	14			

3. Locate the fixed and permanent instantaneous centres by inspection. In Fig. I_{12} & I_{14} are fixed instantaneous centres & I_{23} and I_{34} are permanent instantaneous centres.
4. Locate the neither fixed nor permanent instantaneous centres by Kennedy's theorem. This is done by **circle** diagram as shown in Fig. (b).
- Mark points on a circle equal to the number of links in a mechanism. In the present case, mark 1, 2, 3, and 4 on the circle.
5. Join the points by solid lines to show that these centres are already found.
- In the **circle diagram** [Fig. 6.8 (b)] these lines are 12, 23, 34 and 14 to indicate the centres I_{12} , I_{23} , I_{34} and I_{14} .

6. In order to find the other two instantaneous centres, join two such points that the line joining them forms two adjacent triangles in the circle diagram.
- The line for completing two triangles, should be a common side to the two triangles.

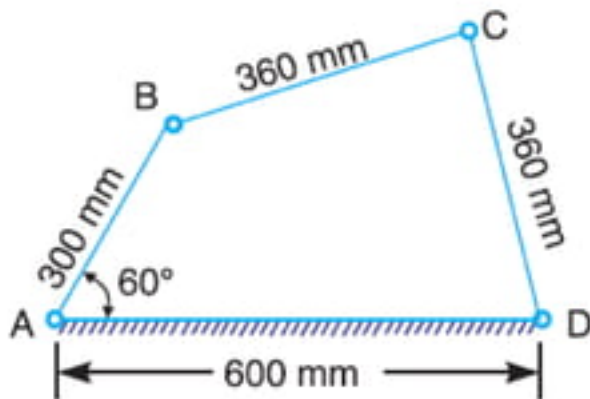


(a) Four bar mechanism.



(b) Circle diagram.

Example 1. In a pin jointed four bar mechanism, as shown in Fig. $AB = 300$ mm, $BC = CD = 360$ mm, and $AD = 600$ mm. The angle $BAD = 60^\circ$. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC



End of Module