

Inversion of Mechanism

We have already discussed that when one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as *inversion of the mechanism*.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

Note: The part of a mechanism which initially moves with respect to the frame or fixed link is called *driver* and that part of the mechanism to which motion is transmitted is called *follower*. Most of the mechanisms are reversible, so that same link can play the role of a driver and follower at different times. For example, in a reciprocating steam engine, the piston is the driver and flywheel is a follower while in a reciprocating air compressor, the flywheel is a driver.

Types of Kinematic Chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view :

1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

These kinematic chains are discussed, in detail, in the following articles.

Four Bar Chain or Quadric Cycle Chain

We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained.

The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. It consists of four links, each of them forms a turning pair at *A*, *B*, *C* and *D*. The four links may be of different lengths. According to **Grashof's law** for a four bar

mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links. A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the

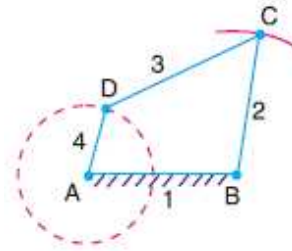


Fig: Four bar chain.

other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof 's law. Such a link is known as **crank** or **driver**. In Fig, AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known as **lever** or **rocker** or **follower** and the link CD (link 3) which connects the crank and lever is called **connecting rod** or **coupler**. The fixed link AB (link 1) is known as **frame** of the mechanism. When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

Inversions of Four Bar Chain

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view :

1. Beam engine (crank and lever mechanism).

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig. In this mechanism, when the crank rotates about the fixed centre A , the lever oscillates about a fixed centre D . The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

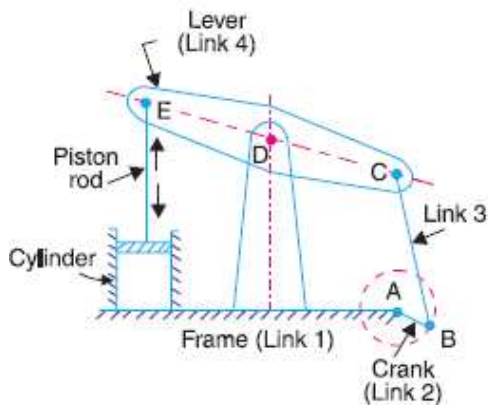


Fig. Beam engine.

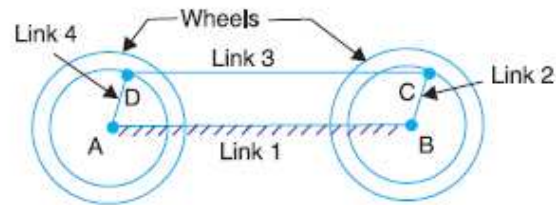


Fig. Coupling rod of a locomotive.

2. Coupling rod of a locomotive (Double crank mechanism). The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in Fig.

In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

3. Watt's indicator mechanism (Double lever mechanism). A *Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig. The four links are: fixed link at A , link AC , link CE and link BFD . It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers. The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

The initial position of the mechanism is shown in Fig by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

Notes-5

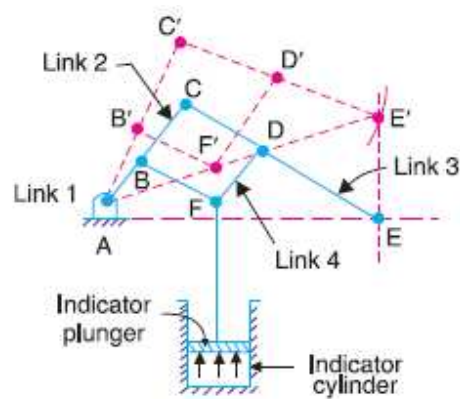


Fig: Watt's indicator mechanism.

Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism.

This type of mechanism converts rotary motion into reciprocating motion and vice versa.

In a single slider crank chain, as shown in Fig, the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.

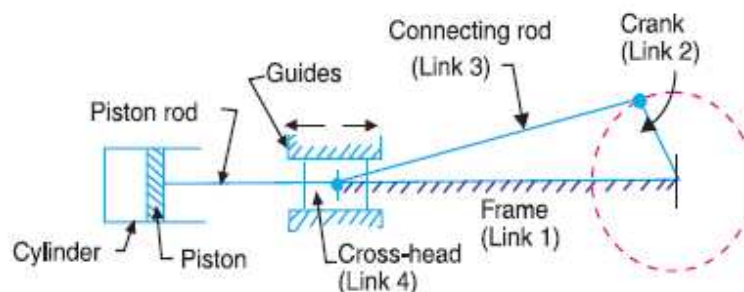


Fig: Single slider crank chain.

The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Application of Kutzbach Criterion to Plane Mechanisms

We have discussed in the previous article that Kutzbach criterion for determining the number of degrees of freedom or movability (n) of a plane mechanism is

$$n = 3(l - 1) - 2j - h$$

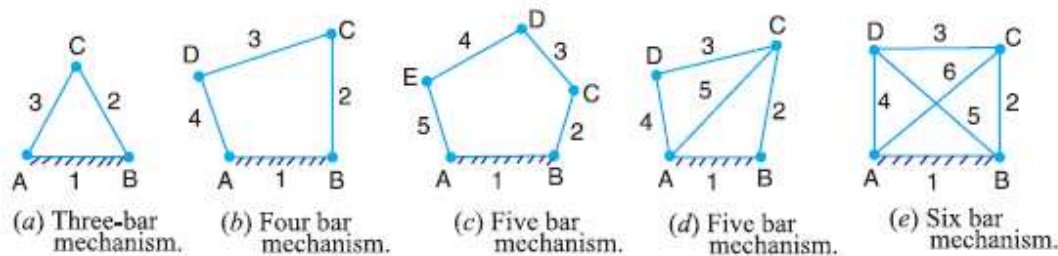


Fig: Plane mechanisms.

The number of degrees of freedom or movability (n) for some simple mechanisms having no higher pair (*i.e.* $h = 0$), as shown in Fig, are determined as follows :

1. The mechanism, as shown in Fig (a), has three links and three binary joints, *i.e.*

$$l = 3 \text{ and } j = 3.$$

$$\square n = 3(3 - 1) - 2 \times 3 = 0$$

2. The mechanism, as shown in Fig (b), has four links and four binary joints, *i.e.*

$$l = 4 \text{ and } j = 4.$$

$$\square n = 3(4 - 1) - 2 \times 4 = 1$$

3. The mechanism, as shown in Fig (c), has five links and five binary joints, *i.e.*

$$l = 5, \text{ and } j = 5.$$

$$\square n = 3(5 - 1) - 2 \times 5 = 2$$

4. The mechanism, as shown in Fig (d), has five links and six equivalent binary joints (because there are two binary joints at B and D , and two ternary joints at A and C), *i.e.*

$$l = 5 \text{ and } j = 6.$$

$$\square n = 3(5 - 1) - 2 \times 6 = 0$$

5. The mechanism, as shown in Fig (e), has six links and eight equivalent binary joints (because there are four ternary joints at A , B , C and D), *i.e.* $l = 6$ and $j = 8$.

- i. $\square n = 3(6 - 1) - 2 \times 8 = -1$

It may be noted that

Notes-5

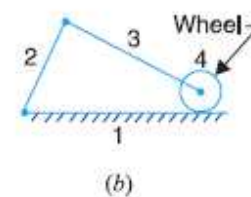
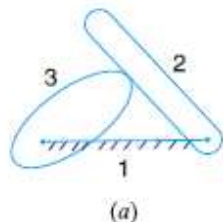
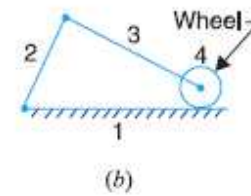
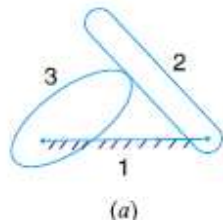
(a) When $n = 0$, then the mechanism forms a structure and no relative motion between the links is possible, as shown in Fig.(a) and (d).

(b) When $n = 1$, then the mechanism can be driven by a single input motion, as shown in Fig (b).

(c) When $n = 2$, then two separate input motions are necessary to produce constrained motion for the mechanism, as shown in Fig (c).

(d) When $n = -1$ or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure, as shown in Fig (e).

The application of Kutzbach's criterion applied to mechanisms with a higher pair or two degree of freedom joints is shown in Fig.



Inversions of Single Slider Crank Chain

We have seen in the previous article that a single slider crank chain is a four-link mechanism. We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinematic chain. It is thus obvious, that four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

1. Pendulum pump or Bull engine. In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.

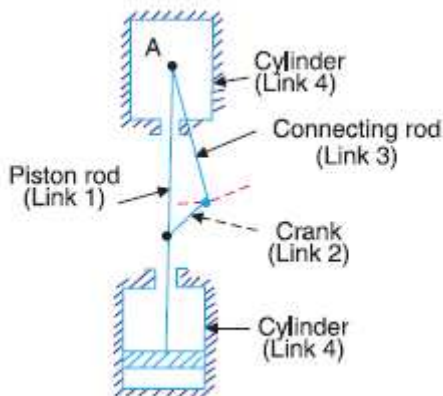


Fig: Pendulum pump.

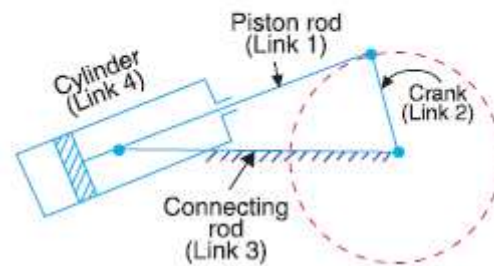


Fig: Oscillating cylinder engine.

2. Oscillating cylinder engine. The arrangement of oscillating cylinder engine mechanism, as shown in Fig, is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

3. Rotary internal combustion engine or Gnome engine. Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre D, as shown in Fig, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

Notes-6

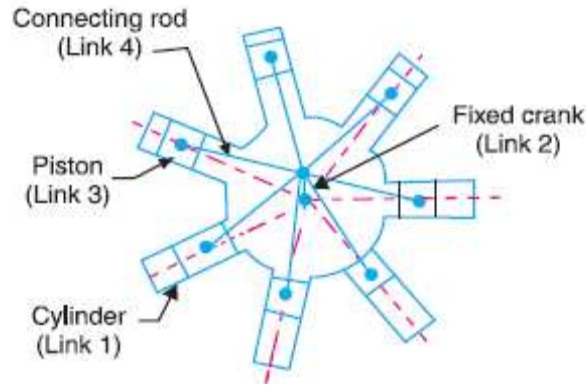


Fig: Rotary internal combustion engine.

4. Crank and slotted lever quick return motion mechanism. This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1R_2 . The line of stroke of the ram (i.e. R_1R_2) is perpendicular to AC produced

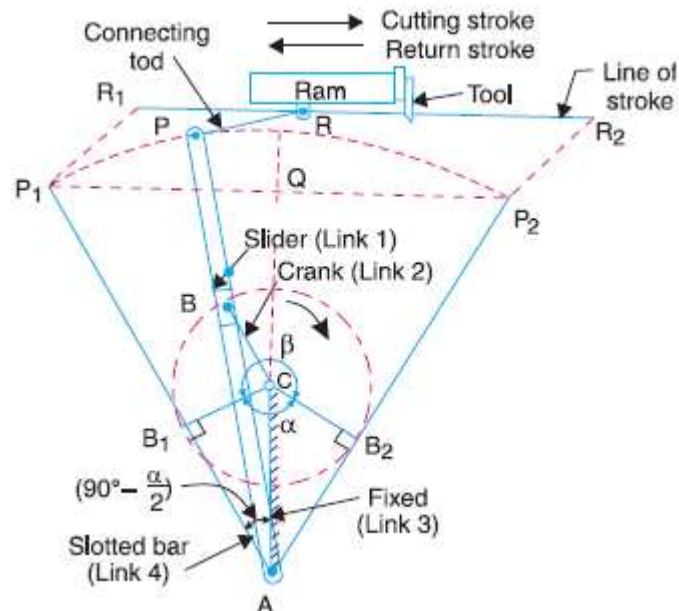


Fig: Crank and slotted lever quick return motion mechanism.

In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates

from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therefore travel of the tool or length of stroke

$$= R_1 R_2 = P_1 P_2 = 2P_1 Q = 2AP_1 \sin \angle P_1 A Q$$

$$= 2AP_1 \sin \left(90 - \frac{\alpha}{2} \right) = 2AP \cos \frac{\alpha}{2} \quad \text{.....}(\because AP_1 = AP)$$

$$2AP \times \frac{CB_1}{AC} \quad \text{.....}(\because \cos \frac{\alpha}{2} = \frac{CB_1}{AC})$$

$$2AP \times \frac{CB}{AC} \quad \text{.....}(\because CB_1 = CB)$$

5. Whitworth quick return motion mechanism. This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD.

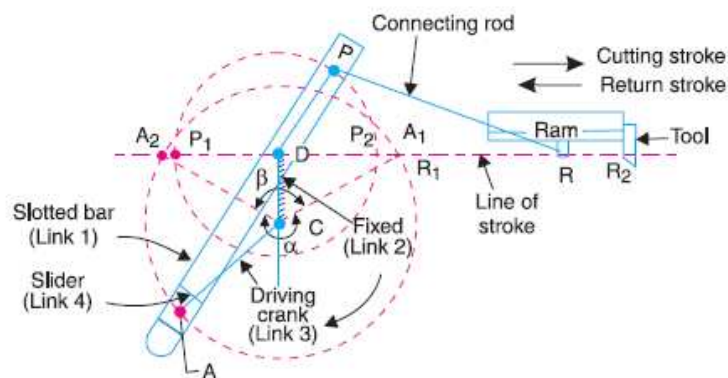


Fig: Whitworth quick return motion mechanism.

Notes-6

When the driving crank CA moves from the position CA₁ to CA₂ (or the link DP from the position DP₁ to DP₂) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 PD. Now when the driving crank moves from the position CA₂ to CA₁ (or the link DP from DP₂ to DP₁) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA₁ to CA₂. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA₂ to CA₁. Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \text{ OR } \frac{360^\circ - \beta}{\beta}$$